

# U4D2\_T Rational Exponents

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U4D2\_T  
Rational E...

U4D2 MCR3UI

## Rational Exponents

$$\begin{array}{l} \text{exponent} \\ \rightarrow m \\ a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m \\ \text{exponential form} \qquad \qquad \qquad \text{radical form} \end{array}$$

Think of  $m$  as the exponent and  
 $n$  as the root.

To Evaluate:

Either:

- Take the 'nth' root of 'a' and then raise the answer to the exponent 'm'

OR

- Raise 'a' to the exponent 'm' and then take the 'nth' root of the answer

\*\*\*Remember all exponent laws apply when simplifying rational exponents.\*\*\*

Example 1: Evaluate....do not use a calculator!

a)  $25^{\frac{3}{2}}$   
 $= (\sqrt{25})^3$   
 $= 5^3$   
 $= 125$

note:  
 $\sqrt{\quad}$  is written  
 $\sqrt{\quad}$

b)  $(-27)^{-\frac{1}{3}}$  *flip the base of the power*  
 $= \frac{1}{\sqrt[3]{-27}}$   
 $= \frac{1}{-3}$   
 $= -\frac{1}{3}$

*3  $\sqrt[3]{-27}$   
 $-27 \times \frac{1}{3}$*

**NOTE:**  
 $(\frac{125}{8})^{-\frac{1}{3}}$   
 $= \sqrt[3]{\frac{8}{125}}$   
 $= \frac{\sqrt[3]{8}}{\sqrt[3]{125}}$   
 $= \frac{2}{5}$

OR  
 $(-27^{\frac{1}{3}})^{-1}$   
 $(\sqrt[3]{-27})^{-1}$

\* note: you CAN take the 'odd' root of a negative number.

c)  $-9^{2.5}$   
 $= -9^{\frac{5}{2}}$   
 $= -(9^{\frac{5}{2}})$   
 $= -(\sqrt{9})^5$   
 $= -(3^5)$   
 $= -243$

d)  $4^{\frac{3}{2}} \div 16^{\frac{1}{4}}$   
 $= (\sqrt{4})^3 \div \sqrt[4]{16}$   
 $= 2^3 \div 2$   
 $= 2^2$   
 $= 4$

$(4^2)^{\frac{1}{4}}$   
 $= 4^{\frac{2}{4}}$   
 $= 4^{\frac{1}{2}}$   
 $= \sqrt{4}$   
 $= 2$

note:  $(-9)^{\frac{5}{2}}$   
 has no real solution

Example: Write using exponents, in fully simplified form.

$$\begin{aligned}
 \text{a) } & \sqrt[3]{\sqrt{2x^4}} \\
 &= \left[ (2x^4)^{\frac{1}{2}} \right]^{\frac{1}{3}} \\
 &= (2x^4)^{\frac{1}{6}} \\
 &= (2)^{\frac{1}{6}} (x^4)^{\frac{1}{6}} \\
 &= 2^{\frac{1}{6}} x^{\frac{2}{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (\sqrt[3]{a^2b^4})^5 \\
 &= \left[ (a^2b^4)^{\frac{1}{3}} \right]^5 \\
 &= (a^2b^4)^{\frac{5}{3}} \\
 &= (a^{\frac{10}{3}}) (b^{\frac{20}{3}}) \\
 &= a^{\frac{10}{3}} b^{\frac{20}{3}}
 \end{aligned}$$



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**Simplifying Expressions Using Exponent Laws**

