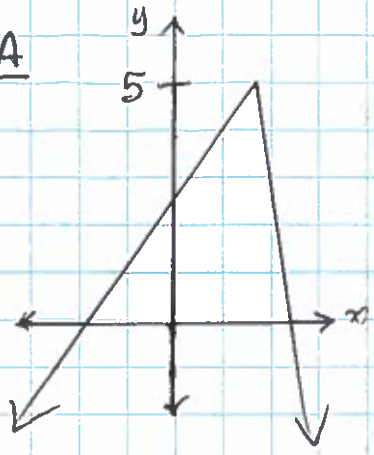
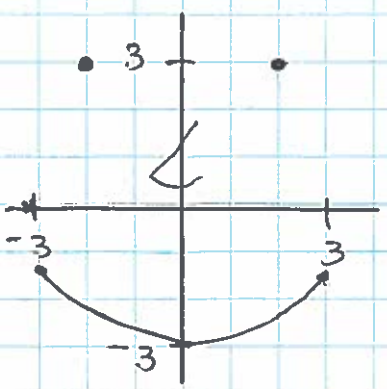
PART A

1a)

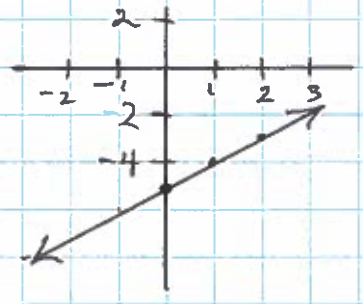


b)



2 a) $y = x - 5$ is a function (sloped lines are functions)

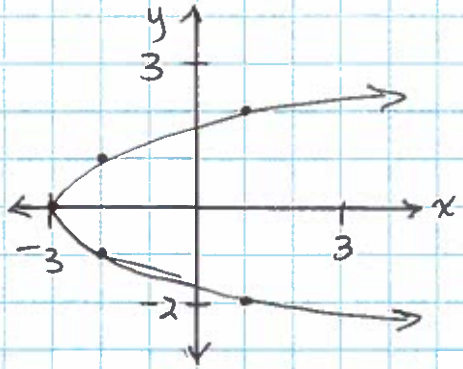
$m = 1, b = -5$



b) $x = y^2 - 3$ is not a function

x	y
-3	0
-2	1
-2	-1

← there are points lined up vertically.

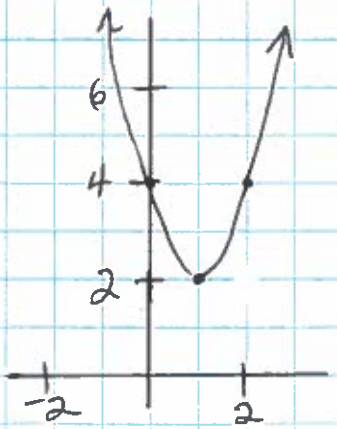


* To graph use Desmos, if necessary.
(A parabola on its side is not a function).

c) $y = 2(x-1)^2 - 2$ is a function. (Quadratic equations are functions).

V(1,2)

stretch factor 2.



2d) $x^2+y^2=4$ is not a function. (Circles are not functions.) sketch is a circle: centre at (0,0) radius = 2.

3. a) More sales means more money taken in.
This is a function
independent variable is number of tickets sold
dependent variable is amount of revenue ($).

b) There are a few ages in each grade, and there a a few possible grades at each age.

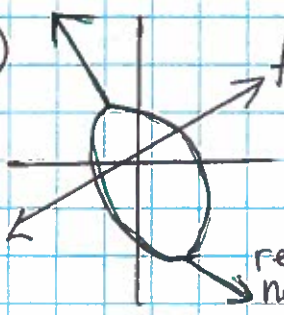
This is not a function.

c) Faster speed results in less time.

