

U5D8_T Proving Trig Identities Cont'd (Reciprocal)

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U5D8_T
Proving Tr...

U5D8 MCR3UI

Warm Up: QUIZ

Proving Trigonometric Identities (Continued)

Recall: QI: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ PI: $\sin^2\theta + \cos^2\theta = 1$

Today, include: Reciprocal Identities (RI):

co-secant: $\csc\theta = \frac{1}{\sin\theta}$ **secant:** $\sec\theta = \frac{1}{\cos\theta}$

co-tangent: $\cot\theta = \frac{1}{\tan\theta}$

Example 1) Prove.

a) $1 - \sin^2\theta = \sin\theta\cos\theta\cot\theta$

LS
 $\cos^2\theta$ (PI)

RS
 $\sin\theta\cos\theta\left(\frac{1}{\tan\theta}\right)$ (RI)

$= \cancel{\sin\theta}\cos\theta\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right)$ (QI)

$= \cos^2\theta$

$= \text{LS}$

∴ $1 - \sin^2\theta = \sin\theta\cos\theta\cot\theta$

$$b) 1 + \tan^2 \theta = \sec^2 \theta$$

LS

$$\frac{1}{1} + \frac{\sin^2 \theta}{\cos^2 \theta} \quad (\text{QI})$$

$$= \frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta} \quad (\text{PI})$$

$$= \sec^2 \theta \quad (\text{RI})$$

$$= \text{RS.}$$

$$\therefore 1 + \tan^2 \theta = \sec^2 \theta$$

$$c) \tan \theta + \cot \theta = \frac{\sec \theta}{\sin \theta}$$

LS

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \quad (\text{QI, RI})$$

$$= \frac{\sin \theta \sin \theta + \cos \theta \cos \theta}{\cos \theta \sin \theta}$$

$$\text{LCD} \\ \cos \theta \sin \theta$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta} \quad (\text{PI})$$

$$\sec \theta \cdot \frac{1}{\sin \theta} \quad (\text{RI})$$

$$= \frac{\sec \theta}{\sin \theta}$$

$$= \text{RS.}$$

$$\therefore \tan \theta + \cot \theta = \frac{\sec \theta}{\sin \theta}$$

U5D8 Worksheet: Trigonometric Identities

Prove each identity.

1. $\sin \theta = \cos \theta \tan \theta$

2. $\csc \theta = \sec \theta \cot \theta$

3. $\cos \theta = \sin \theta \cot \theta$

4. $\sec \theta = \csc \theta \tan \theta$

5. $1 + \csc A = \csc A (1 + \sin A)$

6. $\cot B \sin B \sec B = 1$

7. $\cos C (\sec C - 1) = 1 - \cos C$

8. $1 + \sin D = \sin D (1 + \csc D)$

9. $1 - \sin^2 \theta = \sin \theta \cos \theta \cot \theta$

10. $\csc \theta = \cot^2 \theta + 1$

11. $\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$

12. $\frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{2}{\cos \theta}$

13. $\csc^2 \theta \cos^2 \theta = \csc^2 \theta - 1$

14. $\tan \theta + \cot \theta = \frac{\sec \theta}{\sin \theta}$

15. $\frac{\cot \theta}{\csc \theta} = \cos \theta$