- Simplify
  - $5a^{-3} \times 8a^{-9}$
  - $-24c^5d^3 \div 4c^8d^{-3}$
  - $m^2n^5 \times m^3n^{-7}$
  - $\left(\frac{24c^8d^5}{-8c^2d}\right)\left(\frac{15c^3d^9}{18cd^5}\right)$
  - $\frac{12m^5n^{-2} \times 5m^{-11}n^6}{15m^3n^{-4}}$
  - $(xy^3)^6 \div (x^2y^4)^8$
- Write in radical form, then evaluate.
  - $81^{\frac{3}{4}}$
  - $16^{\frac{-3}{4}}$
  - $625^{0.75}$
  - $4^{\frac{3}{2}}$
  - $8^{\frac{4}{3}}$
- Evaluate. Do not convert fraction answers to decimals.
  - $\left(\frac{1}{9}\right)^{\frac{3}{2}}$
  - $\left(-\frac{1}{32}\right)^{0.8}$
  - $\left(\frac{49}{25}\right)^{\frac{1}{2}}$
  - $\left(-\frac{27}{125}\right)^{\frac{4}{3}}$
  - $\left(\frac{625}{343}\right)^0$
- Evaluate.
  - $32^{\frac{2}{5}} \times 243^{\frac{2}{5}}$
  - $64^{\frac{2}{3}} \times 125^{\frac{1}{3}}$
  - $4^{\frac{5}{2}} \times 81^{\frac{3}{4}}$
- Simplify.
  - $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$
  - $\left(n^{\frac{1}{2}}\right)^{-6}$
  - $x^{\frac{-3}{2}} \div x^{\frac{1}{4}}$
  - $(9a^4b^{-2} \times 4a^2b^{-6})^{\frac{1}{2}}$
  - $8m^{\frac{1}{3}}n^{\frac{-3}{2}}(-2m^{\frac{-2}{3}}n^{\frac{1}{3}})^{-4}$
- Simplify.
  - $\frac{36x^{-2}y^3z^{-4}}{12xy^{-2}z^{-2}}$
  - $\sqrt{\frac{32x^{-5}y^2 \times 18x^2y}{4xy^{-3}}}$
  - $\left(\frac{3x^{-2}y^3}{12xy^{-1}}\right)\left(\frac{10x^4y^{-2}}{5x^{-1}y^2}\right)$
  - $\frac{8^{1-2x} \times 4^{2x+3}}{16^{2-3x}}$
  - $\frac{16^{2m-n} \times 9^{m+3n}}{27^{m+n} \times 8^{m-n}}$
  - $\frac{5^{-200} - 5^{-198}}{5^{-199} + 5^{-200}}$
- Simplify.
  - $\frac{(c^{a+b})(c^{a-b})}{c^2}$
  - $\frac{(x^a)^2(x^b)^2}{(x^{a+b})(x^{a-b})}$
  - $\frac{x^{2a-b} \cdot x^{a-3b}}{(x^{3a+b})^{-2}}$
  - $\frac{(m^{x-1})(m^{2x+5})}{m^{3x-1}}$
  - $\frac{3^{-6a+3-5a}}{3^{-6a+3-7a}}$
- Evaluate.
  - $(5^{\frac{1}{2}} + 2^{\frac{1}{2}})(5^{\frac{1}{2}} - 2^{\frac{1}{2}})$
  - $(8^{\frac{2}{3}} - 5^{\frac{1}{2}})(8^{\frac{2}{3}} + 5^{\frac{1}{2}})$
- Simplify.
  - $\left(\sqrt[3]{49y^{\frac{2}{m}}}\right)^{-1}$
  - $\sqrt[3]{\frac{m^2\sqrt{mn}}{\sqrt{n}}}$
  - $\left(\frac{\sqrt[4]{a^{2n-1}} \times \sqrt[4]{a}}{\sqrt{a}}\right)^2$

**ANSWERS:**

- 1a)  $\frac{40}{a^{12}}$  b)  $\frac{-6d^6}{c^3}$  c)  $\frac{m^5}{n^2}$  d)  $\frac{-5c^8d^8}{2}$  e)  $\frac{4n^8}{m^9}$  f)  $x^2y^2$       2a) 27 b)  $\frac{1}{8}$  c) 125 d)  $\frac{1}{8}$  e) 16
- 3a) 27 b)  $\frac{1}{16}$  c)  $\frac{7}{5}$  d)  $\frac{81}{625}$  e) 1      4a) 36 b) 80 c) 864
- 5a) 1 b)  $\frac{1}{n^3}$  c)  $\frac{1}{x^4}$  d)  $\frac{6a^3}{b^4}$  e)  $\frac{m^3}{2n^6}$       6a)  $\frac{3y^5}{x^3z^2}$  b)  $\frac{12y^3}{x^2}$  c)  $\frac{x^2}{2}$  d)  $2^{10x+1}$  e)  $2^{5m-n}3^{3n-m}$  f) -4
- 7a)  $c^{2a-2}$  b)  $x^{2b}$  c)  $x^{9a-2b}$  d)  $m^5$  e)  $3^a$       8 a) 3 b) 11      9a)  $\frac{1}{7^n y^{mn}}$  b)  $m^{\frac{1}{3}}n^{\frac{1}{3}}$  c)  $a^{n-1}$



