

U4D4_T Exploring Exponential Functions

Monday, April 8, 2019 1:37 PM



U4D4_T
Exploring ...

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Warm Up: Then QUIZ

a) $(2a^2bc^3)(-6a^4bc)^{-2}$ b) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$ c) $\left(\sqrt{\sqrt[7]{\sqrt{x^{\frac{5}{6}}}}}\right)^{21}$

U4D4_MCR3UI

Exploring Properties of Exponential Functions

Investigation:

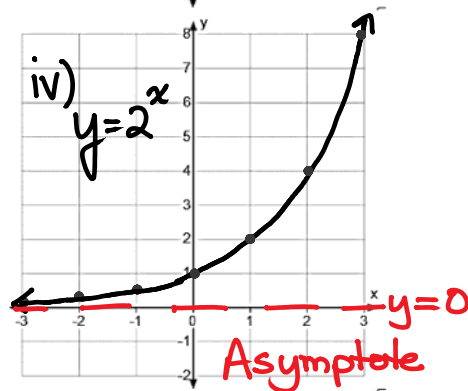
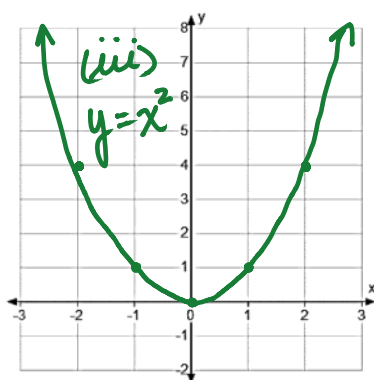
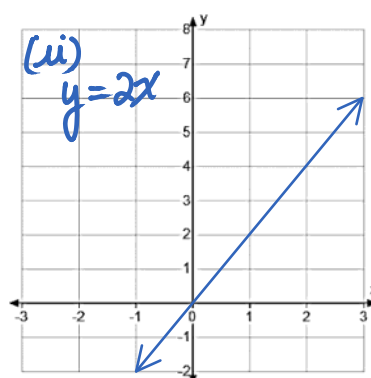
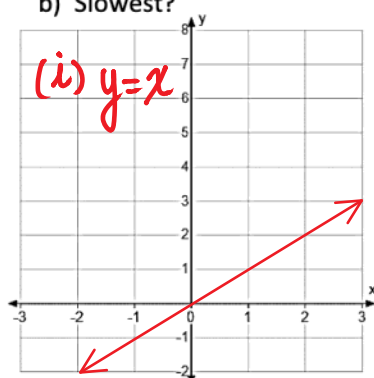
1. Complete the following tables.

i)	x	y=x	ii)	x	y=2x	iii)	x	y=x ²	iv)	x	y=2 ^x
	0	0		0	0		0	0		0	1
	1	1		1	2		1	1		1	2
	2	2		2	4		2	4		2	4
	3	3		3	6		3	9		3	8
	4	4		4	8		4	16		4	16
	5	5		5	10		5	25		5	32
	6	6		6	12		6	36		6	64

2. Which pattern is growing:

a) Fastest?

b) Slowest?



3. Complete the First and second differences.

x	y=x	First Differences	Second Differences
0	0		
1	1	1	
2	2	1	0
3	3	1	0
4	4	1	0
5	5	1	0
6	6	1	0

constant (linear)

x	y=2x	First Differences	Second Differences
0	0		
1	2	2	0
2	4	2	0
3	6	2	0
4	8	2	0
5	10	2	0
6	12	2	0

constant (linear)

x	y=x ²	First Differences	Second Differences
0	0		
1	1	1	2
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2
6	36	11	2

constant (quadratic)

x	y=2 ^x	First Differences	Second Differences
0	1		
1	2	1	1
2	4	2	2
3	8	4	4
4	16	8	8
5	32	16	16
6	64	32	32

$\frac{2}{1} = 2$
 $\frac{4}{2} = 2$
 Common Ratio $\downarrow 2$
 C.R. 2

Find the ratio of consecutive terms in any finite difference column. If there is a common ratio, the function is exponential.

4. What do you notice about the finite differences?

4. What do you notice about the finite differences?

When **FIRST** differences are constant the graph is **LINEAR**.