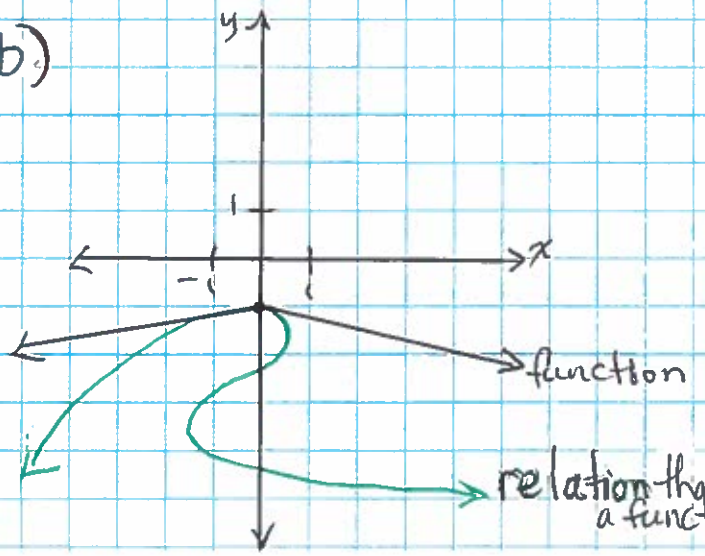
This is a function.

Independent variable speed
Dependent variable time.

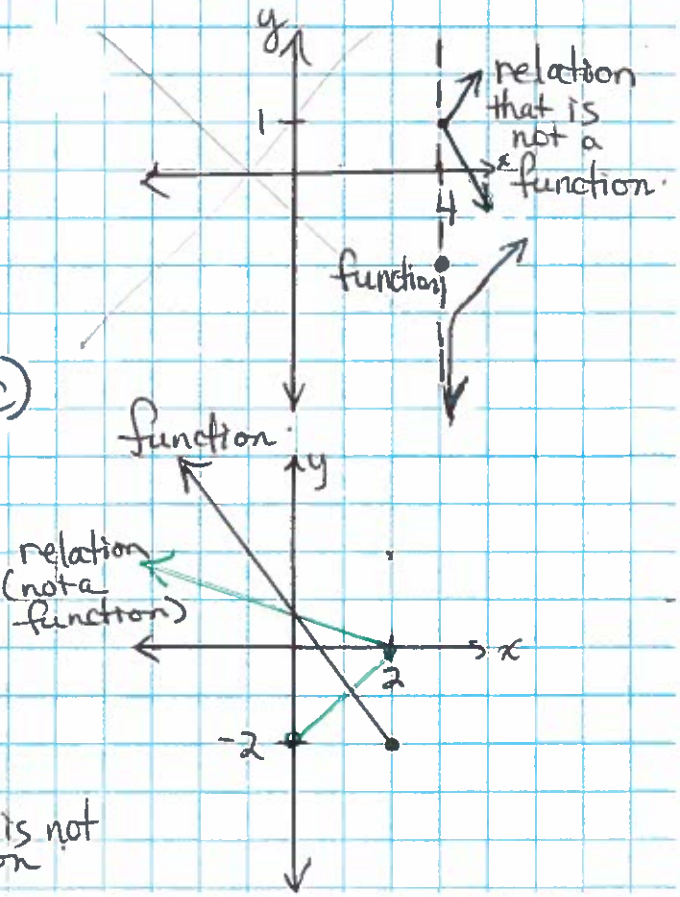
4. a)



b)



c)



5a) The length of each pen is x,

the width of each pen is $\frac{90-4x}{6}$.

Total area is $A(x)$.

$A(x) = 3$(the area of 1 pen)

$A(x) = 3\left(\frac{90-4x}{6}\right)(x)$

$A(x) = 45x - 2x^2$

width must be greater than zero. $90 - 4(22.5) = 0$.

b) $D: \{x \in \mathbb{R},\ 0 < x < 22.5\}$

$R: \{A \in \mathbb{R},\ 0 < A < 253\frac{1}{8}\}$

$-2\left(x^2 - \frac{45}{2}x + \frac{2025}{16} - \frac{2025}{16}\right)$

$= -2\left(x - \frac{45}{4}\right)^2 + \frac{2025}{8}$

$\frac{2025}{8}$: max area $= 253\frac{1}{8}\ m^2$

when $x = \frac{45}{4}$ OR $11\frac{1}{4}\ m$

TRICKY!

