1a) $\cos x \tan x$
$= \cos x \frac{\sin x}{\cos x}$ (QI)
$= \sin x$

b) $\sin^2 x$
$= 1 - \cos^2 x$ (PI).

c) $\cos^2 x$
$= 1 - \sin^2 x$ (PI)

d) $\tan^2 x$
$= \frac{\sin^2 x}{\cos^2 x}$ (QI)

e) $\tan x \sin x$
$= \frac{\sin x}{\cos x} \cdot \sin x$ (QI)
$= \frac{\sin^2 x}{\cos x}$

f) $1 - \sin^2 x$
$= \cos^2 x$ (PI).

g) $\sin x \tan x \cos x$
$= \sin x \frac{\sin x}{\cos x} \cos x$ (QI)
$= \sin^2 x$

h) $1 - \cos^2 x$
$= \sin^2 x$ (PI)

i) $\sin^2 x + \cos^2 x$
$= 1$ (PI).

2 a) $\frac{\sin x}{\tan x} = \cos x$

LS
$\sin x \div \tan x$
$= \sin x \div \frac{\sin x}{\cos x}$ (QI)
$= \sin x \times \frac{\cos x}{\sin x}$
$= \cos x$
$= RS$
$\therefore \frac{\sin x}{\tan x} = \cos x$

b) $\sin x \cos x \tan x = 1 - \cos^2 x$

LS
$\sin x \cos x \frac{\sin x}{\cos x}$ (QI)
$= \sin^2 x$
$= 1 - \cos^2 x$
$= RS$
$\therefore \sin x \cos x \tan x = 1 - \cos^2 x$

c) $\frac{1 - \cos^2 x}{\sin x} = \sin x$

LS
$\frac{1 - \cos^2 x}{\sin x}$
$= \frac{\sin^2 x}{\sin x}$ (PI)
$= \sin x$
$\therefore \frac{1 - \cos^2 x}{\sin x} = \sin x$

d) $\sin^2 x + \frac{\sin x \cos x}{\tan x} = 1$

LS
$\sin^2 x + \sin x \cos x \div \tan x$
$= \sin^2 x + \sin x \cos x \div \frac{\sin x}{\cos x}$ (QI)
$= \sin^2 x + \sin x \cos x \frac{\cos x}{\sin x}$
$= \sin^2 x + \cos^2 x$ (PI)
$= 1$
$= RS$
$\therefore \sin^2 x + \frac{\sin x \cos x}{\tan x} = 1$

2e) $1 + \frac{1}{\tan^2 x} = \frac{1}{\sin^2 x}$

LS

$= 1 + 1 \div \tan^2 x$

$= 1 + 1 \div \frac{\sin^2 x}{\cos^2 x}$ (QI)

$= 1 + 1 \times \frac{\cos^2 x}{\sin^2 x}$

$= 1 + \frac{\cos^2 x}{\sin^2 x}$

$= \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$

$= \frac{1}{\sin^2 x}$ $\quad \therefore 1 + \frac{1}{\tan^2 x} = \frac{1}{\sin^2 x}$

$= RS$

2f) $2\sin^2 x - 1 = \sin^2 x - \cos^2 x$

RS

$\sin^2 x - \cos^2 x$

$= \sin^2 x - (1 - \sin^2 x)$ (PI)

$= \sin^2 x - 1 + \sin^2 x$

$= 2\sin^2 x - 1$

$= LS$

$\therefore 2\sin^2 x - 1 = \sin^2 x - \cos^2 x$

2g) $\frac{1}{\cos x} - \cos x = \sin x \tan x$

LS

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

$= \frac{1 - \cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$ (PI) $\quad \therefore \frac{1}{\cos x} - \cos x = \sin x \tan x$

$= \sin x \cdot \frac{\sin x}{\cos x}$ (QI)

$= \sin x \tan x = RS$

Pg. 398 #2, 4, 13.

2h) $\sin x + \tan x = \tan x(1+\cos x)$

RS

$\frac{\sin x}{\cos x}(1+\cos x)$ (QI)

$= \frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}.\cos x$

$= \frac{\sin x}{\cos x} + \sin x$

$= \tan x + \sin x$ (QI)

$= \sin x + \tan x.$

$= LS$

$\therefore \sin x + \tan x = \tan x(1+\cos x).$

i) $\frac{1}{1-\sin^2 x} = 1+\tan^2 x$

RS

$1+\tan^2 x$

$= 1 + \frac{\sin^2 x}{\cos^2 x}$ (QI)

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$ (PI)

$= \frac{1}{1-\sin^2 x}$ (PI).

