U4D9_T Review

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U4D9 Review: Exponents and Exponential Functions

A: Exponent Laws & Exponential Expressions

1) Evaluate.

~~then evaluate~~.

$$\left(\frac{5}{7}\right)^{-2}$$

$$= \left(\frac{7}{5}\right)^{2}$$

$$= \frac{7^2}{5^2}$$

$$= \frac{49}{25}$$

2) Rewrite in radical form and then evaluate

$$(-64)^{-\frac{2}{3}}$$

$$= \left(\frac{1}{-64}\right)^{\frac{2}{3}}$$

$$= \frac{1}{\left(\sqrt[3]{-64}\right)^2}$$

$$= \frac{1}{(-4)^2}$$

$$= \frac{1}{16}$$

3) Simplify and rewrite using positive exponents.

$$\frac{(2x^{-5}y^3)^2(-6x^4y^{-1})}{3xy^{-7}}$$

$$= \frac{(2)^2(x^{-5})^2(y^3)^2(-6)(x^4)(y^{-1})}{3xy^{-7}}$$

$$= \frac{4x^{-10}y^6(-6)x^4y^{-1}}{3xy^{-7}}$$

$$= -\frac{24}{3}x^{-10+4-1}y^{6-1+7}$$

$$= -8x^{-7}y^{12} = \frac{-8y^{12}}{x^7}$$

Try this one:

$$\left(\frac{-64x^{17}y^{5}}{2x^{2}y^{-5}}\right)^{\frac{3}{5}}$$

note: $\sqrt[5]{-64}$ is a decimal number so simplify inside bracket FIRST.

$= (-32x^{15}y^{10})^{3/5}$

$= (\sqrt[5]{-32})^{3}\, x^{15\times\frac{3}{5}}\, y^{10\times\frac{3}{5}}$

$= (-2)^{3}x^{9}y^{6}$

$= -8x^{9}y^{6}$

4) Rewrite in ~~radical~~ exponential form and simplify.

$$\left(\sqrt[6]{27a^{3}b^{4}}\right)^{2}$$

$= \left[(27a^{3}b^{4})^{\frac{1}{6}}\right]^{2}$

$= (27a^{3}b^{4})^{\frac{1}{3}}$

$= \sqrt[3]{27}\, a^{3\times\frac{1}{3}}\, b^{4\times\frac{1}{3}}$

$= 3ab^{\frac{4}{3}}$ OR $3a\sqrt[3]{b^{4}}$

5) Solve.

$$3^{2k} = 243$$

$3^{2k} = 3^5$

$2k = 5$

$k = \frac{5}{2}$

B: Exponential Functions

1. List the transformations in the order they must be applied.

$\frac{1}{4}(x+4)$

$$f(x) = -\left(\frac{1}{3}\right)^{\left(\frac{1}{4}x+1\right)} - 1$$

1. reflection in x-axis
2. hor. stretch factor 4
3. shift left 4
4. shift down 1

2. Identify each table of values as linear, quadratic, or exponential. Show calculations to help explain/support your answer. For the exponential function(s) state whether it is growth or decay AND determine the equation.

x	y
-2	5.75
-1	5.3
0	4.85
1	4.4
2	3.95

First Differences: -0.45, -0.45, -0.45, -0.45

∴ linear

4^x
$\frac{1}{16}$
$\frac{1}{4}$
1
4
16

x	y	FIRST diff	RATIOS
-2	5.0625	$\frac{3}{16}$	$\frac{3}{4} \div \frac{3}{16} = 4$
-1	5.25	$\frac{3}{4}$	$3 \div \frac{3}{4} = 4$
0	6	3	$\frac{12}{3} = 4$
1	9	12	
2	21		

$5\frac{1}{4}$ $5\frac{1}{16}$

Common ratio base of exponential GROWTH is 4

$f(x) = 4^x$

$y = 4^x + 5$

3. For $y = \frac{1}{2}(4)^{-x} + 2$

State the base/parent function $g(x) = 4^x$

State the transformations in the order that they that must be applied

1. reflection in y-axis
2. Vert. Comp. factor $\frac{1}{2}$
3. shift up 2 UNITS (2 units is 1 'box' ∵ scale is counting by 2's).

