U4D5_T Determining Equations of Exponential Functions

Monday, April 8, 2019 9:07 PM

U4D5_T Determini...

U4D5 MCR3UI

Graphing Exponential Functions and Determining Exponential Equations of the form y=a(b)^x

Warm Up:

Simplify.

a) $\left(\dfrac{2x^2}{yz^3}\right)^2\left(\dfrac{y^2z^3}{2x^4}\right)^3$

$= \dfrac{(2)^2(x^2)^2}{(y)^2(z^3)^2} \times \dfrac{(y^2)^3(z^3)^3}{(2)^3(x^4)^3}$

$= \dfrac{4x^4y^6z^9}{8x^{12}y^2z^6}$

$= \dfrac{y^4z^3}{2x^8}$

b) $81^{-\frac{1}{2}} \div 27^{-\frac{2}{3}}$

multiply by the reciprocal

take the reciprocal of the base.

$= \dfrac{1}{\sqrt{81}} \times 27^{\frac{2}{3}}$

$= \dfrac{(\sqrt[3]{27})^2}{9}$

$= \dfrac{3^2}{9}$

$= 1$

long way

$\dfrac{1}{\sqrt{81}} \div \dfrac{1}{(\sqrt[3]{27})^2}$

$= \dfrac{1}{\sqrt{81}} \times \dfrac{(\sqrt[3]{27})^2}{1}$

c) $\dfrac{(y^{x-1})(y^{2x+5})}{y^{3x-1}}$

$= \dfrac{y^{(x-1)+(2x+5)}}{y^{(3x-1)}}$

$= y^{3x+4-(3x-1)}$

$= y^5$

d) $y = \frac{1}{8}(2)^{n-1}$

$y = (2)^{-3}(2)^{n-1}$

$y = 2^{-3+n-1}$

$y = 2^{n-4}$

e) $y = 12(3)^{n+2}$

$y = 4(3)^1(3)^{n+2}$

$y = 4(3)^{n+3}$

f) $y = \frac{(2)^{n-1}(4)^n}{(8)^{n-4}}$

$y = \frac{2^{n-1}(2^2)^n}{(2^3)^{n-4}}$

$y = \frac{2^{n-1+2n}}{2^{3n-12}}$

$y = 2^{3n-1-(3n-12)}$

$y = 2^{11}$

$y = 2048.$

Graphing Base Exponential Functions

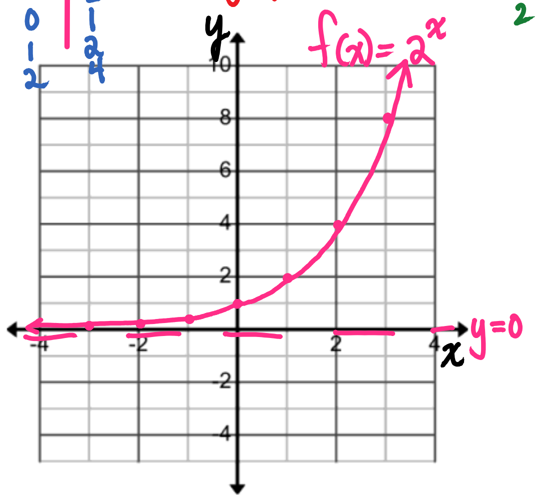
$f(x) = 2^x$ $\qquad$ $g(x) = \left(\frac{1}{3}\right)^x$

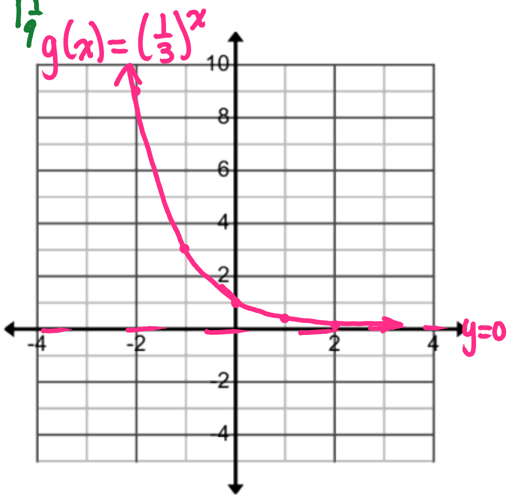
x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

get so you can graph these without a table.

x	y
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$

note: $\left(\frac{1}{3}\right)^x$ is 3^{-x}





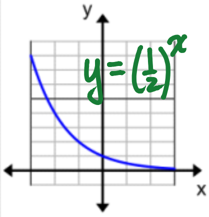
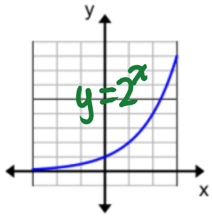
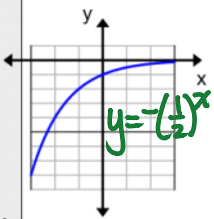
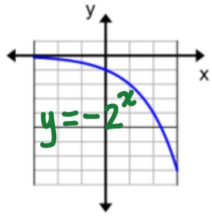
Determining the Equation of an Exponential Function