$= LS.$

$\therefore \frac{1}{1-\sin^2 x} = 1+\tan^2 x.$

j) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

RS

$2\cos^2 x - 1$

$= 2\cos^2 x - (\sin^2 x + \cos^2 x)$

$= 2\cos^2 x - \sin^2 x - \cos^2 x$

$= \cos^2 x - \sin^2 x$

$= LS$

$\therefore \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

alternate.

LS

$\cos^2 x - \sin^2 x$

$= \cos^2 x - (1-\cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$

$= RS.$

2k) $\sin^2 x + \cos^2 x + \tan^2 x = \frac{1}{\cos^2 x}$

LS

$\underbrace{\sin^2 x + \cos^2 x} + \tan^2 x$

$= 1 + \frac{\sin^2 x}{\cos^2 x}$ (PI, QI)

$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$ (PI)

$=$ RS. $\therefore \sin^2 x + \cos^2 x + \tan^2 x = \frac{1}{\cos^2 x}$

2 l) $\frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$

RS

$\tan x \div (1 + \tan x)$

$= \frac{\sin x}{\cos x} \div \left(1 + \frac{\sin x}{\cos x}\right)$ (QI)

$= \frac{\sin x}{\cos x} \div \left(\frac{\cos x + \sin x}{\cos x}\right)$

$= \frac{\sin x}{\cos x} \times \frac{\cos x}{\cos x + \sin x}$

$= \frac{\sin x}{\cos x + \sin x}$

$= \frac{\sin x}{\sin x + \cos x}$

$=$ LS

$\therefore \frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$

Pg 398 #2, 4, 13

2 m) $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x-\sin^2 x}$

LS

$(1+\tan^2 x) \div (1-\tan^2 x)$

$= \left(1+\dfrac{\sin^2 x}{\cos^2 x}\right) \div \left(1-\dfrac{\sin^2 x}{\cos^2 x}\right)$ (QI)

$= \left(\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}\right) \div \left(\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)$

$= \dfrac{\cos^2 x+\sin^2 x}{\cos^2 x} \cdot \dfrac{\cos^2 x}{\cos^2 x-\sin^2 x}$

$= \dfrac{1}{\cos^2 x-\sin^2 x}$ (PI)

$=$ RS.

$\therefore \dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x-\sin^2 x}$

4. a) $\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \dfrac{1}{\sin^2 x\cos^2 x}$

LS

$\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$

$= \dfrac{\cos^2 x+\sin^2 x}{\sin^2 x\cos^2 x}$

$= \dfrac{1}{\sin^2 x\cos^2 x}$ (PI)

$=$ RS.

$\therefore \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \dfrac{1}{\sin^2 x\cos^2 x}$

b) LS

$\tan x + \dfrac{1}{\tan x}$

$= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$ (PI)

$= \dfrac{\sin^2 x+\cos^2 x}{\cos x\sin x}$

$= \dfrac{1}{\sin x\cos x}$ (PI)

$=$ RS.

RS

$\dfrac{1}{\sin x\cos x}$

$\therefore \tan x + \dfrac{1}{\tan x} = \dfrac{1}{\sin x\cos x}$

Pg 398 # 4, 13.

4c) $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$

LS

$\frac{1(1+\cos x) + 1(1-\cos x)}{(1-\cos x)(1+\cos x)}$

$= \frac{1+\cos x + 1 - \cos x}{1-\cos^2 x}$

$= \frac{2}{1-\cos^2 x}$

$= \frac{2}{\sin^2 x}$ (PI)

$= RS$

$\therefore \frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$

d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

LS

$(\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= \sin^2 x + \cos^2 x + 2\sin x\cos x$

$= 1 + 2\sin x\cos x$ (PI)

$= RS.$

$\therefore (\sin x + \cos x)^2 = 1 + 2\sin x\cos x.$

e) $(1-\cos^2 x)\left(1 + \frac{1}{\tan^2 x}\right) = 1$

LS

$1 + \frac{1}{\tan^2 x} - \cos^2 x - \frac{\cos^2 x}{\tan^2 x}$

$= 1 + \frac{\cos^2 x}{\sin^2 x} - \cos^2 x - \cos^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$ (QI)