**Simplifying Expressions Using Exponent Laws**

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  - $\left(\frac{24c^8d^5}{-8c^2d}\right)\left(\frac{15c^3d^9}{18cd^5}\right)$
  - $\frac{12m^5n^{-2} \times 5m^{-11}n^6}{15m^3n^{-4}}$
  - $(xy^{\frac{2}{3}})^6 \div (x^{\frac{1}{2}}y^{\frac{1}{4}})^8$
- Write in radical form, then evaluate.
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  - $64^{\frac{2}{3}} \times 125^{\frac{1}{3}}$
  - $4^{\frac{5}{2}} \times 81^{\frac{3}{4}}$
- Simplify.
  - $a^{\frac{1}{2}} \times a^{-\frac{1}{2}}$
  - $\left(n^{\frac{1}{2}}\right)^{-6}$
  - $x^{\frac{3}{2}} \div x^{-\frac{1}{4}}$
  - $(9a^4b^{-2} \times 4a^2b^{-6})^{\frac{1}{2}}$
  - $8m^{\frac{1}{3}}n^{-\frac{3}{2}}(-2m^{-\frac{2}{3}}n^{\frac{1}{3}})^{-4}$
- Simplify.
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  - $\frac{(m^{x-1})(m^{2x+5})}{m^{3x-1}}$
  - $\frac{3^{-6a+3-5a}}{3^{-6a+3-7a}}$
- Evaluate.
  - $(5^{\frac{1}{2}} + 2^{\frac{1}{2}})(5^{\frac{1}{2}} - 2^{\frac{1}{2}})$
  - $(8^{\frac{2}{3}} - 5^{\frac{1}{2}})(8^{\frac{2}{3}} + 5^{\frac{1}{2}})$
- Simplify.
  - $\left(\sqrt{49y^{\frac{2}{m}}}\right)^{-\frac{1}{n}}$
  - $\sqrt[3]{\frac{m^{\frac{1}{2}}\sqrt{mn}}{\frac{1}{\sqrt{n}}}}$
  - $\left(\frac{\sqrt[4]{a^{2n-1}} \times \sqrt[4]{a}}{\sqrt{a}}\right)^2$

**ANSWERS:**

- 1a)  $\frac{40}{a^{12}}$  b)  $\frac{-6d^6}{c^3}$  c)  $\frac{m^5}{n^2}$  d)  $\frac{-5c^8d^8}{2}$  e)  $\frac{4n^8}{m^9}$  f)  $x^2y^2$  2a) 27 b)  $\frac{1}{8}$  c) 125 d)  $\frac{1}{8}$  e) 16
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- 5a) 1 b)  $\frac{1}{n^3}$  c)  $\frac{1}{x^4}$  d)  $\frac{6a^3}{b^4}$  e)  $\frac{m^3}{2n^6}$  6a)  $\frac{3y^5}{x^3z^2}$  b)  $\frac{12y^3}{x^2}$  c)  $\frac{x^2}{2}$  d)  $2^{10x+1}$  e)  $2^{5m-n}3^{3n-m}$  f) -4
- 7a)  $c^{2a-2}$  b)  $x^{2b}$  c)  $x^{9a-2b}$  d)  $m^5$  e)  $3^a$  8 a) 3 b) 11 9a)  $\frac{1}{7^n y^{mn}}$  b)  $m^{\frac{1}{3}}n^{\frac{1}{3}}$  c)  $a^{n-1}$

$$\begin{array}{lll}
 \text{1a) } 5a^{-3} \times 8a^{-9} & \text{b) } -24c^5d^3 \div 4c^8d^{-3} & \text{c) } m^2n^5 \times m^3n^{-7} \\
 = 40a^{-12} & = -6c^{-8}d^{3+3} & = m^5n^2 \\
 = \frac{40}{a^{12}} & = -\frac{6d^6}{c^8} & = \frac{m^5}{n^2}
 \end{array}$$

$$\begin{array}{ll}
 \text{d) } \left( \frac{24c^8d^5}{-8c^2d} \right) \left( \frac{15c^3d^9}{18cd^5} \right) & \text{e) } \frac{12m^5n^{-2} \times 5m^{-1}n^6}{15m^3n^{-4}} \\
 = \left( \frac{-5}{2} \right) (c^6d^4) (c^2d^4) & = \frac{4m^{-6}n^4}{m^3n^{-4}} \\
 = -\frac{5c^8d^8}{2} & = 4m^{-9}n^{4+4} \\
 & = \frac{4n^8}{m^9}
 \end{array}$$