1. Complete the chart to compare the effect of changing the value of a in $y=a(2^x)$.

	$f(x)=2^x$	$y=3(2)^x$	$y=0.5(2)^x$	$y=-(2)^x$	$y=-3(2)^x$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y > 0$	$y > 0$	$y > 0$	$y < 0$	$y < 0$
y-intercept	$y=1$ or $(0,1)$	$y=3$ or $(0,3)$	$y=0.5$ OR $(0,0.5)$	$y=-1$ or $(0,-1)$	$y=-3$ or $(0,-3)$
asymptote	$y=0$	$y=0$	$y=0$	$y=0$	$y=0$
Inc./dec.	Increasing	Inc.	Inc.	Dec.	Decreasing

$\underbrace{y=3(2)^x,\ y=0.5(2)^x}_{b>1,\ a>0}$ $\qquad$ $\underbrace{y=-(2)^x,\ y=-3(2)^x}_{b>1,\ a<0}$

2. Summary: Exponential Equations of the form $y=a(b)^x$

	$0 < b < 1$	$b > 1$
$a > 0$ (i.e. a is positive)	 Decreasing on $x \in R$ D={ $x \in \mathbb{R}$ } R={ $y > 0$ } Horizontal Asymptote: $y=0$ y-intercept: y= a E.g. $y=2(\frac{1}{3})^x$ $a>0$ $0<b<1$	 Increasing on $x \in R$ D={ $x \in \mathbb{R}$ } R={ $y > 0$ } Horizontal Asymptote: $y=0$ y-intercept: y= a E.g. $y=3(2)^x$ $a>0$ $b>1$
$a < 0$ (i.e. a is negative)	 Increasing on $x \in R$ D={ $x \in \mathbb{R}$ } R={ $y < 0$ } Horizontal Asymptote: $y=0$ y-intercept: y= a E.g. $y=-2(\frac{3}{4})^x$ $a<0$ $0<b<1$	 Decreasing on $x \in R$ D={ $x \in \mathbb{R}$ } R={ $y < 0$ } Horizontal Asymptote: $y=0$ y-intercept: y= a E.g. $y=-3(2)^x$ $a<0$ $b>1$

"for all"

$y = (-1)^x$ Not | $\boxed{b>0}$ $\forall$ exponential functions.

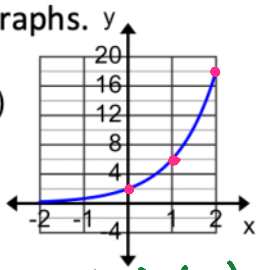
x	y
-2	1
-1	-1
0	1
1	-1
2	1
0.5	DNE

yuck!

You will not be dealing with this mess!

3. Determine the exponential equation in the form $y=a(b)^x$, for the given graphs.

a)



$(0,2)\ (1,6)$

$\times 3 = b$

a

$a = 2$ (y-int)

x	y
0	2
1	6
2	18

$> \times 3$

$> \times 3$

$a = 2, b = 3$

$y = a(b)^x$

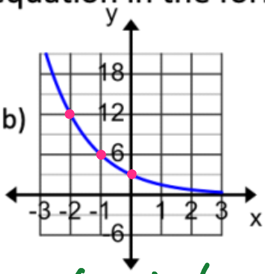
$6 = 2(b)^1$

$3 = b$

$a = 2, b = 3$

$y = 2(3)^x$

b)



$(-1,6)\ (0,3)$

$\times \frac{3}{6} = \frac{1}{2}$ $b = \frac{1}{2}$

$a = 3$

x	y
-2	12
-1	6
0	3

$> \times \frac{1}{2}$

$> \times \frac{1}{2}$

$a = 3, b = \frac{1}{2}$

$y = a(b)^x$

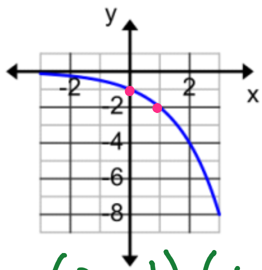
$6 = 3(b)^{-1}$

$(2)^{-1} = (b^{-1})^{-1}$

$\frac{1}{2} = b$, $a = 3$

$\boxed{y = 3\left(\frac{1}{2}\right)^x}$

c)



$(0,-1)\ (1,-2)$

$a = -1$

$\times 2$

$y = -(2)^x$

OR

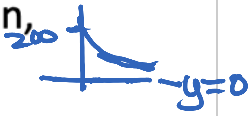
$y = -2^x$

4. Write an Exponential Function given the properties within each situation below:

i) A bacteria colony doubles every hour. The initial population contained 5 bacteria. Write a function to relate the population of bacteria to the time, in hours.

$P(t) = 5(2)^t$, where P is the population of bacteria, t is the time in hours.

ii) A radioactive sample has a half-life of 3 days. The initial sample is 200 mg. Write a function to relate the amount remaining, in milligrams, to the time, in days. Then, determine the range for the radioactive sample.



time (days)	Number of half-life time periods	Population mg
0	0	200
3	1	100
6	2	50

$P(t) = 200\left(\frac{1}{2}\right)^{t/3}$

where P is the number of mg of the sample remaining, t is the number of days.

$y = a\left(\frac{1}{2}\right)^{\frac{t}{h}}$ ← the time it takes to halve in same units as t.

Range: $\{0 < P \leq 200\}$