$= 1 - \cos^2 x + \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$

$= \sin^2 x + \frac{\cos^2 x}{\sin^2 x}(1-\cos^2 x)$ (PI)

$= \sin^2 x + \frac{\cos^2 x(\cancel{\sin^2 x})}{\cancel{\sin^2 x}}$ (PI)

$= \sin^2 x + \cos^2 x = 1 \overset{(PI)}{=} RS$

$\therefore (1-\cos^2 x)\left(1 + \frac{1}{\tan^2 x}\right) = 1,$

Pg 398 # 4f – j, 13.

4f) $\dfrac{1+2\sin x\cos x}{\sin x+\cos x} = \sin x+\cos x$

LS

$= \dfrac{1+2\sin x\cos x}{\sin x+\cos x}$

$= \dfrac{\sin^2 x+\cos^2 x+2\sin x\cos x}{\sin x+\cos x}$ (PI)

$= \dfrac{\sin^2 x+2\sin x\cos x+\cos^2 x}{\sin x+\cos x}$

$= \dfrac{(\sin x+\cos x)^2}{\sin x+\cos x}$

$= \sin x+\cos x$

$= RS$

$\therefore \dfrac{1+2\sin x\cos x}{\sin x+\cos x} = \sin x+\cos x.$

4 g) $\dfrac{\sin x}{1-\cos x} - \dfrac{1+\cos x}{\sin x} = 0$

LS

$\dfrac{\sin x(\sin x)-(1+\cos x)(1-\cos x)}{(1-\cos x)(\sin x)}$

$= \dfrac{\sin^2 x-(1-\cos^2 x)}{\sin x\,(1-\cos x)}$

$= \dfrac{\sin^2 x-\sin^2 x}{\sin x(1-\cos x)}$ (PI)

$= 0$

$= RS$

$\therefore \dfrac{\sin x}{1-\cos x} - \dfrac{1+\cos x}{\sin x} = 0.$

Pg 398 #4h-j, 13.

4h) $\sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$

LS

$\sin^2 x(1-\sin^2 x)$

$= (1-\cos^2 x)(\cos^2 x)$ (PI)

$= \cos^2 x - \cos^4 x$ $\therefore \sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$

$= RS$

4i) $(1+\tan^2 x)(1-\cos^2 x) = \tan^2 x$

LS

$(1+\tan^2 x)(1-\cos^2 x)$

$= \underbrace{1-\cos^2 x} + \tan^2 x - \tan^2 x \cos^2 x$

$= \sin^2 x + \tan^2 x - \frac{\sin^2 x}{\cos^2 x}\cos^2 x$ (PI, QI)

$= \sin^2 x + \tan^2 x - \sin^2 x$

$= \tan^2 x$

$= RS$ $\therefore (1+\tan^2 x)(1-\cos^2 x) = \tan^2 x$.

j) $\frac{\sin x - 1}{\sin x + 1} = \frac{-\cos^2 x}{(\sin x+1)^2}$

LS $\frac{\sin x - 1}{\sin x + 1} \times \frac{\sin x + 1}{\sin x + 1}$

$= \frac{\sin^2 x - 1}{(\sin x + 1)^2}$

$= \frac{-(1-\sin^2 x)}{(\sin x+1)^2}$

$= \frac{-\cos^2 x}{(\sin x+1)^2}$ $\therefore \frac{\sin x - 1}{\sin x + 1} = \frac{-\cos^2 x}{(\sin x+1)^2}$

$= RS.$

13. $x = a\cos\theta - b\sin\theta$, $y = a\sin\theta + b\cos\theta$

Prove $x^2 + y^2 = a^2 + b^2$

LS

$(a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= (a^2\cos^2\theta - 2ab\sin\theta\cos\theta + b^2\sin^2\theta) + (a^2\sin^2\theta + 2ab\sin\theta\cos\theta + b^2\cos^2\theta)$.

* don't ~~forget~~ to "HAVE a BLAST!

$= a^2\cos^2\theta + a^2\sin^2\theta - 2ab\sin\theta\cos\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta + b^2\cos^2\theta$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= a^2(1) + b^2(1)$ (PI)

$= a^2 + b^2$

$=$ RS. Q.E.D