$$\begin{array}{ll}
 \text{f) } (xy^{\frac{2}{3}})^6 \div (x^{\frac{1}{2}}y^{\frac{1}{4}})^8 & \text{2a) } 81^{\frac{3}{4}} \\
 = \frac{(x)^6 (y^{\frac{2}{3}})^6}{(x^{\frac{1}{2}})^8 (y^{\frac{1}{4}})^8} & = (\sqrt[4]{81})^3 \\
 = \frac{x^6 y^4}{x^4 y^2} & = 3^3 \\
 = x^2 y^2 & = 27
 \end{array}$$

$$\begin{array}{l}
 \text{2c) } 625^{0.75} \\
 = 625^{\frac{3}{4}} \\
 = (\sqrt[4]{625})^3 \\
 = 5^3 \\
 = 125
 \end{array}$$

$$\begin{array}{ll}
 \text{2d) } 4^{-\frac{3}{2}} & \text{e) } 8^{\frac{4}{3}} \\
 = \frac{1}{(\sqrt{4})^3} & = (\sqrt[3]{8})^4 \\
 = \frac{1}{2^3} & = 2^4 \\
 = \frac{1}{8} & = 16
 \end{array}$$

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$$3a) \left(\frac{1}{9}\right)^{-3/2}$$

$$= 9^{3/2}$$

$$= (\sqrt{9})^3$$

$$= 3^3$$

$$= 27$$

$$b) \left(-\frac{1}{32}\right)^{0.8}$$

$$= \left(-\frac{1}{32}\right)^{4/5}$$

$$= \frac{1}{(\sqrt[5]{-32})^4}$$

$$= \frac{1}{(-2)^4}$$

$$= \frac{1}{16}$$

$$c) \left(\frac{49}{25}\right)^{1/2}$$

$$= \frac{\sqrt{49}}{\sqrt{25}}$$

$$= \frac{7}{5}$$

$$d) \left(-\frac{27}{125}\right)^{4/3}$$

$$= \frac{(\sqrt[3]{-27})^4}{(\sqrt[3]{125})^4}$$

$$= \frac{(-3)^4}{5^4}$$

$$= \frac{81}{625}$$

$$3e) \left(\frac{625}{343}\right)^0$$

$$= 1$$

$$4a) 32^{2/5} \times 243^{2/5}$$

$$= (\sqrt[5]{32})^2 \times (\sqrt[5]{243})^2$$

$$= 2^2 \times 3^2$$

$$= 4 \times 9$$

$$= 36$$

$$b) 64^{2/3} \times 125^{1/3}$$

$$= (\sqrt[3]{64})^2 \times \sqrt[3]{125}$$

$$= 4^2 \times 5$$

$$= 16 \times 5$$

$$= 80$$

$$4c) 4^{5/2} \times 81^{3/4}$$

$$= (\sqrt{4})^5 \times (\sqrt[4]{81})^3$$

$$= 2^5 \times 3^3$$

$$= 32 \times 27$$

$$= 864$$

$$5a) a^{1/2} \times a^{-1/2}$$

$$= a^0$$

$$= 1$$

$$b) \left(n^{1/2}\right)^{-6}$$

$$= n^{-3}$$

$$= \frac{1}{n^3}$$

$$c) x^{-3/2} \div x^{-1/4}$$

$$= x^{-3/2 - (-1/4)}$$

$$= x^{-6/4 + 1/4}$$

$$= x^{-5/4}$$

$$= \frac{1}{(\sqrt[4]{x})^5}$$

$$\textcircled{R} \frac{1}{x^{5/4}}$$

$$\begin{aligned}
 5d) & (9a^4b^{-2} \times 4a^2b^{-6})^{\frac{1}{2}} \\
 &= (36a^6b^{-8})^{\frac{1}{2}} \\
 &= (\sqrt{36})(a^6)^{\frac{1}{2}}(b^{-8})^{\frac{1}{2}} \\
 &= 6a^3b^{-4} \\
 &= \frac{6a^3}{b^4}
 \end{aligned}$$