6. a) $y = -x + 3$
- sloped line
$D: \{x \in \mathbb{R}\}$
$R: \{y \in \mathbb{R}\}$

b) $y = (x+1)^2 - 4$
- parabola $V(-1,-4)$ ∪
$D: \{x \in \mathbb{R}\}$
$R: \{y \in \mathbb{R}, y \geq -4\}$

c) $y = -3x^2 + 1$
- parabola $V(0,1)$ ∩
$D: \{x \in \mathbb{R}\}$
$R: \{y \in \mathbb{R} \mid y \leq 1\}$

d) $x^2 + y^2 = 9$
$D = \{x \in \mathbb{R}, -3 \leq x \leq 3\}$
$R = \{y \in \mathbb{R}, -3 \leq y \leq 3\}$
- circle, radius 3

e) $y = \frac{1}{x+3}$
$D = \{x \in \mathbb{R}, x \neq -3\}$
$R = \{y \in \mathbb{R}, y \neq 0\}$

f) $y = \sqrt{2x+1}$
↑ "principal" (positive) root
$2x + 1 \geq 0$
$x \geq -\frac{1}{2}$
∴ $D: \{x \in \mathbb{R}, x \geq -\frac{1}{2}\}$
$R: \{y \in \mathbb{R}, y \geq 0\}$

7. a) $f(x) = \frac{2}{5}x + 11$

$f(4) = \frac{2(4)}{5} + \frac{55}{5}$
$= \frac{8+55}{5}$
$= \frac{63}{5}$

$f(-5) = \frac{2(-5)}{5} + 11$
$= -2 + 11$
$= 9$

$f\left(-\frac{2}{3}\right) = \frac{2}{5}\left(-\frac{2}{3}\right) + 11$
$= \frac{-4}{15} + \frac{165}{15}$
$= \frac{161}{15}$

$f(x) = 3x^2 + 2x + 1$

$f(4) = 3(4)^2 + 2(4) + 1$
$= 48 + 8 + 1$
$= 57$

$f(-5) = 3(25) + 2(-5) + 1$
$= 75 - 10 + 1$
$= 66$

$f\left(\frac{-2}{3}\right) = 3\left(\frac{4}{9}\right) + 2\left(\frac{-2}{3}\right) + 1$
$= \frac{4}{3} - \frac{4}{3} + 1$
$= 1$

$f(x) = 2(x+4)^2$

$f(4) = 2(4+4)^2$
$= 2(8)^2$
$= 2(64)$
$= 128$

$f(-5) = 2(-5+4)^2$
$= 2(-1)^2$
$= 2$

$f\left(-\frac{2}{3}\right) = 2\left(-\frac{2}{3} + \frac{12}{3}\right)^2$
$= 2\left(\frac{10}{3}\right)^2$
$= \frac{200}{9}$

7b) $f(x)=-6$ $f(4)=-6$ $f(-5)=-6$ $f\left(-\frac{2}{3}\right)=-6$

c) $f(x)=\frac{1}{x}$

$f(4)=\frac{1}{4}$ $f(-5)=-\frac{1}{5}$ $f\left(-\frac{2}{3}\right)=-\frac{3}{2}$

d) $f(x)=\sqrt{x+5}$

$f(4)=\sqrt{4+5}=\sqrt{9}=3$

$f(-5)=\sqrt{-5+5}=\sqrt{0}=0$

$f\left(-\frac{2}{3}\right)=\sqrt{-\frac{2}{3}+\frac{15}{3}}=\sqrt{\frac{13}{3}}=\frac{\sqrt{13}}{\sqrt{3}}$ OR $\frac{\sqrt{13}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{39}}{3}$

a) $f(3)=-12$

$-12=a(3)$

$a=-4$

$\therefore f(x)=-4x$

~~b) $f(x)=3x$ c) $f(x)=\frac{2}{3}x$ d) $f(x)=-x$~~

	Domain	Range	Function?	Explain.
8a)	$D:\{5,6,7,8,9\}$	$R:\{5,6,7,8,9\}$	Yes	No 2 points have same x-value
b)	$D:\{3,4,5,6\}$	$R:\{-1\}$	Yes	"
c)	$D:\{1\}$	$R:\{-14,-8,0,6,11\}$	No.	Points are lined up vertically.
d)	$D:\{1,3,4,5,11\}$	$R:\{1,4,5,9,11\}$	Yes.	No 2 points have same x-value.
e)	$D:\{1,2,3\}$	$R:\{-2,-1,0,1,2\}$	No.	Some of the points are lined up vertically.

