U5D4_T The CAST Rule

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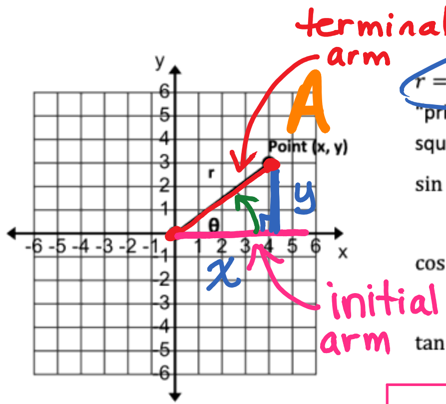
U5D4_T The CAST ...

U5D4 **CAST Rule** Warm Up: Skill Reflection #3

CAST

Given angle θ formed by a **terminal arm** in **standard position**. (For standard position the "initial arm" is the positive x-axis, the terminal arm is found by joining the origin to the point, the angle θ is the angle measured from the initial arm rotating counter-clockwise to the terminal arm.)

If we have a Point in Quadrant I with x>0, y>0:



$r = \sqrt{x^2 + y^2}$ The square root sign means take the "principal square root" which means the positive square root so r is always positive.

$\sin\theta = \frac{y}{r}$ ← Opp ← Hyp

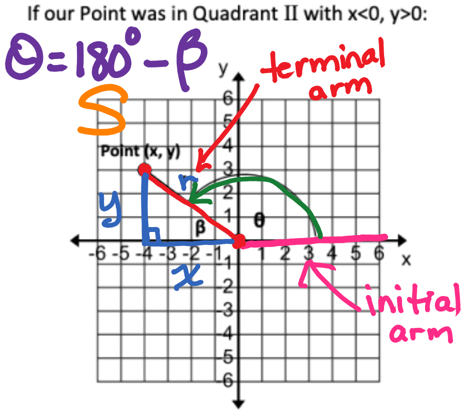
$\cos\theta = \frac{x}{r}$ ← Adj ← Hyp

$\tan\theta = \frac{y}{x}$ ← Opp ← Adj

Note: All three ratios are positive in Quadrant I
0°<θ<90°

$\boxed{\sin\theta > 0,\ \cos\theta > 0, \tan\theta > 0}$

If our Point was in Quadrant II with x<0, y>0:



θ is the "principal angle"

β is the "related acute angle" θRAA

$\sin\theta = \frac{y}{r}$ $\frac{+}{+}$ ⊕

$\cos\theta = \frac{x}{r}$ $\frac{-}{+}$ ⊖

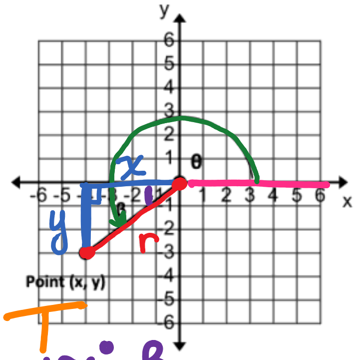
$\tan\theta = \frac{y}{x}$ $\frac{+}{-}$ ⊖

Since r>0 and in Quadrant II, x<0 and y>0,

Sine is the only positive ratio in quadrant II 90°<θ<180°

$\boxed{\sin\theta > 0}$ $\cos\theta < 0, \tan\theta < 0$

If our Point was in Quadrant III with x<0, y<0:



$\theta = 180° + \beta$

θ is the "principal angle"
β is the "related acute angle"

$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

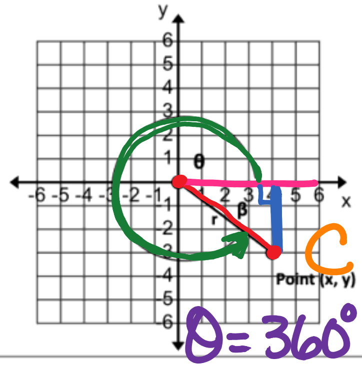
$\tan\theta = \frac{y}{x} = \frac{-}{-} = +$

Since r>0 and in Quadrant III,
x<0 and y<0,

Tangent is the only positive ratio
in quadrant III 180°<θ<270°

$\sin\theta < 0$, $\cos\theta < 0$, $\boxed{\tan\theta > 0}$

If our Point was in Quadrant IV with x>0, y<0:



$\theta = 360° - \beta$

θ is the "principal angle"
β is the "related acute angle"

$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

$\tan\theta = \frac{y}{x} = \frac{-}{+} = -$

Since r>0 and in Quadrant IV,
x>0 and y<0,

Cosine is the only positive ratio
in quadrant IV 270°<θ<360°

$\sin\theta < 0$, $\boxed{\cos\theta > 0}$ $\tan\theta < 0$

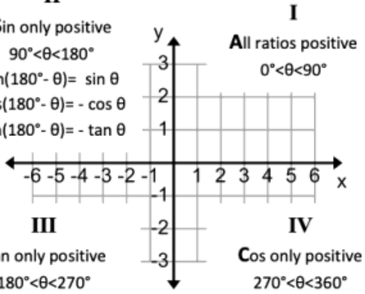
NOTE: θ is measured from the positive x-axis, rotating counter-clockwise to the terminal arm. However, to use simple trigonometric ratios, we need a Rignt-Triangle, hence we place β, the related acute angle between the x-axis and the terminal arm. In this way, we form a right-triangle with the terminal arm and the x-axis with β as the reference angle inside the triangle.