$$\begin{aligned}
 e) & 8m^{\frac{1}{3}}n^{-\frac{3}{2}}(-2m^{-\frac{2}{3}}n^{\frac{1}{3}})^{-4} \\
 &= \frac{8m^{\frac{1}{3}}n^{-\frac{3}{2}}}{(-2)^4(m^{-\frac{2}{3}})^4(n^{\frac{1}{3}})^4} \\
 &= \frac{8m^{\frac{1}{3}}n^{-\frac{3}{2}}}{16m^{-\frac{8}{3}}n^{\frac{4}{3}}} \\
 &= \frac{1}{2}m^{\frac{1}{3}+\frac{8}{3}}n^{-\frac{3}{2}-\frac{4}{3}} \\
 &= \frac{1}{2}m^{\frac{9}{3}}n^{-\frac{9}{6}-\frac{8}{6}} \\
 &= \frac{m^3}{2n^{\frac{17}{6}}}
 \end{aligned}$$

$$\begin{aligned}
 6a) & \frac{36x^{-2}y^3z^{-4}}{12xy^2z^{-2}} \\
 &= 3x^{-3}y^5z^{-2} \\
 &= \frac{3y^5}{x^3z^2}
 \end{aligned}$$

$$\begin{aligned}
 b) & \sqrt{\frac{32x^{-5}y^2 \times 18x^2y}{4xy^{-3}}} \\
 &= \sqrt{\frac{144x^{-3}y^3}{xy^{-3}}} \\
 &= \sqrt{144(x^{-4})^{\frac{1}{2}}(y^6)^{\frac{1}{2}}} \\
 &= 12x^{-2}y^3 \\
 &= \frac{12y^3}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 c) & \left(\frac{3x^{-2}y^3}{12xy^{-1}}\right)\left(\frac{10x^4y^{-2}}{5x^{-1}y^2}\right) \\
 &= \frac{1}{4}x^{-3}y^4(2)(x^5)(y^{-4}) \\
 &= \frac{x^2}{2}
 \end{aligned}$$

$$\begin{aligned}
 6d) & \frac{8^{1-2x} \times 4^{2x+3}}{16^{2-3x}} \\
 &= \frac{(2^3)^{1-2x} \times (2^2)^{2x+3}}{(2^4)^{2-3x}} \\
 &= \frac{2^{3-6x+4x+6}}{2^{8-12x}} \\
 &= 2^{9-2x-(8-12x)}
 \end{aligned}$$

$$\begin{aligned}
 &= 2^{1+10x} \\
 &\textcircled{2} 2^{10x+1} \\
 &= 2^{8m-4n-(3m-3n)} \times 3^{2m+6n-(3m+3n)} \\
 &= 2^{5m-n} \times 3^{-m+3n} \\
 &= 2^{5m-n} 3^{3n-m}
 \end{aligned}$$

$$\begin{aligned}
 6e) & \frac{16^{2m-n} \times 9^{m+3n}}{27^{m+n} \times 8^{m-n}} \\
 &= \frac{(2^4)^{2m-n} \times (3^2)^{m+3n}}{(3^3)^{m+n} \times (2^3)^{m-n}}
 \end{aligned}$$

$$\begin{aligned}
 &= 2^{8m-4n-(3m-3n)} \times 3^{2m+6n-(3m+3n)} \\
 &= 2^{5m-n} \times 3^{-m+3n} \\
 &= 2^{5m-n} 3^{3n-m}
 \end{aligned}$$



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$$\begin{aligned}
 \text{6f)} \quad & \frac{5^{-200} - 5^{-198}}{5^{-199} + 5^{-200}} \times \frac{5^{200}}{5^{200}} \\
 &= \frac{5^0 - 5^2}{5^1 + 5^0} \\
 &= \frac{1 - 25}{5 + 1} \\
 &= \frac{-24}{6} \\
 &= -4
 \end{aligned}$$

$$\begin{aligned}
 \text{7a)} \quad & \frac{(c^{a+b})(c^{a-b})}{c^2} \\
 &= c^{2a-2} \\
 \text{7b)} \quad & \frac{(x^a)^2 (x^b)^2}{(x^{a+b})(x^{a-b})} \\
 &= \frac{x^{2a} x^{2b}}{x^{2a}} \\
 &= x^{2b}
 \end{aligned}$$