9. a) Yes. all points have different x-values. (horizontally lined up).

b) Yes. all points have different x-values.

c) No. some of the points are lined up vertically (they have the same x-values).

d) No., all points are vertically lined up.

10. a) Graph is just one point

b) Graph is a vertical line.

c) Graph is a horizontal line.

challenge

12. ① $f(x)+2g(x)=12x^2+3x+8$

② $2f(x)+3g(x)=18x^2+6x+13$ $\quad f(2)+g(3)=?$

① $\times 2$ $\quad 2f(x)+4g(x)=24x^2+6x+16$

$-② + ① \times 2$ $\quad \boxed{g(x)=6x^2+3}$

sub $g(x)$ into ①

$f(x)+2(6x^2+3)=12x^2+3x+8$

$f(x)=12x^2+3x+8-12x^2-6$

$\boxed{f(x)=3x+2}$

$f(2)+g(3)=(3(2)+2)+(6(3)^2+3)$

$=(8)+(6(9)+3)$

$=8+57$

$=65$

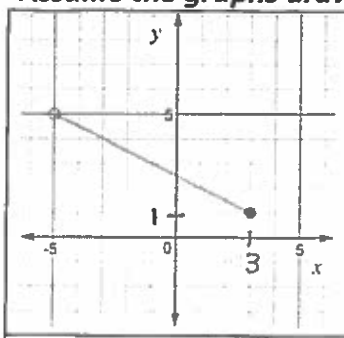
$\therefore f(2)+g(3)=65$

U3D1 Worksheet Part B **Functions, Relations, Domain & Range**

1. State the Domain and Range of each of the given relations in the space provided.
Assume the graphs drawn to the edge of the grid continue on infinitely.

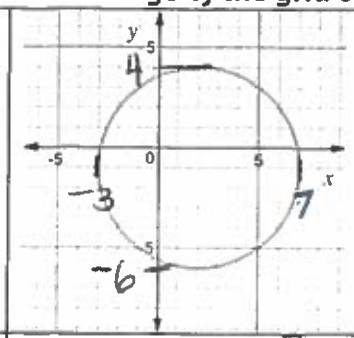
$x, y \in \mathbb{R}$ ∀

note:



