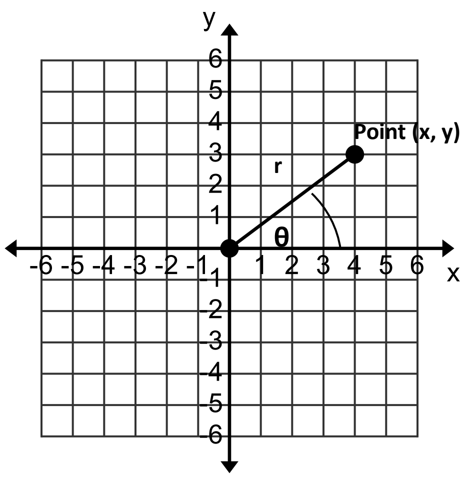
U5D4 **CAST Rule** Warm Up: Skill Reflection #3

CAST

Given angle θ formed by a **terminal arm** in **standard position**. (For standard position the "initial arm" is the positive x-axis, the terminal arm is found by joining the origin to the point, the angle θ is the angle measured from the initial arm rotating counter-clockwise to the terminal arm.)

If we have a Point in Quadrant I with x>0, y>0:



$r = \sqrt{x^2 + y^2}$ The square root sign means take the "principal square root" which means the positive square root so r is always positive.

$\sin\theta = \frac{y}{r}$

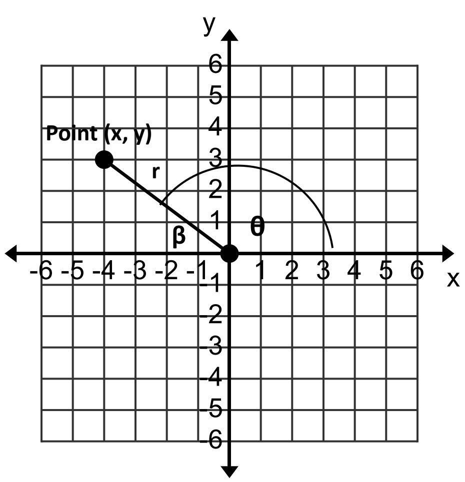
$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$\sin\theta > 0,\ \cos\theta > 0, \tan\theta > 0$

Note: All three ratios are positive in Quadrant I
0°<θ<90°

If our Point was in Quadrant II with x<0, y>0:



θ is the "principal angle"
β is the "related acute angle"

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

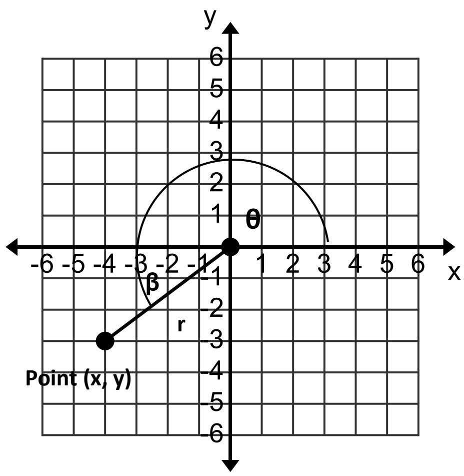
$\tan\theta = \frac{y}{x}$

Since r>0 and in Quadrant II, x<0 and y>0,

Sine is the only positive ratio in quadrant II 90°<θ<180°

$\sin\theta > 0,\ \cos\theta < 0, \tan\theta < 0$

If our Point was in Quadrant III with x<0, y<0:



θ is the "principal angle"

β is the "related acute angle"

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

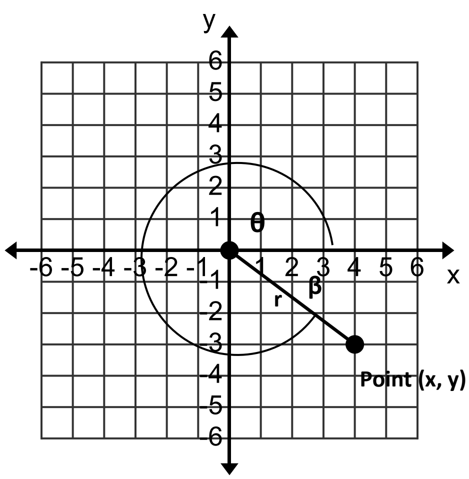
$$\tan\theta = \frac{y}{x}$$

Since r>0 and in Quadrant III, x<0 and y<0,

Tangent is the only positive ratio in quadrant III 180°<θ<270°

$$\sin\theta < 0,\ \cos\theta < 0, \tan\theta > 0$$

If our Point was in Quadrant IV with x>0, y<0:



θ is the "principal angle"

β is the "related acute angle"

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

Since r>0 and in Quadrant IV, x>0 and y<0,

Cosine is the only positive ratio in quadrant IV 270°<θ<360°

$$\sin\theta < 0,\ \cos\theta > 0, \tan\theta < 0$$

NOTE: θ is measured from the positive x-axis, rotating counter-clockwise to the terminal arm. However, to use simple trigonometric ratios, we need a Rignt-Triangle, hence we place β, the related acute angle between the x-axis and the terminal arm. In this way, we form a right-triangle with the terminal arm and the x-axis with β as the reference angle inside the triangle.

II

Sin only positive

90°<θ<180°

sin(180°- θ)= sin θ

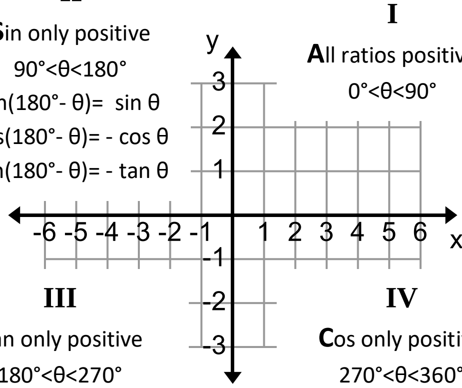
cos(180°- θ)= - cos θ

tan(180°- θ)= - tan θ

I

All ratios positive

0°<θ<90°



III

Tan only positive

180°<θ<270°

sin(180°+ θ)= - sin θ

cos(180°+ θ)= - cos θ

tan(180°+ θ)= tan θ

IV

Cos only positive

270°<θ<360°

sin(360°- θ)= - sin θ

cos(360°- θ)= cos θ

tan(360°- θ)= - tan θ

The CAST rule:

To determine whether a trig ratio will be positive or negative, you may use the CAST rule.

If you know which quadrant θ lies in then…

← See grid

Example 1: The point P(-3, -6) lies on the terminal arm of an angle θ in standard position.
a) Determine the exact values of sin θ, cos θ and tan θ.
b) Determine the principal angle, θ.

Example 2: Find θ if cos θ = -0.6784, and $0° \leq \theta \leq 180°$.

Example 3: Angle θ is in standard position in quadrant II and $0° \leq \theta \leq 360°$. Given the trig ratio, find:

a) the exact values of the other two trig ratios. $sin\theta = \frac{3}{4}$

b) the principal angle, θ

Example 4: Find θ if tan θ = -0.5238, if $0° \leq \theta \leq 360°$.

U5D4 Homework: # p. 281 #1, 4, p. 348 #1abef + principal angle, #2abef + principal angle, #6, (Where it says $0 \leq \theta \leq 2\pi$ treat as 0º ≤ θ ≤ 360º), p. 348 #1a) $\cos\theta = \frac{8}{17}$ (book error)