When **SECOND** differences are constant the graph is

QUADRATIC.

When **THE RATIO** of consecutive terms in any finite difference column is constant the graph is **EXPONENTIAL**.

(when there is a common ratio, the graph is exponential)

5. Complete the following tables.

i)
ii)

x	$y=3^x$	First Differences	Second Differences
0	1		
1	3	2	4
2	9	6	12
3	27	18	36
4	81	54	

Handwritten notes for table i):
 $6 \div 3 = 3$
 $18 \div 6 = 3$
 $54 \div 18 = 3$
 $\frac{12}{4} = 3$
 $\frac{36}{12} = 3$
 C.R. 3

x	$y=0.5^x$	First Differences	Second Differences
0	1		
1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{4}$
2	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{8}$
3	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{16}$
4	$\frac{1}{16}$	$-\frac{1}{16}$	

Handwritten notes for table ii):
 $\frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$
 $\frac{1}{16} \div \frac{1}{8} = \frac{1}{2}$
 $\frac{1}{16} - \frac{1}{8} = -\frac{1}{16}$
 $\frac{1}{16} - \frac{1}{16} = -\frac{1}{16}$
 C.R. $\frac{1}{2}$

Common Ratio 3.

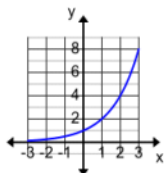
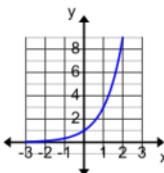
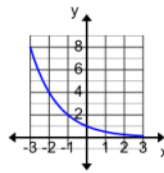
6. How do $y = 3^x$ and $y = 0.5^x$ compare with $y = 2^x$?

$y = 3^x$ is growing faster than $y = 2^x$
 $y = 0.5^x$ is getting smaller as x increases.

7. Complete the following chart.

Note: Interval of increase means...For what x-values is the graph increasing?

Interval of decrease means...For what x-values is the graph decreasing?

	$y=2^x$	$y=3^x$	$y=0.5^x$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y > 0$	$y > 0$	$y > 0$
x-intercepts?	<hr/>	<hr/>	<hr/>
y-intercept	(0,1) $y=1$	(0,1)	(0,1)
Interval of increase	$x \in \mathbb{R}$	$x \in \mathbb{R}$	<hr/>
Interval of decrease	<hr/>	<hr/>	$x \in \mathbb{R}$
Description of graph	<ul style="list-style-type: none"> • always increasing 	<ul style="list-style-type: none"> • always increasing • grows faster than $y=2^x$ 	<ul style="list-style-type: none"> • always decreasing • reflection of $y=2^x$ in y-axis. <p>NOTE: $2^{-x} = \left(\frac{1}{2}\right)^x = 0.5^x$</p>
Sketch of graph			
Asymptotes ?	$y=0$	$y=0$	$y=0$

8. Sam's mom told him that if he consistently does all of his chores, each day she will give him double the amount that was given the previous day. She gives him \$0.50 the first day.

(a) Assuming Sam does his chores consistently, how much money will his mom give him on the fourth day?

Day 1	\$0.50
Day 2	\$1
Day 3	\$2
Day 4	\$4

\$4 on fourth day.

(b) Sam is saving up to buy a new \$300 graphics card for his computer. On what day can he buy his graphics card?

Day	Mom gives Sam	Sam's Total
Day 1	\$0.50	\$0.50
Day 2	1	1.50
Day 3	2	3.50
Day 4	4	7.50

8 15.50
16 31.50
32 63.50

64 127.50
128 255.50
256 over 300.

On 10th day he can buy his graphics card.

Properties of Exponential Functions:

NOTE: (Base must be positive)

- As the independent variable increases by a constant amount, the dependent variable increases by a **common factor**.
(As the independent variable increases by one, the dependent variable increases by a **common factor** equal to the **base** of the exponential function.)
- The **ratio** of consecutive finite differences is a constant.
- For bases **greater** than 1, the graph's **slope increases** at a constant rate (the slope of the graph gets steeper as x increases)
- For bases **between** 0 and 1, the graph's **slope decreases** at a constant rate (the slope of the graph gets less steep as x increases)
- $b^0 = 1, \forall b \in \mathbb{R}, b \neq 0$