MCR 3UI **Applications of Sinusoidal & Other Trigonometric Functions**

1. The human ear interprets the amplitude of a sound wave as loudness. Thus, a sound wave modelled by y = 2sinkt is louder than a sound wave modelled by y = sinkt. Because the human ear does not operate on a linear scale, the perceived loudness ratio is not actually 2:1. When an instrument is played, the sound wave spreads out in a spherical pattern. The amplitude decreases as the square of the distance. Suppose that a sound wave can be modelled as y = 64 sinkt at a distance of 1 m from its source. At a distance of 2 m, a = $\frac{64}{2^2}$ = 16. The modelling equation becomes y = 16 sin kt.

a) Write the modelling equation at a distance of 4 m from the source.
b) Write the modelling equation at a distance of 8 m from the source.
c) How far from the source does the modelling equation become $y = \frac{1}{4}\sin kt$?

2. Is y=tanx a function? Explain how you know. What is the domain and range of y = tanx on the interval $0° \leq x \leq 360°$?

3. At the end of a dock, high tide of 14 m is recorded at 9:00 a.m. Low tide of 6 m is recorded at 3:00 p.m. A sinusoidal function can model the water depth versus time.
a) Construct a model for the water depth using a cosine function, where time is measured in hours past high tide.
b) Construct a model for the water depth using a sine function, where time is measured in hours past high tide.
c) Construct a model for the water depth using a sine function, where time is measured in hours past low tide.
d) Construct a model for the water depth using a cosine function, where time is measured in hours past low tide.

4. The height, h, in metres, above the ground of a rider on a Ferris wheel after t seconds can be modelled by the sine function: h(t) = 10 sin [3(t-30)]+12.
a) Graph the function.
b) Determine the maximum and minimum heights of the rider above the ground.
c) Determine the height of the rider above the ground after 30 seconds.
d) Determine the time required for the Ferris wheel to complete one revolution.

5. The population, P, of a lakeside town with a large number of seasonal residents can be modelled using the function P(t) = 5000 sin[30(t-7)] +8000, where t is the number of months after New Year's Day.
a) Find the maximum and minimum values for the population over a whole year.
b) When is the population a maximum? When is it a minimum?
c) What is the population on September 30?
d) When is the population about 10 000?

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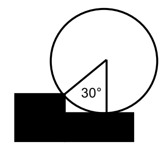
6. The movement of a piston in an automobile engine can be modelled by the function y = 50sin10800t+20, where y is the distance, in centimetres, from the crankshaft and t is the time, in seconds.
a) What is the period of the motion?
b) Determine the maximum, minimum and amplitude.
c) When do the maximum and minimum values occur?
d) What is the vertical position of the piston at t = $\frac{1}{120}$ seconds?

7. Smog is a generic term used to describe pollutants in the air. A smog alert is usually issued when the air quality index is greater than 50. Air quality can vary throughout the day, increasing when more cars are on the road. Consider a model of the form I = 30 sin[15(t-4)] +25, where I is the value of the air quality index and t measure the time after midnight, in hours.
a) What is the period of the modelled function? Explain why this makes sense.
b) Determine the maximum, minimum and amplitude.
c) When do the maximum and minimum occur?
d) During what time interval would a smog alert be issued, according to this model?



8. A Ferris wheel has a diameter of 20 m and is 4 m above ground level at its lowest point. Assume that a rider enters a car from a platform that is located 30 degrees around the rim before the car reaches its lowest point.(Hint: position the centre of the wheel at the origin, then apply tranlation)
a) Model the rider's height above the ground versus angle using a transformed sine function.
b) Model the rider's height above the ground versus angle using a transformed cosine function.
c) Suppose that the platform is moved to 60 degrees around the rim from the lowest position of the car. How will the equations in parts a) and b) change? Write the new equations.

9. Suppose that the centre of the Ferris wheel in question 8 is moved upward 2 m, but the platform is left in place at a point 30 degrees before the car reaches its lowest point. How do the equations in parts a) and b) of question 8 change? Write the new equations.

Answers: (answers may vary due to the periodic nature of the sinusoidal functions)
1 a) y=4sinkt b) y=sinkt c) 16 m
2. Yes. D=$\{x \in R | x \neq 90°, x \neq 270°\}$, R=$\{y \in R\}$
3 a) y=4cos30x + 10 b) y = 4sin[30(x+3)]+10 c) y = 4sin[30(x-3)]+10 d) y = 4cos[30(x-6)]+10
4. b) Max = 22 m, min = 2 m c) 12 m d) 120seconds
5 a) max 13 000, min 3000 b) max: end of Oct at t = 10, min: end of Apr at t = 4 c) 12 330
d) Jan 6, t = 6 days and Aug 24, t = 7 months 24 days.
6a) $\frac{1}{30}s$ b) max 70 cm, min -30 cm, amp 50 cm c) max at $\frac{1}{120}$s, min at $\frac{1}{40}$s d) 70 cm
7a) period 24 hours Sample answer: Smog is often created from human activity that generally repeats from day to day, so a period of 24 hours is appropriate. b) max 55, min -5, amp 30 c) max at 10 a.m., min at 10 p.m.
d) sample answer: An interval around 10 a.m. (precisely: 7:46 a.m. until 12:14 p.m.)
8 a) y = 10sin(x+240)+14 b) y = 10cos(x+150)+14 c) Phase shift is an additional 30 degrees y = 10sin(x+210)+14 b) y = 10cos(x+120)+14
9a) y = 10sin(x+240)+16 b) y = 10cos(x+150)+16