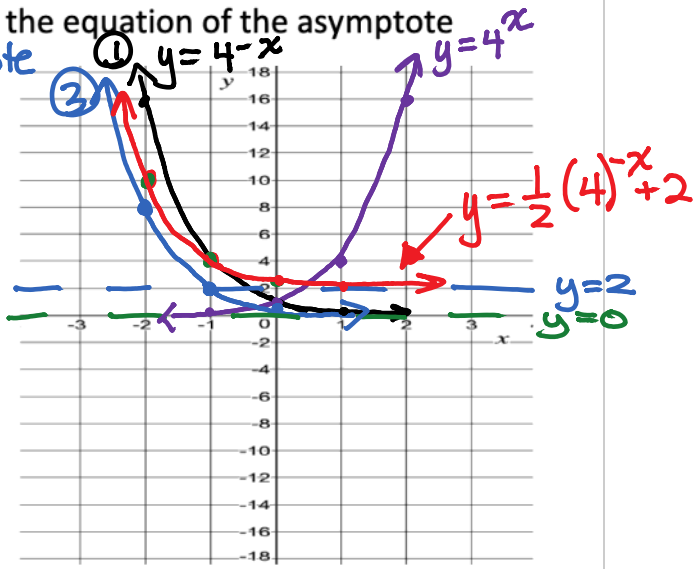
State the x and y-intercepts, and the equation of the asymptote

* y-int $y = \frac{1}{2}(4)^0 + 2$

$y = 2.5$

x-int NONE

Asymptote $y = 2$



Graph the new function

State the domain and range

D: $\{x \in \mathbb{R}\}$

R: $\{y > 2\}$

Is the function increasing or decreasing?

decreasing.

4. The town of Vanessa is growing exponentially at a rate of 4.5% each year. Growth Factor 1.045

a) If the population of Vanessa is now 15 000, how many people will be living there in 42 months?

$t = 42$

there in 42 months?)

$P(t) = 15000(1.045)^t$ $\quad t = \frac{42}{12}$

$P(\frac{42}{12}) = 15000(1.045)^{(42 \div 12)}$

NOTE: t, time in years, P(t) population

$P(\frac{42}{12}) = 17498.397...$

∴ we would expect the population to be 17498.

b) How many years would it take for the population to quadruple? (accurate to nearest tenth of a year)

number of months.

$15000(1.045)^t = 60000$

t	P(t)
20	36175.
30	56179.7
31	58707 ←
32	61349
31.5	60014
31.4	59750

31.5 years is 31 years, 6 months

∴ it would take 31 years, 6 months to quadruple.

5. A 500g sample of plutonium-243 has a half-life of 12 days.

a) Determine an equation to model this situation.

$M(t) = 500\left(\frac{1}{2}\right)^{t/12}$, where $M(t)$ is the mass in grams, after t days.

b) Determine how many grams of plutonium-243 remain after 6 weeks.

$t = 6 \times 7$

$t = 42$ days

$M(42) = 500\left(\frac{1}{2}\right)^{\frac{42}{12}}$

$M(42) \doteq 44$

$\therefore$ 44g would remain.

c) Determine how long it would take for only one-quarter of the original sample to remain. (accurate to the nearest day)

$500\left(\frac{1}{2}\right)^{\frac{t}{12}} = 125$

t	$M(t)$
12	250
24	125

OR

$\left(\frac{1}{2}\right)^{\frac{t}{12}} = \frac{1}{4}$

$\left(\frac{1}{2}\right)^{\frac{t}{12}} = \left(\frac{1}{2}\right)^{2}$

$\frac{t}{12} = 2$

$t = 24$

$\therefore$ it will take 24 days.