U5D7_T Proving Trig Identities

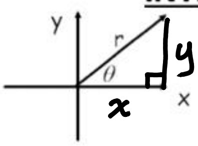
Sunday, April 28, 2019 2:06 PM

U5D7_T
Proving Tr...

U5D7 Warm Up: Skill Reflection #5

INTRODUCTION TO TRIG IDENTITIES

Recall:



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

recall: $x^2+y^2=r^2$

Example 1. Simplify (using terminal arm)

a) $\frac{\sin\theta}{\cos\theta}$ ← means ÷

$= \left(\frac{y}{r}\right) \div \left(\frac{x}{r}\right)$

$= \frac{y}{r} \times \frac{r}{x}$

$= \frac{y}{x}$

$= \tan\theta$

b) $sin^2\theta + cos^2\theta$

$= (\sin\theta)^2 + (\cos\theta)^2$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{x^2+y^2}{r^2}$

$= \frac{r^2}{r^2}$

$= 1$

Proving Trigonometric Identities

Quotient Identity (QI):

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

Pythagorean Identity (PI):

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

In example 1, we discovered some trig "identities". They are called identities because they remain equal regardless of the value of θ.

You have seen "identities" before; for example,

$(x-5)(x+5) = x^2 - 25$ is considered an identity.
If we are simplifying an expression, we can replace $(x-5)(x+5)$ with $x^2 - 25$ or we could replace $x^2 - 25$ with $(x-5)(x+5)$ without changing the value of the expression.

We use identities to write an expression in a more convenient form.

Tips for Proving Trig Identities

1. Need to show the LS = RS by manipulating **one side at a time only**.
(For grade 11 most of the proofs can be proven using one side of the expression only. In grade 12, you will usually need to work with both sides.)

2. Keep LS separate from RS at all times. NEVER 'move' anything from one side to the other (like you do when solving equations)!

3. Start by trying to work with the most complicated looking side first.

4. Remember you can manipulate the identities just like regular algebraic equations…
$sin^2\theta + cos^2\theta = 1 \quad \Rightarrow sin^2\theta = 1 - cos^2\theta \Rightarrow$
$cos^2\theta = 1 - sin^2\theta$
Any of these are called the Pythagorean Identity (PI).

5. May need to use a common denominator when adding or subtracting identities in fraction form.

Example 2. Write an equivalent expression for:

a) $tanxcosx$

$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}$ (QI)

$= \sin x$

b) $sin^2\theta$

$= 1-\cos^2\theta$ (PI)

c) $\frac{1}{tan^2\theta}$

'reciprocal' of $\tan^2\theta$, $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$ (QI)

You may (but don't need to) write:

$1 \div \tan^2\theta$

$= 1 \div \frac{\sin^2\theta}{\cos^2\theta}$ (QI)

$= 1 \times \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

Example 3. Prove the following identities:

a) $\frac{\cos x \tan x}{\sin x} = 1$

LS

$\frac{\cos x}{\sin x} \cdot \tan x$

$= \frac{\cos x}{\sin x} \times \frac{\sin x}{\cos x}$ (QI)

$= 1$

$=$ R.S.

Q.E.D.

$\therefore \boxed{\frac{\cos x \tan x}{\sin x} = 1}$

b) $1 - \sin^2\theta = \frac{\sin\theta\cos\theta}{\tan\theta}$

LS

$\cos^2\theta$ (PI)

RS

$\sin\theta\cos\theta \cdot \frac{1}{\tan\theta}$

$= \sin\theta\cos\theta \frac{\cos\theta}{\sin\theta}$ (QI)

$= \cos^2\theta$

$=$ L.S.

Q.E.D

$\therefore \boxed{1 - \sin^2\theta = \frac{\sin\theta\cos\theta}{\tan\theta}}$

c) $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}$

LS

$= \frac{1(1+\cos x) + 1(1-\cos x)}{(1-\cos x)(1+\cos x)}$

$= \frac{1 + \cancel{\cos x} + 1 - \cancel{\cos x}}{1 - \cos^2 x}$

$= \frac{2}{\sin^2 x}$ (PI)

$=$ R.S.

Q.E.D.

$\therefore \boxed{\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{\sin^2 x}}$

d) $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

LS

$$\frac{\cos\theta}{1+\sin\theta} \times \frac{(1-\sin\theta)}{1-\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta} = R.S.$$

$$\therefore \boxed{\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}}$$

e) $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{2}{\cos\theta}$

LS

$$\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2\cos\theta}{\cos^2\theta} \quad (PI)$$

$$= \frac{2}{\cos\theta}$$

$$= R.S.$$

$$\therefore \boxed{\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = \frac{2}{\cos\theta}}$$

Homework: p. 398 #1, 2bcgl, 4abei