

U5D6 Special Triangles Blackboard Lesson

U5D6 Homework: Pg 348 #3, 7bcf, 8, 11, Worksheet 5-3

U5D7: Work Period Extra Practice Worksheet 5-5

U5D6

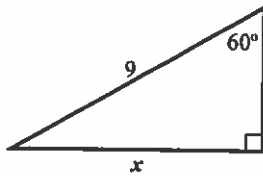
MCR 3U1 – *Trigonometric Ratios of Special Angles* Worksheet 5-3

1. Find the exact value of the following:

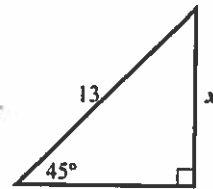
- | | |
|--|--|
| a) $\sin^2 45^\circ + 2 \sin 30^\circ \sec 30^\circ$ | b) $\sin 30^\circ \sin 45^\circ \sin 60^\circ$ |
| c) $\sin^2 30^\circ + \cos^2 30^\circ$ | d) $\sin 30^\circ \cos 30^\circ + \sin 60^\circ \cos 60^\circ$ |
| e) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$ | f) $2 \sin 30^\circ \cos 30^\circ$ |
| g) $5 \sec 30^\circ \tan 60^\circ$ | h) $\cos^2 60^\circ + 3 \sec^2 30^\circ$ |
| i) $3 \sin^2 45^\circ + 4 \cos^2 45^\circ$ | j) $\sin 60^\circ \cos 60^\circ \tan 60^\circ$ |
| k) $\sec^2 45^\circ \csc^2 45^\circ - 1$ | l) $\sin 30^\circ + \cos 60^\circ + \sec 60^\circ$ |

2. State the exact value for x in each of the following:

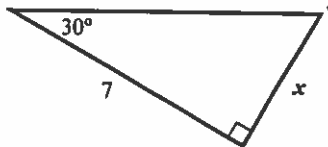
a)



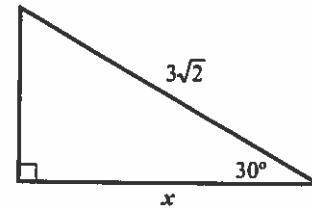
b)



c)



d)



3. Prove the following

- | | |
|--|--|
| a) $\sin^2 30^\circ + \cos^2 30^\circ = \sin^2 60^\circ + \cos^2 60^\circ$ | b) $1 + \tan^2 45^\circ = \sec^2 45^\circ$ |
| c) $\csc^2 60^\circ = 1 + \cot^2 60^\circ$ | d) $\cos 60^\circ \sec 30^\circ = \tan 30^\circ$ |

ANSWERS

- | | | | | | |
|-----------------------------|---------------------------|--------------------------|--------------------------|------|-------------------------|
| 1a) $\frac{3+4\sqrt{3}}{6}$ | b) $\frac{\sqrt{6}}{8}$ | c) 1 | d) $\frac{\sqrt{3}}{2}$ | e) 1 | f) $\frac{\sqrt{3}}{2}$ |
| g) 10 | h) $\frac{17}{4}$ | i) $\frac{7}{2}$ | j) $\frac{3}{4}$ | k) 3 | l) 3 |
| 2a) $\frac{9\sqrt{3}}{2}$ | b) $\frac{13\sqrt{2}}{2}$ | c) $\frac{7\sqrt{3}}{3}$ | d) $\frac{3\sqrt{6}}{2}$ | | |

Worksheet 5-3

a) $\sin^2 45^\circ + 2 \sin 30^\circ \sec 30^\circ$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$
reciprocal of $\cos 30^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + 2 \left(\frac{1}{2}\right) \left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{1}{2} + \frac{2}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2\sqrt{3}} + \frac{4}{2\sqrt{3}} \quad \text{OR} \quad = \frac{1}{2} + \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$= \frac{4+\sqrt{3}}{2\sqrt{3}}$$

$$= \frac{1}{2} + \frac{2\sqrt{3}}{3}$$

$$= \frac{3+4\sqrt{3}}{6}$$

b) $\sin 30^\circ \sin 45^\circ \sin 60^\circ$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4\sqrt{2}}$$

$$= \frac{\sqrt{3}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}}{8}$$

c) $\sin^2 30^\circ + \cos^2 30^\circ$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