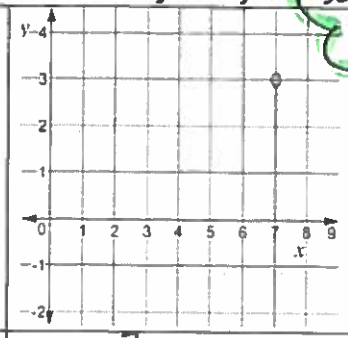
D: $\{-5 < x \leq 3\}$

R: $\{1 \leq y < 5\}$



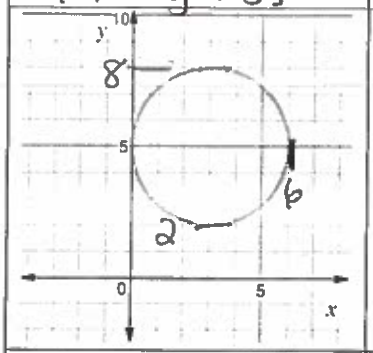
D: $\{-3 \leq x \leq 7\}$

R: $\{-6 \leq y \leq 4\}$



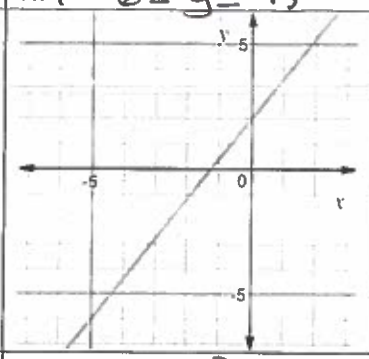
D: $\{7\}$

R: $\{y < 3\}$



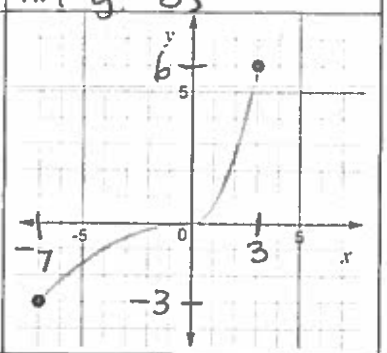
D: $\{0 \leq x \leq 6\}$

R: $\{2 \leq y \leq 8\}$



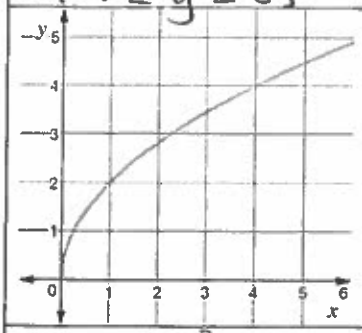
D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$



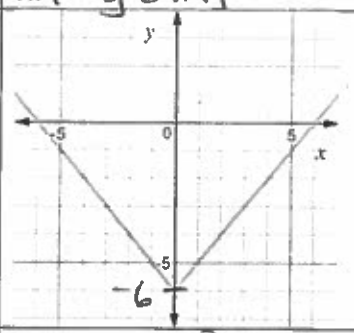
D: $\{-7 \leq x \leq 3\}$

R: $\{-3 \leq y \leq 6\}$



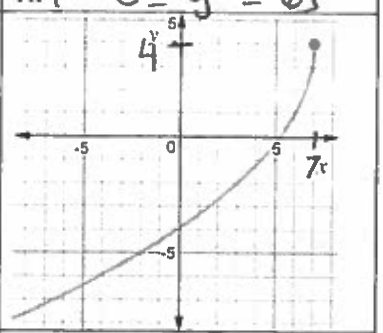
D: $\{x \geq 0\}$

R: $\{y \geq 0\}$



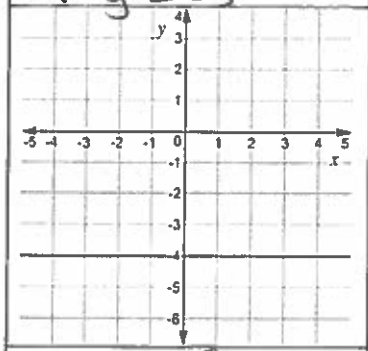
D: $\{x \in \mathbb{R}\}$

R: $\{y \geq -6\}$



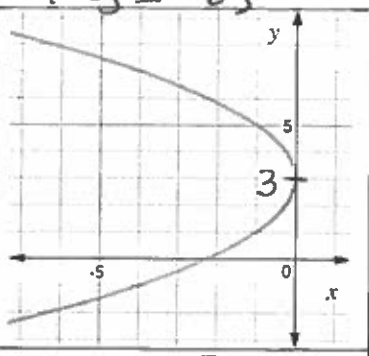
D: $\{x \leq 7\}$

R: $\{y \leq 4\}$



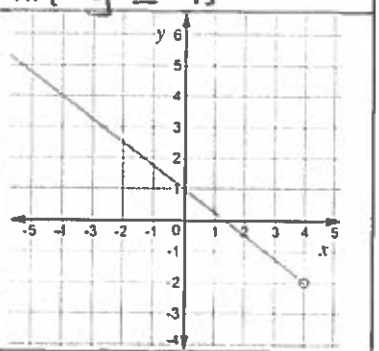
D: $\{x \in \mathbb{R}\}$

R: $\{-4\}$



D: $\{x \leq 3\}$

R: $\{y \in \mathbb{R}\}$



D: $\{x < 4\}$

R: $\{y > -2\}$