

U6D8_T_SolvingTrigEquations

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U6D8 MCR3UI

Solving Trigonometric Equations

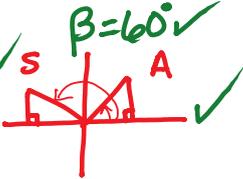
Warm Up: Give all possible values of θ from $0^\circ \leq \theta \leq 360^\circ$

a) $\sin\theta = 0.8081$ 
 $\theta = \sin^{-1}(0.8081)$
 $\theta = 54^\circ$ or $\theta = 180^\circ - 54^\circ$
 $\theta = 126^\circ$

b) $\cos\theta = -\frac{\sqrt{3}}{2}$ 
 $\beta = 30^\circ$ $\theta = 180^\circ - 30^\circ$ $\theta = 180^\circ + 30^\circ$
 $\theta = 150^\circ$ or $\theta = 210^\circ$

Solving trig equations is similar to solving regular algebraic equations:

$2\sin\theta - \sqrt{3} = 0$ is similar to solving $2x - \sqrt{3} = 0$, where $x = \sin\theta$

$2\sin\theta = \sqrt{3}$ 
 $\sin\theta = \frac{\sqrt{3}}{2}$ $\beta = 60^\circ$ ✓
 $2x = \sqrt{3}$
 $x = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ$ or $\theta = 180^\circ - 60^\circ$
 $\theta = 120^\circ$

Example 1: Solve for θ for $0^\circ \leq \theta \leq 360^\circ$

a) $4\cos\theta = \cos\theta + 2$ (combine like terms and isolate $\cos\theta$)

$4\cos\theta - \cos\theta = 2$
 $3\cos\theta = 2$
 $\cos\theta = \frac{2}{3}$ ✓

$\beta = \cos^{-1}(2 \div 3)$
 $\beta = 48^\circ$ ✓



$\theta = 48^\circ$ or 312° ✓

b) $2\sin^2\theta - 6 = 0$

$$2\sin^2\theta = 6 \quad | \quad 2(\sin^2\theta - 3) = 0$$

$$\sin^2\theta = 3 \quad | \quad (\sin\theta - \sqrt{3})(\sin\theta + \sqrt{3}) = 0$$

$$\sin\theta = \pm\sqrt{3} \quad | \quad \sin\theta = \pm\sqrt{3}$$

Recall: $R: \{y \mid y \in \mathbb{R}, -1 \leq y \leq 1\}$

\therefore there is no solution.

c) $3\sin^2\theta + 3\sin\theta = 0$

$$3\sin\theta(\sin\theta + 1) = 0$$

$$\sin\theta = 0 \text{ or } \sin\theta = -1$$

$\theta = 0^\circ, 180^\circ, 270^\circ \text{ or } 360^\circ$

$$\sin\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin\theta = \frac{-3 \pm \sqrt{9 - 0}}{6}$$

$$\sin\theta = \frac{-3 \pm 3}{6}$$

$$\sin\theta = 0 \text{ or } -1$$

d) $2\cos^2\theta - 1 = 0$

$$2\cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{2} \quad \text{NOTE: } \sqrt{1} = 1$$

$$\cos\theta = \pm \frac{1}{\sqrt{2}}$$

Remember the \pm

$\theta = 45^\circ$

$\theta = 45^\circ, 135^\circ, 225^\circ, \text{ or } 315^\circ$

e) $2\sin^2\theta - 7\sin\theta + 3 = 0$

(similar to solving $2x^2 - 7x + 3 = 0$)

$$\cancel{2} \cancel{1} \cancel{3} \quad \begin{matrix} 1 & 1 & 3 \\ 2 & 1 & 1 \end{matrix}$$

$$(\sin\theta - 3)(2\sin\theta - 1) = 0$$

$$\sin\theta = 3 \text{ or } \sin\theta = \frac{1}{2}$$

$\theta = 30^\circ$
or 150°

$$\sin\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin\theta = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\sin\theta = \frac{7 \pm 5}{4}$$

$$\sin\theta = 3 \text{ or } \frac{1}{2}$$

U6D8 Practice: Page 408 #2, 3acef, 5abc. Solve in degrees.