

$$\begin{aligned} \text{1a) } \cos x \tan x &= \cos x \frac{\sin x}{\cos x} \text{ (QI)} \\ &= \sin x \end{aligned}$$

$$\text{b) } \sin^2 x = 1 - \cos^2 x \text{ (PI)}$$

$$\text{c) } \cos^2 x = 1 - \sin^2 x \text{ (PI)}$$

$$\text{d) } \tan^2 x = \frac{\sin^2 x}{\cos^2 x} \text{ (QI)}$$

$$\text{e) } \tan x \sin x = \frac{\sin x}{\cos x} \cdot \sin x \text{ (QI)}$$

$$\text{f) } 1 - \sin^2 x = \cos^2 x \text{ (PI)}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$\begin{aligned} \text{g) } \sin x \tan x \cos x &= \sin x \frac{\sin x}{\cos x} \cos x \text{ (QI)} \\ &= \sin^2 x \end{aligned}$$

$$\text{h) } 1 - \cos^2 x = \sin^2 x \text{ (PI)}$$

$$\text{i) } \sin^2 x + \cos^2 x = 1 \text{ (PI)}$$

$$\begin{aligned} \text{2 a) } \frac{\sin x}{\tan x} &= \cos x \\ \text{LS} \\ \sin x \div \tan x &= \sin x \div \frac{\sin x}{\cos x} \text{ (QI)} \\ &= \sin x \times \frac{\cos x}{\sin x} \\ &= \cos x \\ &= \text{RS} \end{aligned}$$

$$\therefore \frac{\sin x}{\tan x} = \cos x$$

$$\begin{aligned} \text{b) } \sin x \cos x \tan x &= 1 - \cos^2 x \\ \text{LS} \\ \sin x \cos x \frac{\sin x}{\cos x} \text{ (QI)} &= \sin^2 x \end{aligned}$$

$$= \sin^2 x$$

$$= 1 - \cos^2 x$$

$$= \text{RS}$$

$$\therefore \sin x \cos x \tan x = 1 - \cos^2 x$$

$$\text{d) } \sin^2 x + \frac{\sin x \cos x}{\tan x} = 1$$

$$\begin{aligned} \text{LS} \\ \sin^2 x + \sin x \cos x \div \tan x &= \sin^2 x + \sin x \cos x \frac{\cos x}{\sin x} \text{ (QI)} \end{aligned}$$

$$= \sin^2 x + \sin x \cos x \frac{\cos x}{\sin x} \text{ (QI)}$$

$$= \sin^2 x + \sin x \cos x \frac{\cos x}{\sin x}$$

$$= \sin^2 x + \cos^2 x \text{ (PI)}$$

$$\begin{aligned} \text{c) } \frac{1 - \cos^2 x}{\sin x} &= \sin x \\ \text{LS} \\ \frac{1 - \cos^2 x}{\sin x} &= \frac{\sin^2 x}{\sin x} \text{ (PI)} \\ &= \sin x \end{aligned}$$

$$= \sin x$$

$$\therefore \frac{1 - \cos^2 x}{\sin x} = \sin x = 1 \text{ RS} \therefore \sin^2 x + \frac{\sin x \cos x}{\tan x} = 1$$

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$$2e) 1 + \frac{1}{\tan^2 x} = \frac{1}{\sin^2 x}$$

$$\stackrel{LS}{=} 1 + 1 \div \tan^2 x$$

$$= 1 + 1 \div \frac{\sin^2 x}{\cos^2 x} \quad (QI)$$

$$= 1 + 1 \times \frac{\cos^2 x}{\sin^2 x}$$

$$= 1 + \frac{\cos^2 x}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} \quad \because 1 + \frac{1}{\tan^2 x} = \frac{1}{\sin^2 x}$$

$$= RS$$

$$2f) 2\sin^2 x - 1 = \sin^2 x - \cos^2 x$$

$$\stackrel{RS}{=} \sin^2 x - \cos^2 x$$

$$= \sin^2 x - (1 - \sin^2 x) \quad (PI)$$

$$= \sin^2 x - 1 + \sin^2 x$$

$$= 2\sin^2 x - 1$$

$$= LS$$

$$\therefore 2\sin^2 x - 1 = \sin^2 x - \cos^2 x$$

$$2g) \frac{1}{\cos x} - \cos x = \sin x \tan x$$

$$\stackrel{LS}{=} \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x} \quad (PI)$$

$$= \sin x \cdot \frac{\sin x}{\cos x} \quad (QI)$$

$$= \sin x \tan x = RS$$

$$\therefore \frac{1}{\cos x} - \cos x = \sin x \tan x$$

$$2h) \sin x + \tan x = \tan x (1 + \cos x)$$

$$\begin{aligned} & \frac{RS}{\sin x (1 + \cos x) (QI)} \\ & \frac{\sin x}{\cos x} \\ & = \frac{\sin x}{\cos x} + \frac{\sin x \cdot \cos x}{\cos x} \\ & = \frac{\sin x}{\cos x} + \sin x \\ & = \tan x + \sin x \quad (QI) \\ & = \sin x + \tan x. \\ & = LS \end{aligned}$$