$$\begin{aligned}
 \text{7c)} \quad & \frac{x^{2a-b} x^{a-3b}}{(x^{3a+b})^{-2}} \\
 &= \frac{x^{3a-4b}}{x^{-6a-2b}} \\
 &= x^{9a-2b}
 \end{aligned}$$

$$\begin{aligned}
 \text{7d)} \quad & \frac{m^{x-1} (m^{2x+5})}{m^{3x-1}} \\
 &= m^{x+2x-3x-1+5+1} \\
 &= m^5
 \end{aligned}$$

$$\begin{aligned}
 \text{7e)} \quad & \frac{3^{-6a} + 3^{-5a}}{3^{-6a} + 3^{-7a}} \\
 &= \frac{3^{-6a} (3^0 + 3^a)}{3^{-7a} (3^a + 3^0)} \\
 &= 3^{-6a+7a} \frac{(3^a+1)}{(3^a+1)} \\
 &= 3^a
 \end{aligned}$$

(OR)

$$\begin{aligned}
 & \frac{3^{-6a} + 3^{-5a}}{3^{-6a} + 3^{-7a}} \times \frac{3^{7a}}{3^{7a}} \\
 &= \frac{3^a + 3^{2a}}{3^a + 3^0} \\
 &= \frac{3^a (1 + 3^a)}{(1 + 3^a)} \\
 &= 3^a
 \end{aligned}$$

$$\begin{aligned}
 8a) & (5^{\frac{1}{2}} + 2^{\frac{1}{2}})(5^{\frac{1}{2}} - 2^{\frac{1}{2}}) \quad (\text{difference of squares}) \\
 & = (5^{\frac{1}{2}})(5^{\frac{1}{2}}) - (2^{\frac{1}{2}})(2^{\frac{1}{2}}) \\
 & = (5^{\frac{1}{2}})^2 - (2^{\frac{1}{2}})^2 \\
 & = 5 - 2 \\
 & = 3
 \end{aligned}$$

$$\begin{aligned}
 8b) & (8^{\frac{2}{3}} - 5^{\frac{1}{2}})(8^{\frac{2}{3}} + 5^{\frac{1}{2}}) \\
 & = (8^{\frac{2}{3}})^2 - (5^{\frac{1}{2}})^2 \\
 & = 8^{\frac{4}{3}} - 5^1 \\
 & = (\sqrt[3]{8})^4 - 5 \\
 & = 2^4 - 5 \\
 & = 16 - 5 \\
 & = 11
 \end{aligned}$$

$$\begin{aligned}
 9a) & \left( \sqrt[4]{49y^{\frac{2}{m}}} \right)^{-\frac{1}{n}} \\
 & = (\sqrt[4]{49})^{-\frac{1}{n}} \left[ (y^{\frac{2}{m}})^{\frac{1}{4}} \right]^{-\frac{1}{n}} \\
 & = 7^{-\frac{1}{n}} y^{-\frac{1}{mn}} \\
 & = \frac{1}{7^{\frac{1}{n}} y^{\frac{1}{mn}}}
 \end{aligned}$$

$$\begin{aligned}
 9b) & \sqrt[3]{\frac{m^{\frac{1}{2}} \sqrt{mn}}{\frac{1}{\sqrt{n}}}} \\
 & = \left[ \frac{m^{\frac{1}{2}} (m)^{\frac{1}{2}} (n)^{\frac{1}{2}}}{(n)^{-\frac{1}{2}}} \right]^{\frac{1}{3}} \\
 & = (mn)^{\frac{1}{3}} \\
 & = m^{\frac{1}{3}} n^{\frac{1}{3}}
 \end{aligned}$$

$$\begin{aligned}
 9c) & \left( \frac{\sqrt[4]{a^{2n-1}} \times \sqrt[4]{a}}{\sqrt{a}} \right)^2 \\
 & = \left( \frac{(a^{2n-1})^{\frac{1}{4}} \times a^{\frac{1}{4}}}{a^{\frac{1}{2}}} \right)^2 \\
 & = \frac{(a^{\frac{2n-1}{4}})^2 \times (a^{\frac{1}{4}})^2}{(a^{\frac{1}{2}})^2} = a^{n-\frac{1}{2}+\frac{1}{2}-1} \\
 & = \frac{(a^{\frac{2n-1}{2}})(a^{\frac{1}{2}})}{a^1} = a^{n-1}
 \end{aligned}$$