d) $\sin 30^\circ \cos 30^\circ + \sin 60^\circ \cos 60^\circ$

$$= \frac{1}{2} \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

e) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

f) $2 \sin 30^\circ \cos 30^\circ$

$$= 2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{2}$$

g) $5 \sec 30^\circ \tan 60^\circ$

$$= 5 \left(\frac{2}{\sqrt{3}}\right) (\sqrt{3})$$

$$= 10$$

h) $\cos^2 60^\circ + 3 \sec^2 30^\circ$

$$= \left(\frac{1}{2}\right)^2 + 3 \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= \frac{1}{4} + 3 \left(\frac{4}{3}\right)$$

$$= \frac{1}{4} + 4$$

$$= \frac{1}{4} + \frac{16}{4}$$

$$= \frac{17}{4}$$

$$\begin{aligned}
 \text{i)} \quad & 3\sin^2 45^\circ + 4\cos^2 45^\circ \\
 &= 3\left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 \\
 &= 3\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right) \\
 &= \frac{3}{2} + \frac{4}{2} \\
 &= \boxed{\frac{7}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & \sin 60^\circ \cos 60^\circ \tan 60^\circ \\
 &= \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{1} \\
 &= \boxed{\frac{3}{4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & \sec^2 45^\circ \csc^2 45^\circ - 1 \\
 &= \left(\frac{\sqrt{2}}{1}\right)^2 \times \left(\frac{\sqrt{2}}{1}\right)^2 - 1 \\
 &= (2)(2) - 1 \\
 &= 4 - 1 \\
 &= \boxed{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{l)} \quad & \sin 30^\circ + \cos 60^\circ + \sec 60^\circ \\
 &= \frac{1}{2} + \frac{1}{2} + \frac{2}{1} \\
 &= 1 + 2 \\
 &= \boxed{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{2 a)} \quad & \sin 60^\circ = \frac{x}{9} \\
 & x = 9 \sin 60^\circ \\
 & x = 9\left(\frac{\sqrt{3}}{2}\right) \\
 & \boxed{x = \frac{9\sqrt{3}}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \sin 45^\circ = \frac{x}{13} \\
 & x = 13 \sin 45^\circ \\
 & x = 13\left(\frac{1}{\sqrt{2}}\right) \\
 & \boxed{x = \frac{13}{\sqrt{2}}} \quad \text{OR } x = \frac{13 \times \sqrt{2}}{\sqrt{2} \sqrt{2}} \\
 & \boxed{x = \frac{13\sqrt{2}}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad & \tan 30^\circ = \frac{x}{7} \\
 & x = 7 \tan 30^\circ \\
 & x = 7\left(\frac{1}{\sqrt{3}}\right) \\
 & x = \frac{7 \times \sqrt{3}}{\sqrt{3} \sqrt{3}} \\
 & \boxed{x = \frac{7\sqrt{3}}{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{d)} \quad & \cos 30^\circ = \frac{x}{3\sqrt{2}} \\
 & x = 3\sqrt{2} \cos 30^\circ \\
 & x = 3\sqrt{2} \left(\frac{\sqrt{3}}{2}\right) \\
 & \boxed{x = \frac{3\sqrt{6}}{2}}
 \end{aligned}$$

Worksheet 5-3

Pg ③

$$3.a) \quad \frac{LS}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

$$\frac{RS}{\sin^2 60^\circ + \cos^2 60^\circ}$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

$$LS = RS$$

$$\therefore \sin^2 30^\circ + \cos^2 30^\circ = \sin^2 60^\circ + \cos^2 60^\circ$$

$$b) \quad \frac{LS}{1 + \tan^2 45^\circ}$$

$$= 1 + (1)^2$$

$$= 2$$

$$\frac{RS}{\sec^2 45^\circ}$$

$$= \left(\frac{\sqrt{2}}{1}\right)^2$$

$$= 2$$

$$LS = RS$$

$$\therefore 1 + \tan^2 45^\circ = \sec^2 45^\circ$$

$$c) \quad \csc^2 60^\circ = 1 + \cot^2 60^\circ$$

$$\frac{LS}{\frac{1}{\sin^2 60^\circ}}$$

$$= \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= \frac{4}{3}$$

$$\frac{RS}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}$$

$$LS = RS$$

$$\therefore \csc^2 60^\circ = 1 + \cot^2 60^\circ$$

$$d) \quad \cos 60^\circ \sec 30^\circ = \tan 30^\circ$$

$$\frac{LS}{\frac{1}{2} \left(\frac{2}{\sqrt{3}}\right)}$$

$$= \frac{1}{\sqrt{3}}$$

$$\frac{RS}{\frac{1}{\sqrt{3}}}$$

$$LS = RS$$

$$\therefore \cos 60^\circ \sec 30^\circ = \tan 30^\circ$$