**MCR3UI** **Trig Identities** **2012-2013**

Prove each identity.

1. $\sin\theta = \cos\theta \tan\theta$
2. $\csc\theta = \sec\theta \cot\theta$
3. $\cos\theta = \sin\theta \cot\theta$
4. $\sec\theta = \csc\theta \tan\theta$
5. $1 + \csc A = \csc A\,(1 + \sin A)$
6. $\cot B \sin B \sec B = 1$
7. $\cos C\,(\sec C - 1) = 1 - \cos C$
8. $1 + \sin D = \sin D\,(1 + \csc D)$
9. $1 - \sin^2\theta = \sin\theta \cos\theta \cot\theta$
10. $\csc^2\theta = \cot^2\theta + 1$
11. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$
12. $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{2}{\cos\theta}$
13. $\csc^2\theta \cos^2\theta = \csc^2\theta - 1$
14. $\tan\theta + \cot\theta = \dfrac{\sec\theta}{\sin\theta}$
15. $\dfrac{\cot\theta}{\csc\theta} = \cos\theta$

**MCR3UI** **Trig Identities** **2012-2013**

Prove each identity.

16. $\sin\theta = \cos\theta \tan\theta$
17. $\csc\theta = \sec\theta \cot\theta$
18. $\cos\theta = \sin\theta \cot\theta$
19. $\sec\theta = \csc\theta \tan\theta$
20. $1 + \csc A = \csc A\,(1 + \sin A)$
21. $\cot B \sin B \sec B = 1$
22. $\cos C\,(\sec C - 1) = 1 - \cos C$
23. $1 + \sin D = \sin D\,(1 + \csc D)$
24. $1 - \sin^2\theta = \sin\theta \cos\theta \cot\theta$
25. $\csc^2\theta = \cot^2\theta + 1$
26. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$
27. $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{2}{\cos\theta}$
28. $\csc^2\theta \cos^2\theta = \csc^2\theta - 1$
29. $\tan\theta + \cot\theta = \dfrac{\sec\theta}{\sin\theta}$
30. $\dfrac{\cot\theta}{\csc\theta} = \cos\theta$

# Handout. Trig Identities. U6L8 Pg 1 of 3

1. LS: $\sin\theta$

RS: $\cos\theta\tan\theta$
$= \cos\theta \frac{\sin\theta}{\cos\theta}$ (QI)
$= \sin\theta$

LS = RS
$\therefore \sin\theta = \cos\theta\tan\theta$

2. LS: $\csc\theta$

RS: $\sec\theta\cot\theta$
$= \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$ (RI)
$= \frac{1}{\sin\theta}$
$= \csc\theta$

LS = RS
$\therefore \csc\theta = \sec\theta\cot\theta$.

3. LS: $\cos\theta$

RS: $\sin\theta\cot\theta$
$= \sin\theta \cdot \frac{\cos\theta}{\sin\theta}$ (RI)
$= \cos\theta$

LS = RS
$\therefore \cos\theta = \sin\theta\cot\theta$.

4. LS: $\sec\theta$

RS: $\csc\theta\tan\theta$
$= \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$ (RI)
$= \frac{1}{\cos\theta}$
$= \sec\theta$.

LS = RS
$\therefore \sec\theta = \csc\theta\tan\theta$.

5. LS: $1 + \csc A$

RS: $\csc A(1 + \sin A)$ (distributive property)
$= \csc A + \csc A \sin A$
$= \csc A + \frac{1}{\sin A} \sin A$ (RI).
$= \csc A + 1$

LS = RS
$\therefore 1 + \csc A = \csc A(1 + \sin A)$

6. LS: $\cot B \sin B \sec B$

RS: $1$
$= \frac{\cos B}{\sin B} \cdot \sin B \cdot \frac{1}{\cos B}$ (RI).
$= 1$

LS = RS
$\therefore \cot B \sin B \sec B = 1$.