II

Sin only positive
90°<θ<180°
sin(180°- θ)= sin θ
cos(180°- θ)= - cos θ
tan(180°- θ)= - tan θ

I

All ratios positive
0°<θ<90°



III

Tan only positive
180°<θ<270°
sin(180°+ θ)= - sin θ
cos(180°+ θ)= - cos θ
tan(180°+ θ)= tan θ

IV

Cos only positive
270°<θ<360°
sin(360°- θ)= - sin θ
cos(360°- θ)= cos θ
tan(360°- θ)= - tan θ

The CAST rule:

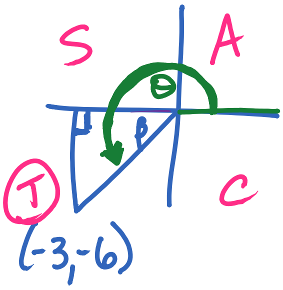
To determine whether a trig ratio will be positive or negative, you may use the CAST rule.

If you know which quadrant θ lies in then...

← See grid

Example 1: The point P(-3, -6) lies on the terminal arm of an angle θ in standard position.

a) Determine the exact values of sin θ, cos θ and tan θ.



$x = -3$

$y = -6$

$r = \sqrt{9+36}$

$r = \sqrt{45}$

$r = 3\sqrt{5}$

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-6}{3\sqrt{5}}$ $\quad \cos\theta = \frac{-3}{3\sqrt{5}}$ $\quad \tan\theta = \frac{-6}{-3}$

$\sin\theta = \frac{-2}{\sqrt{5}}$ $\quad \cos\theta = \frac{-1}{\sqrt{5}}$ $\quad \tan\theta = 2$

b) Determine the principal angle, θ.

First find the related acute angle, β by using the absolute value of any one of the primary trig ratios.

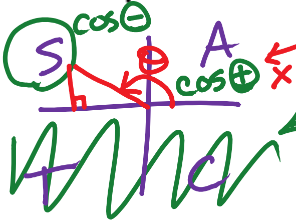
$\beta = \sin^{-1}(2 \div \sqrt{5})$ $\quad \beta = \cos^{-1}(1 \div \sqrt{5})$ $\quad \beta = \tan^{-1}(2)$

$\beta \doteq 63^\circ$ $\quad \beta \doteq 63^\circ$ $\quad \beta \doteq 63^\circ$

$\theta = 180^\circ + 63^\circ$

$\theta = 243^\circ$

Example 2: Find θ if cos θ = -0.6784, and 0° ≤ θ ≤ 180°.



$\beta = \cos^{-1}(0.6784)$

$\beta \doteq 47°$

$\theta = 180° - 47°$

$\boxed{\theta = 133°}$

Example 3: Angle θ is in standard position in quadrant II and ~~0° ≤ θ ≤ 360°~~. Given the trig ratio, find:

$90° < \theta < 180°$

a) the exact values of the other two trig ratios.

$sin\theta = \frac{3}{4}$ ← y ← r

$x^2 + y^2 = r^2$

$x = -\sqrt{r^2 - y^2}$

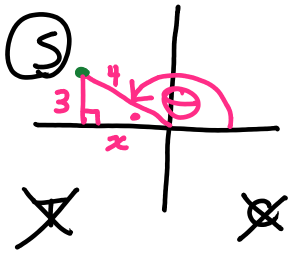
$x = -\sqrt{16 - 9}$

$x = -\sqrt{7}$

$\cos\theta = \frac{-\sqrt{7}}{4}$

$\tan\theta = \frac{3}{-\sqrt{7}}$

$\tan\theta = \frac{-3}{\sqrt{7}}$



b) the principal angle, θ

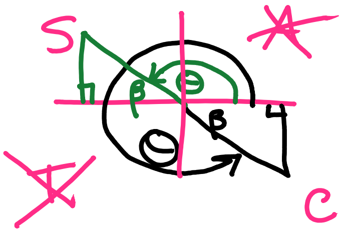
$\beta = \sin^{-1}(0.75)$

$\beta \doteq 49°$

$\theta \doteq 180° - 49°$

$\boxed{\theta \doteq 131°}$

Example 4: Find θ if tan θ = -0.5238, if $0° \leq \theta \leq 360°$.



$\beta = \tan^{-1}(0.5238)$

$\beta \doteq 28°$

$\theta \doteq 180° - 28°$

$\boxed{\theta \doteq 152°}$ OR $\boxed{\theta \doteq 332°}$

$\theta \doteq 360° - 28°$

U5D4 Homework: # p. 281 #1, 4, p. 348 #1abef + principal angle, #2abef + principal angle, #6, (Where it says $0 \leq \theta \leq 2\pi$ treat as $0º \leq \theta \leq 360º$), p. 348 #1a) $\cos\theta = \frac{8}{17}$ (book error)