$$\therefore \sin x + \tan x = \tan x (1 + \cos x).$$

$$(i) \frac{1}{1 - \sin^2 x} = 1 + \tan^2 x$$

$$\begin{aligned} & \frac{RS}{1 + \tan^2 x} \\ & = 1 + \frac{\sin^2 x}{\cos^2 x} \quad (QI) \\ & = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ & = \frac{1}{\cos^2 x} \quad (PI) \\ & = \frac{1}{1 - \sin^2 x} \quad (PI) \\ & = LS. \end{aligned}$$

$$\therefore \frac{1}{1 - \sin^2 x} = 1 + \tan^2 x.$$

$$j) \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\begin{aligned} & \frac{RS}{2\cos^2 x - 1} \\ & = 2\cos^2 x - (\sin^2 x + \cos^2 x) \\ & = 2\cos^2 x - \sin^2 x - \cos^2 x \\ & = \cos^2 x - \sin^2 x \\ & = LS \end{aligned}$$

$$\therefore \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

alternate.

$$\begin{aligned} & \frac{LS}{\cos^2 x - \sin^2 x} \\ & = \cos^2 x - (1 - \cos^2 x) \\ & = \cos^2 x - 1 + \cos^2 x \\ & = 2\cos^2 x - 1 \\ & = RS. \end{aligned}$$

$$2k) \sin^2 x + \cos^2 x + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\begin{aligned} & \text{LS} \\ & \sin^2 x + \cos^2 x + \tan^2 x \\ &= 1 + \frac{\sin^2 x}{\cos^2 x} \quad (\text{PI, QI}) \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \quad (\text{PI}) \\ &= \text{RS} \quad \therefore \sin^2 x + \cos^2 x + \tan^2 x = \frac{1}{\cos^2 x} \end{aligned}$$

$$2d) \frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$$

$$\begin{aligned} & \text{RS} \\ & \tan x \div (1 + \tan x) \\ &= \frac{\sin x}{\cos x} \div \left(1 + \frac{\sin x}{\cos x}\right) \quad (\text{QI}) \\ &= \frac{\sin x}{\cos x} \div \left(\frac{\cos x + \sin x}{\cos x}\right) \\ &= \frac{\sin x}{\cos x} \times \frac{\cos x}{\cos x + \sin x} \\ &= \frac{\sin x}{\cos x + \sin x} \\ &= \frac{\sin x}{\sin x + \cos x} \\ &= \text{LS} \end{aligned}$$

$$\therefore \frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$$

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$$2m) \frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

$$\begin{aligned} &\text{LS} \\ &(1 + \tan^2 x) \div (1 - \tan^2 x) \\ &= \left(1 + \frac{\sin^2 x}{\cos^2 x}\right) \div \left(1 - \frac{\sin^2 x}{\cos^2 x}\right) \quad (\text{QI}) \end{aligned}$$

$$\begin{aligned} &= \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right) \div \left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right) \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\cos^2 x - \sin^2 x} \\ &= \frac{1}{\cos^2 x - \sin^2 x} \quad (\text{PI}) \end{aligned}$$

= RS.

$$\therefore \frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

$$4. (a) \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$$

$$\begin{aligned} &\text{LS} \\ &\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x} \end{aligned}$$

$$= \frac{1}{\sin^2 x \cos^2 x} \quad (\text{PI})$$

= RS.

$$\therefore \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{1}{\sin^2 x \cos^2 x}$$

$$(b) \text{LS} \quad \tan x + \frac{1}{\tan x}$$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \quad (\text{PI})$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\sin x \cos x} \quad (\text{PI})$$

= RS.

RS

$$\frac{1}{\sin x \cos x}$$

$$\therefore \tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$$

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$$4c) \frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$$

$$\begin{aligned} & \frac{1(1+\cos x) + 1(1-\cos x)}{(1-\cos x)(1+\cos x)} \\ &= \frac{1+\cos x + 1-\cos x}{1-\cos^2 x} \end{aligned}$$

$$= \frac{2}{1-\cos^2 x}$$

$$= \frac{2}{\sin^2 x} \quad (\text{PI})$$

$$= \text{RS} \quad \therefore \frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$$

$$d) (\sin x + \cos x)^2 = 1 + 2\sin x \cos x$$

$$\begin{aligned} & \frac{\text{LS}}{(\sin x + \cos x)^2} \\ &= \sin^2 x + 2\sin x \cos x + \cos^2 x \end{aligned}$$

$$\begin{aligned} &= \sin^2 x + \cos^2 x + 2\sin x \cos x \\ &= 1 + 2\sin x \cos x \quad (\text{PI}) \end{aligned}$$