7. LS: $\cos C(\sec C - 1)$

RS: $1 - \cos C$
$= \cos C \sec C - \cos C$
$= \cos C \cdot \frac{1}{\cos C} - \cos C$ (RI)
$= 1 - \cos C$

LS = RS
$\therefore \cos C(\sec C - 1) = 1 - \cos C$

8. LS: $1 + \sin D$

RS: $\sin D(1 + \csc D)$
$= \sin D + \sin D \csc D$
$= \sin D + \sin D \frac{1}{\sin D}$ (RI)
$= \sin D + 1$
$= 1 + \sin D$

LS = RS
$\therefore 1 + \sin D = \sin D(1 + \csc D)$.

# Handout Trig Identities.

9. $1-\sin^2\theta = \sin\theta\cos\theta\cot\theta$

LS: $1-\sin^2\theta$

RS: $\sin\theta\cos\theta\cot\theta$

$= \sin\theta\cos\theta\frac{\cos\theta}{\sin\theta}$ (RI)

$= \cos^2\theta$

$= 1-\sin^2\theta$

$= LS$

$LS = RS$

$\therefore 1-\sin^2\theta = \sin\theta\cos\theta\cot\theta$

10. $\csc^2\theta = \cot^2\theta + 1$

LS: $\csc^2\theta$

RS: $\cot^2\theta + 1$

$= \frac{\cos^2\theta}{\sin^2\theta} + 1$ (RI)

$= \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta}$

$= \csc^2\theta$ (RI)

$= LS$

$LS = RS \therefore \csc^2\theta = \cot^2\theta + 1$

11. $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

LS:

$\frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$ (PI)

$= \frac{1-\sin\theta}{\cos\theta}$

$= RS$

$LS = RS \therefore \frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

12. $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{2}{\cos\theta}$

LS:

$\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{1-\sin^2\theta}$

$= \frac{2\cos\theta}{\cos^2\theta}$ (PI).

~~LS = RS~~

$= \frac{2}{\cos\theta}$

$= RS$

$\therefore \frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{2}{\cos\theta}$

13. $\csc^2\theta\cos^2\theta = \csc^2\theta - 1$

RS:

$\csc^2\theta - 1$

$= \csc^2\theta - \left(\frac{\csc^2\theta}{\csc^2\theta}\right)$

$= \csc^2\theta\left(1 - \frac{1}{\csc^2\theta}\right)$

$= \csc^2\theta(1-\sin^2\theta)$ RI

$= \csc^2\theta(\cos^2\theta)$ PI

$= LS$

$\therefore \csc^2\theta\cos^2\theta = \csc^2\theta - 1.$

(13) easier approach

LS:

$\frac{1}{\sin^2\theta} \cdot \cos^2\theta$ (RI)

$= \cot^2\theta$ (RI)

RS:

$\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}$

$= \frac{1-\sin^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$

$LS = RS$

$\therefore \csc^2\theta\cos^2\theta = \csc^2\theta - 1 = \cot^2\theta$

14. $\tan\theta + \cot\theta = \dfrac{\sec\theta}{\sin\theta}$

LS

$\tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ (QI, RI)

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta} \cdot \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\sin\theta} \cdot \sec\theta$ (RI)

$= \dfrac{\sec\theta}{\sin\theta}$ $\therefore \tan\theta + \cot\theta = \dfrac{\sec\theta}{\sin\theta}$.

$=$ RS.

15. $\dfrac{\cot\theta}{\csc\theta} = \cos\theta$.

LS

$\dfrac{\cot\theta}{\csc\theta}$

$= \cot\theta \cdot \dfrac{1}{\csc\theta}$

$= \dfrac{\cos\theta}{\sin\theta} \cdot \sin\theta$ (RI).

$= \cos\theta$

$=$ RS $\therefore \dfrac{\cot\theta}{\csc\theta} = \cos\theta$.