U7D1-T SEQUENCES AND SERIES

Friday, December 8, 2017 11:42 AM

U7D1-T SEQUENC...

QUIZ: DEC. 14 (Thurs)
Unit Test Dec. 21 (Thurs).

U7D1 SEQUENCES AND SERIES

Introduction

A function can be used to generate a sequence of numbers :

Example: $f(x) = x^2$ generates

$f(1) =$ 1 $f(2) =$ 4 $f(3) =$ 9

$f(4) =$ 16

We have the sequence 1, 4, 9, 16 . . . . .

Thus a sequence is the set of numbers generated by a function, f(x),

if x is restricted to the Natural Numbers.

N = { 1, 2, 3, 4, ... }

Each element in a sequence is referred to as a term. We use

t with a subscript to indicate a specific term.

1, 4, 9, 16 . . .

i.e., $t_1 =$ 1 $t_2 =$ 4 $t_3 =$ 9 $t_4 =$ 16 . . .

. . $f(1) = 1$ $f(2) = 4$ $f(3) = 9$ $f(4) = 16$

$t_1 = 1$ $t_2 = 4$

Types of Sequences

1. **Finite Sequences** :

sets of a limited number of numbers that follow a mathematical pattern

e.g., 1, 4, 9, 16, 25 **has exactly five terms**

2. **Infinite Sequences** :

sets of an unlimited number of numbers that follow a mathematical pattern

e.g., 1, 4, 9, 16, 25, 36 . . . . . an **unlimited number of terms**

In general, sequences can be generated using functions that utilize individual or combined mathematical operations, or even previous numbers in the sequence.

1. Arithmetic Sequences: **sets of numbers with a common difference generated from a linear function, f(n), with $n \in \mathbb{N}$**

E.g., $t_n = n + 6$

generates the sequence: 7, 8, 9, ...

2. Geometric Sequences: **sets of numbers with a common ratio generated from an exponential function, f(n), with $n \in \mathbb{N}$.**

E.g.,

$-(3)^n$

$t_n = -3^n$ not $(-3)^n$

generates the sequence: −3, −9, −27, ...

3. Recursive Sequences: **sets of numbers generated by using previous numbers in the sequence.**

E.g., to find any term, add the previous two terms.

$t_{k+2} = t_k + t_{k+1}$, *where* $t_1 = 1$ *and* $t_2 = 1$

generates the sequence

1, 1, 2, 3, 5, 8, 13, 21

Fibonacci Sequence

$t_n = 2 \times t_{n-1}$,

$t_1 = 7$

7, 14, 28, 56, ...

Examples:

1. Write the first 3 terms for the following sequences:

a) $t_n = n^3 - 5$

$t_1 = 1^3 - 5 = -4$ $\quad t_2 = 2^3 - 5 = 3$

$t_3 = 3^3 - 5 = 22$

$\boxed{-4, 3, 22}$

b) $t_n = n^2 + 2n$

$t_1 = 1^2 + 2(1) = 3$ $\quad t_2 = 2^2 + 2(2) = 8$

$t_3 = 3^2 + 2(3) = 15$

$\boxed{3, 8, 15}$

c) $t_k = t_{k-1} + k$, where $t_1 = 5$

$t_2 = t_1 + 2$ $\quad t_3 = t_2 + 3$ $\quad t_4 = 10 + 4$

k

$t_2 = 7$ $\quad t_3 = 10$ $\quad t_4 = 14$

To find any term, take the term before and add the current term number.

$\boxed{5, 7, 10, 14}$

first three terms

2. Write the general term for each of the following.

a) 5, 6, 7, 8 . . . .

+1 +1 +1 ← slope

$\boxed{t_n = n + 4}$

first differences are all the same so linear

x	y	1st diff
1	2	
		3
2	5	
		3
3	8	
		3
4	11	

Δx: $1 < 2$, $1 < 3$, $1 < 4$

Δy

b) 2, 5, 8, 11 . . .

+3 +3 +3 ← linear, slope = 3

$\boxed{t_n = 3n - 1}$

x=0

$y = ab^x$

n=1 n=2

with n=1

$n=1$ $n=2$

c) 1, 3, 9, 27, ...

$\times 3$ $\times 3$ $\times 3$ ← common ratio

starts with $n=1$

$t_n = 1(3)^{n-1}$

$\boxed{t_n = 3^{n-1}}$

$x \neq 0$

$y = ab^x$

initial value

$x=0$

multiplying factor from ratio column

d) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$\boxed{t_n = \frac{n}{n+1}}$

$\frac{2}{1}$

e) $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$

perfect squares.

$\boxed{t_n = \frac{n+1}{n^2}}$

$\times 8$ $\times 2$ $\times 4$

f) $\frac{2}{1}, \frac{15}{8}, \frac{7}{4}, \frac{13}{8}, \frac{3}{2}, \ldots$

try a common denominator

$= \frac{16}{8}, \frac{15}{8}, \frac{14}{8}, \frac{13}{8}, \frac{12}{8}, \ldots$

-1 -1 -1 -1

$t_n = \frac{-n+17}{8}$

$\boxed{t_n = \frac{17-n}{8}}$

g) 4, 7, 10, 13 . . .

3 3 3 ← first diff 3

$t_n = 3n+1$

$n^2 = 1\ 4\ 9\ 16$

h) -3, 0, 5, 12, ...

3 5 7

2 2 ← 2nd differences same ⇒ quadratic

$2a$

an^2

$\boxed{t_n = n^2 - 4}$

$2\ 8\ 18\ 32\ 50\ 2n^2$

i) 3, 9, 19, 33, 51, ...

6 10 14 18

4 4 4

$\div 2$

quadratic $a=2$

$= ax^2 + bx + c$

$\boxed{t_n = 2n^2 + 1}$

U7D1 HW: Handout