U4D3_T Solving Exponential Equations

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U4D3_T Solving Ex...

U4D3 MCR3UI **Solving Exponential Equations**

Warm Up: Simplify.

a) $3^x \cdot 3^4$

$= 3^{x+4}$

b) $\sqrt[2]{\sqrt[5]{x^4y^6}}$

$= \left[(x^4y^6)^{\frac{1}{5}}\right]^{\frac{1}{2}}$

$= (x^4)^{\frac{1}{10}}(y^6)^{\frac{1}{10}}$

exponential form

$= x^{\frac{2}{5}}y^{\frac{3}{5}}$

OR

radical form

$\sqrt[5]{x^2y^3}$

c) $\left(\frac{125}{27}\right)^{-\frac{2}{3}}$

$= \left(\frac{27}{125}\right)^{\frac{2}{3}}$

$= \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{125})^2}$

$= \frac{3^2}{5^2}$

$= \frac{9}{25}$

Solving Exponential Equations

Method 1: Using a common base

If there is a common base, you can equate the exponents. This gives a linear equation that you can solve.

a) $4^x = 4^5$

$\boxed{x = 5}$

b) $2^{x+3} = 2^{2x-1}$

$x + 3 = 2x - 1$

$x - 2x = -1 - 3$

$-x = -4$

$\boxed{x = 4}$

x	2^{x+3}	2^{2x-1}
1	2^4	2^1
2	2^5	2^3
⋮		
4	2^7	2^7

$\therefore x = 4$

Method 1 con't: If the bases are NOT the same, you can either make them the same (This is not required for the grade 11 curriculum) OR

Method 2: you can use a table of values to figure out the value of the unknown (trial and error).

c) $3^x = 27$

$3^x = 3^3$

$\boxed{x = 3}$

d) $4^{3k} = 64$

note: $64 = 4^3$

Method 1:

$4^{3k} = 4^3$ ← common base 4

equate exponents

$3k = 3$

solve for k.

$\boxed{k = 1}$

Method 2:

k	4^{3k}
0	$4^0 = 1$
1	$4^3 = \boxed{64}$
2	
3	

$\therefore \boxed{k = 1}$

e) $4^x = 8^{x-1}$

rewrite 4 & 8 as powers of 2

Method 1:

$(2^2)^x = (2^3)^{x-1}$

$2^{2x} = 2^{3x-3}$) use exponent rules to simplify

$2x = 3x - 3$) equate exponents

$\boxed{x = 3}$) isolate x

Method 2:

x	4^x	8^{x-1}
0	4^0	8^{-1}
5	$4^5 = 1024$	$8^4 = 4096$
$\boxed{3}$	$4^3 = 64$	$8^2 = 64$

$\therefore \boxed{x = 3}$

Examples Involving Rationals

a) $3^{3x-1} = \frac{1}{81}$ $\frac{1}{81} = 3^{-4}$

$3^{3x-1} = 3^{-4}$

$3x - 1 = -4$

$3x = -3$

$\boxed{x = -1}$

x	3^{3x-1}
0	3^{-1}
$\boxed{-1}$	$3^{-3-1} = 3^{-4} = \frac{1}{81}$
-2	

$\therefore \boxed{x = -1}$

b) $27(3^{3x+1}) = 9$) $\div 27$

$3^{3x+1} = \frac{9}{27}$

$3^{3x+1} = \frac{1}{3}$

$3^{3x+1} = 3^{-1}$

$3x + 1 = -1$

$3x = -2$

$\boxed{x = -\frac{2}{3}}$

OR

$27(3^{3x+1}) = 9$

$3^3(3^{3x+1}) = 3^2$

$3^{3x+1+3} = 3^2$

$3^{3x+4} = 3^2$

c) $2(5^{k+1}) = 1250$) $\div 2$

$5^{k+1} = 625$

$5^{k+1} = 5^4$

k	$2(5^{k+1})$
1	$2(5^2) = 50$

$$5^{k+1} = 5^4$$
$$k+1 = 4$$
$$k = 3$$

k	$2(5^{k+1})$
1	$2(5^2)=50$
2	$2(5^3)=250$
$\boxed{3}$	$2(5^4)=1250$ ✓

$\therefore \boxed{k=3}$

$$3^{3x+4} = 3^2$$
$$3x+4 = 2$$
$$3x = -2$$
$$\boxed{x = -\frac{2}{3}}$$

Example Involving Common Factor

$3^{x+2} - 3^x = 216$

Common Factor

$$3^x(3^2 - 3^0) = 216$$

Simplify

$$3^x(9-1) = 216$$
$$3^x(8) = 216$$

$\div 8$ $\div 8$

$$3^x = 27$$
$$3^x = 3^3$$
$$\boxed{x = 3}$$

RECALL:

$$x^5 - x^2$$
$$= x^2(x^3 - 1)$$

x	$3^{x+2} - 3^x$
1	$3^3 - 3^1 = 27 - 1$ ✗
2	$3^4 - 3^2 = 81 - 9$ ✗
3	$3^5 - 3^3 = 243 - 27 = 216$ ✓

$\therefore x = 3$