U6D7_T_Trigonometric Applications

Tuesday, May 14, 2019 6:44 PM

U6D7_T_Trigonometr...

U6D7 MCR 3UI Warm Up:

a) Write two equations of a sinusoidal function (one sine and one cosine) that has amplitude 3, period 90° and a maximum at (0, 5).

$a = 3$ $\quad k = \frac{360°}{90°}$ $\quad$ max 5

$k = 4$ $\quad c = \text{max} - a$

$c = 2$

Begins at max

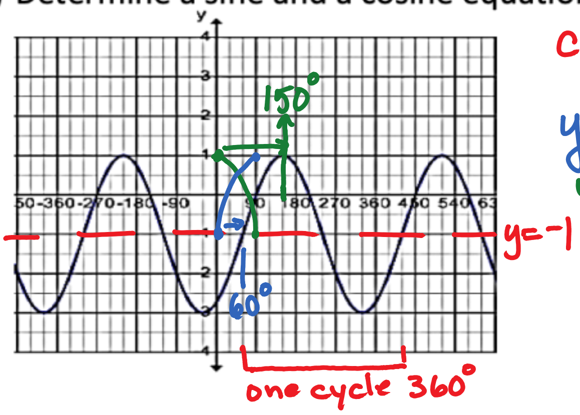
$y = 3\cos 4x + 2$

for sine shift left $\frac{1}{4}$ cycle

$\Rightarrow$ left $\frac{90°}{4} = 22.5°$

$y = 3\sin 4(x + 22.5°) + 2$

b) Determine a sine and a cosine equation for the graph



$c = -1$ $\quad a = 2$

$y = 2\sin(x - 60°) - 1$

$y = 2\cos(x - 150°) - 1$

Trigonometric Applications

When given sinusoidal word problems, it often helps to graph what you know. You may also need to use the formulas below to help determine the values needed to create the equation (if one is not provided for you).

$y = a \sin k(x-d) + c$

Amplitude (|a|) = $\frac{Max - Min}{2}$ **k =** $\frac{360°}{Period}$ or **Period =** $\frac{360°}{k}$

Equation of Sinusoidal Axis:

$y = c$ **or** $y = \frac{Max+Min}{2}$ **or** $y = Max - |a|$

$y = Max - Amplitude$

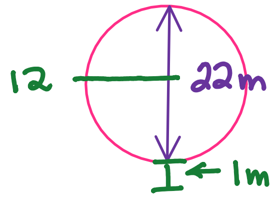
Maximum = $c + |a|$
$c + amplitude$

Minimum = $c - |a|$
$c - amplitude$

<u>Example 1</u>: The height of a chair on a Ferris wheel in motion can be modelled with a sinusoidal function. The diameter of the wheel is 22m and the passengers board from a platform 1 m above the ground. The ride lasts 3 minutes and the wheel makes 6 complete revolutions.

a) Graph the function.

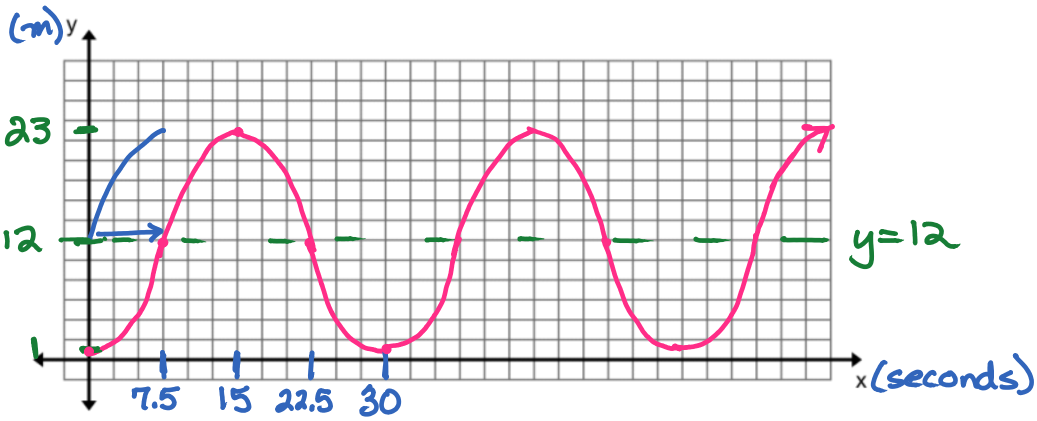
b) Determine the equation of the relation.



