U5D5_T Trig Ratios of Special Angles and CAST Rule

Thursday, April 25, 2019   1:16 PM

U5D5_T
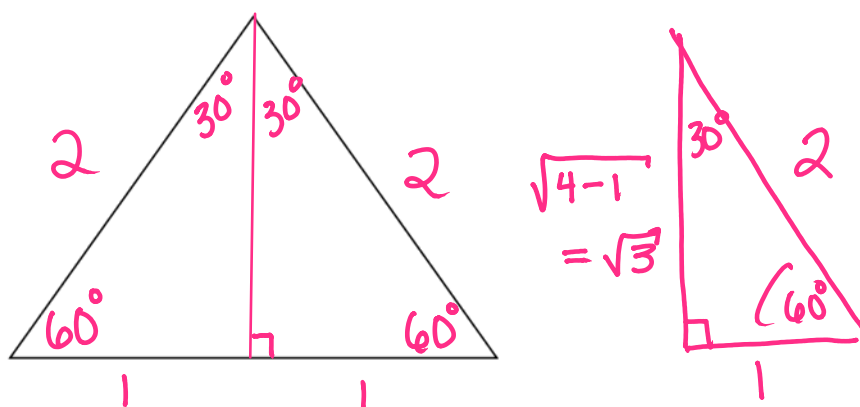Trig Ratio...

U5D5 MCR3UI   THE CAST RULE AND SPECIAL ANGLES

Special Angles - 30° and 60°

Construct an Equilateral triangle with side lengths 2.
Drop a vertical line from the top angle to the opposite
side creating two Right-Triangles. The altitude will bisect
the opposite side since it is an equilateral triangle.
Determine the length of the altitude using the
Pythagorean Theorem.

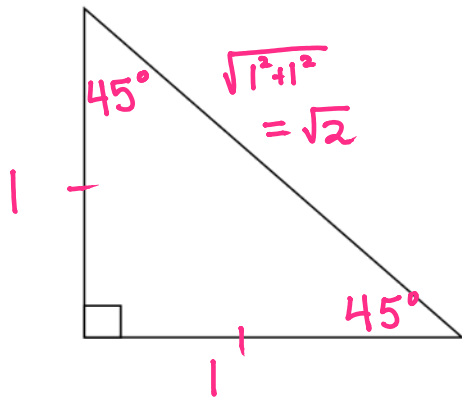$$\sin 30° = \frac{1}{2} \qquad \cos 30° = \frac{\sqrt{3}}{2} \qquad \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin 60° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2} \qquad \tan 60° = \sqrt{3}$$

Special Angles: 45°

Construct an Isosceles Right-Triangle and determine all 3 trig ratios of the non-90 angle. (Tip: The equal side lengths are 1 unit each)

45° $\sqrt{1^2+1^2}$
       $= \sqrt{2}$

1

45°

1

$\sin 45° = \dfrac{1}{\sqrt{2}}$ $\cos 45° = \dfrac{1}{\sqrt{2}}$ $\tan 45° = 1$

NOTE: You should memorize these triangles/ratios, or at least be able to construct them! These angles will be used frequently.

$$\sin 30° = \frac{1}{2} \qquad \cos 30° = \frac{\sqrt{3}}{2} \qquad \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin 60° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2} \qquad \tan 60° = \sqrt{3}$$

$$\sin 45° = \frac{1}{\sqrt{2}} \qquad \cos 45° = \frac{1}{\sqrt{2}} \qquad \tan 45° = 1$$
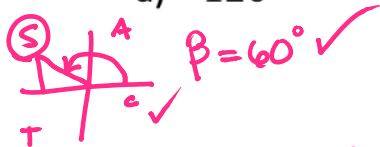
For each of the following examples, complete **WITHOUT a calculator**!

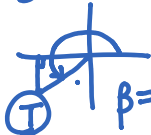**Example 1**: Determine the exact values of the three primary trig ratios of:
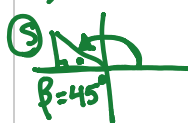
a) 120° b) 210° c) 135°

$\beta = 60°$ ✓

$\sin 120°$ $\cos 120°$ $\tan 120°$
$= \sin 60°$ ✓ $= -\cos 60°$ ✓ $= -\tan 60°$ ✓
$= \frac{\sqrt{3}}{2}$ ✓ $= -\frac{1}{2}$ ✓ $= -\sqrt{3}$ ✓

b) 210°

$\beta = 30°$

$\sin 210°$ $\cos 210°$ $\tan 210°$
$= -\sin 30°$ $= -\cos 30°$ $= \tan 30°$
$= -\frac{1}{2}$ $= -\frac{\sqrt{3}}{2}$ $= \frac{1}{\sqrt{3}}$

c) 135°

$\beta = 45°$

$\sin 135°$ $\cos 135°$ $\tan 135°$
$= \sin 45°$ $= -\cos 45°$ $= -\tan 45°$
$= \frac{1}{\sqrt{2}}$ $= -\frac{1}{\sqrt{2}}$ $= -1$

Example 2: Determine the exact value of

$sec^2 60° - tan45° sin30°$

$$= \frac{1}{(\cos 60°)^2} - (1)\left(\frac{1}{2}\right)$$

$$= 2^2 - \frac{1}{2}$$

$$= 4 - \frac{1}{2}$$

$$= \frac{7}{2}$$

$(\sec 60°)^2$

$= sec^2 60°$

note:

$\cos 60°$

$= \frac{1}{2}$
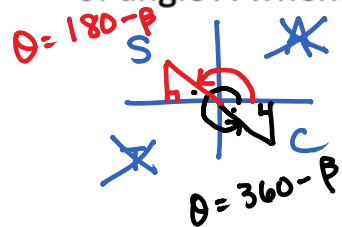
so $\frac{1}{\cos 60°}$

$= \frac{2}{1}$

Example 3: If $0° \leq \theta \leq 360°$, determine all possible measures of angle θ when: $\cos\theta = -\frac{1}{\sqrt{2}}$

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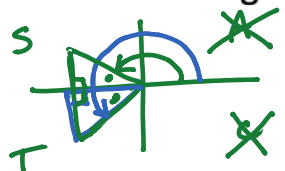
$\beta = 45°$

$\therefore \theta = 135°$ or $225°$

Example 4: If $0° \leq A \leq 360°$, find the possible measures of angle A when: $\tan A = -1$

$\theta = 180 - \beta$

S ✳

$\beta = 45°$

✳

C

$\theta = 360 - \beta$

$\boxed{\theta = 135° \text{ OR } 315°}$