$$= \text{RS.}$$

$$\therefore (\sin x + \cos x)^2 = 1 + 2\sin x \cos x.$$

$$e) (1 - \cos^2 x) \left(1 + \frac{1}{\tan^2 x} \right) = 1$$

$$\begin{aligned} & \frac{\text{LS}}{1 + \frac{1}{\tan^2 x} - \cos^2 x - \frac{\cos^2 x}{\tan^2 x}} \\ &= 1 + \frac{\cos^2 x}{\sin^2 x} - \cos^2 x - \cos^2 x \cdot \frac{\cos^2 x}{\sin^2 x} \quad (\text{QI}) \end{aligned}$$

$$= 1 - \cos^2 x + \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$$

$$= \sin^2 x + \frac{\cos^2 x}{\sin^2 x} (1 - \cos^2 x) \quad (\text{PI})$$

$$= \sin^2 x + \frac{\cos^2 x}{\sin^2 x} (\sin^2 x) \quad (\text{PI}) \quad \therefore (1 - \cos^2 x) \left(1 + \frac{1}{\tan^2 x} \right) = 1,$$

$$= \sin^2 x + \cos^2 x = 1 \quad (\text{PI}) = \text{RS}$$

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$$4f) \frac{1 + 2 \sin x \cos x}{\sin x + \cos x} = \sin x + \cos x$$

LS

$$= \frac{1 + 2 \sin x \cos x}{\sin x + \cos x}$$

$$= \frac{\sin^2 x + \cos^2 x + 2 \sin x \cos x}{\sin x + \cos x} \quad (\text{PI})$$

$$= \frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x}{\sin x + \cos x}$$

$$= \frac{(\sin x + \cos x)^2}{\sin x + \cos x}$$

$$= \sin x + \cos x$$

$$= \text{RS}$$

$$\therefore \frac{1 + 2 \sin x \cos x}{\sin x + \cos x} = \sin x + \cos x.$$

$$4g) \frac{\sin x}{1 - \cos x} - \frac{1 + \cos x}{\sin x} = 0$$

LS

$$\frac{\sin x (\sin x) - (1 + \cos x)(1 - \cos x)}{(1 - \cos x)(\sin x)}$$

$$= \frac{\sin^2 x - (1 - \cos^2 x)}{\sin x (1 - \cos x)}$$

$$= \frac{\sin^2 x - \sin^2 x}{\sin x (1 - \cos x)} \quad (\text{PI})$$

$$= 0$$

$$= \text{RS}$$

$$\therefore \frac{\sin x}{1 - \cos x} - \frac{1 + \cos x}{\sin x} = 0.$$

4h) Pg 398 #4h-j, 13.

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$$\sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$$

$$\text{LS} \frac{\sin^2 x (1 - \sin^2 x)}{}$$

$$= (1 - \cos^2 x)(\cos^2 x) \quad (\text{PI})$$

$$= \cos^2 x - \cos^4 x$$

$$= \text{RS}$$

$$\therefore \sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$$

$$4i) (1 + \tan^2 x)(1 - \cos^2 x) = \tan^2 x$$

$$\text{LS} \frac{(1 + \tan^2 x)(1 - \cos^2 x)}{}$$

$$= \underbrace{1 - \cos^2 x + \tan^2 x - \tan^2 x \cos^2 x}$$

$$= \sin^2 x + \tan^2 x - \frac{\sin^2 x \cos^2 x}{\cos^2 x} \quad (\text{PI, QI})$$

$$= \sin^2 x + \tan^2 x - \sin^2 x$$

$$= \tan^2 x$$

$$= \text{RS}$$

$$\therefore (1 + \tan^2 x)(1 - \cos^2 x) = \tan^2 x.$$

$$j) \frac{\sin x - 1}{\sin x + 1} = \frac{-\cos^2 x}{(\sin x + 1)^2}$$

$$\text{LS} \frac{\sin x - 1}{\sin x + 1} \times \frac{\sin x + 1}{\sin x + 1}$$

$$= \frac{\sin^2 x - 1}{(\sin x + 1)^2}$$

$$= \frac{-(1 - \sin^2 x)}{(\sin x + 1)^2}$$

$$= \frac{-\cos^2 x}{(\sin x + 1)^2}$$

$$= \text{RS.}$$

$$\therefore \frac{\sin x - 1}{\sin x + 1} = \frac{-\cos^2 x}{(\sin x + 1)^2}$$

Pg 398 #13. (Pg. 401) ^{actually.}

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13. $x = a \cos \theta - b \sin \theta$, $y = a \sin \theta + b \cos \theta$

Prove $x^2 + y^2 = a^2 + b^2$

LS

$$(a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2$$
$$= (a^2 \cos^2 \theta - 2ab \sin \theta \cos \theta + b^2 \sin^2 \theta) + (a^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta).$$

* don't forget to
"HAVE a BLAST!"

$$= a^2 \cos^2 \theta + a^2 \sin^2 \theta - 2ab \sin \theta \cos \theta + 2ab \sin \theta \cos \theta + b^2 \sin^2 \theta + b^2 \cos^2 \theta$$
$$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 (1) + b^2 (1) \quad (\text{PI})$$

$$= a^2 + b^2$$

$$= \text{RS.} \quad \text{Q.E.D.}$$