Fill in the information you know…

Amplitude: 11m Period: 30 seconds Minimum: 1 m Maximum: 23m

Sinusoidal Axis: $y=12$ Phase Shift: Starts at: minimum



$a = 11$ $\quad k = \dfrac{360^\circ}{30\text{ sec}}$ $\quad c = 12$

$\ominus$ for cosine

$k = 12^\circ/\text{sec}$

$y = -11\cos 12x + 12$

for sine, $a = 11$, $d = 7.5$ (Right), $k = 12$, $c = 12$

$y = 11\sin 12(x - 7.5) + 12$

Example 2: At Hopewell Rocks in New Brunswick, the high and low tide depths are recorded over a period of time. A model of the form of D(t) = 20 sin[30(t - 6)] + 22 represents the depth of water versus time on a particular day, where d(t) is the depth of water in feet and t is the time in hours after midnight.

Fill in the information you know...

Amplitude: 20

Period: $\frac{360}{30} = 12$

Sinusoidal Axis: $y = 22$

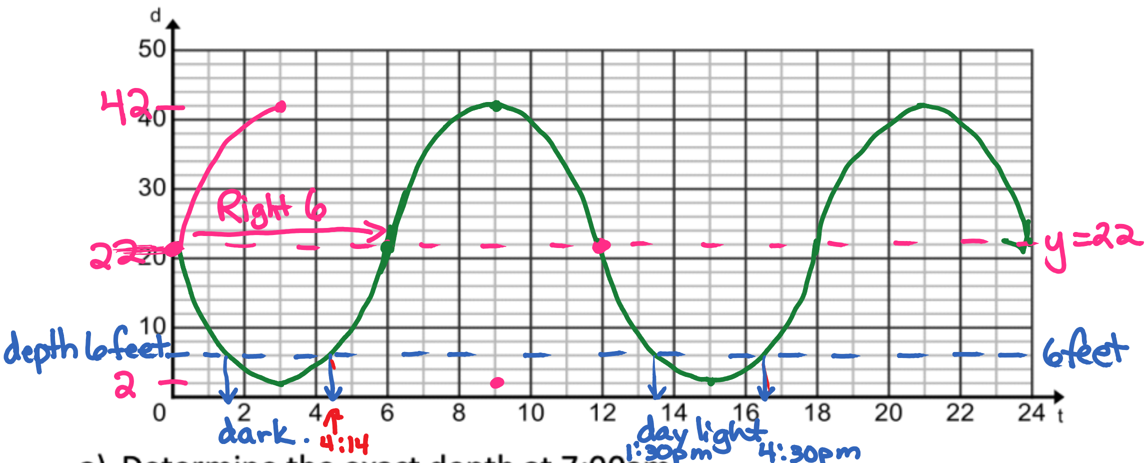
Minimum: $22 - 20 = 2$

Maximum: $22 + 20 = 42$

Phase Shift: R 6

Starts at: midnight

Graph the function.



a) Determine the exact depth at 7:00am. ← 7 hours after midnight

$D(7) = 20\sin[30(7-6)] + 22$

$D(7) = 20\sin 30 + 22$

$D(7) = 20\left(\frac{1}{2}\right) + 22$

$D(7) = 32$

∴ the water depth is 32 feet at 7:00 a.m.

b) It is considered safe to explore the ocean floor and rock formations during low tide as long as the water depth (at the point where it is measured) is less than 6 feet. Using your graph estimate during what time interval (during daylight) it would be safe to walk the ocean floor at Hopewell Rocks.

* from graph approximately 1:30p.m – 4:30p.m

EXACT (BONUS) $D(t) < 6$

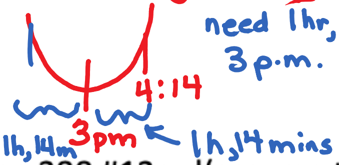
$$20\sin[30(t-6)] + 22 = 6 \quad -22$$
$$20\sin[30(t-6)] = -16 \quad \div 20$$
$$\sin[30(t-6)] = -0.8$$
$$30(t-6) = \sin^{-1}(-0.8)$$
$$t = \frac{\sin^{-1}(-0.8)}{30} + 6$$
$$t = 4.2289965\ldots$$

4 hrs

$0.2289965\ldots \times 60$ mins/hr $\doteq$ 14 minutes

4:14 now look at graph.

Use symmetry



need 1hr, 14min before 3p.m.

$\therefore$ it is safe between 1:16 pm and 4:14pm.

U6D7 Practice: Page 388 #12a-d(assume t=0 is low tide), 14, 16, worksheet