## Ambiguous Case of Sine Law

From last day, (Example $2 \ln \Delta E F D, e=6.7 \mathrm{~cm}, \mathrm{~d}=18.8 \mathrm{~cm}$, and $F=42$ degrees.)


Try these... Use calculator:

| $\operatorname{Sin} 10^{\circ}=$ | $\sin 20^{\circ}$ |  |
| :--- | :--- | :--- |
| $\operatorname{Sin} 170^{\circ}=$ | $\sin 160^{\circ}=$ | $\sin 1^{\circ}=$ |
| What is the pattern? |  | $\sin 179^{\circ}=$ |
| Now, try: $\quad \sin ^{-1}(0.1736)=$ | $\sin ^{-1}(0.3420)=$ | $\sin ^{-1}(0.0175)=$ |

*The calculator does not know whether you are looking for the $\qquad$ angle or the $\qquad$ angle so you
MUST consider BOTH possibilities.

* The calculator always gives the $\qquad$ angle when using the $\sin ^{-1}$ button.
Back to example above,
If $D=60^{\circ}$ then $E=180^{\circ}-42^{\circ}-60^{\circ}=$
Recall: The largest angle is across from the largest side, the smallest angle is across from the smallest side, etc.
Example 1: $\mathrm{b}=4, \mathrm{a}=3, \mathrm{~A}=30^{\circ}$ in $\triangle \mathrm{ABC}$. Solve the triangle.
*We cannot use cosine law. Why?
*Side $b$ is larger than side a so angle $B$ will be larger than $A$. It is possible that $B$ is Obtuse.
Write solution on reverse.
Example 2: $b=4, a=1, A=20^{\circ}$ in $\triangle A B C$. Solve the triangle.
Write solution on reverse.
Example 3: In $\triangle P Q R, Q=38^{\circ}, q=28 \mathrm{~cm}, r=45 \mathrm{~cm}$. Determine the values of angles $P$ and $R$.
Example 4: $b=4, a=5, A=53^{\circ}$ in $\triangle A B C . B=?, C=$ ?


## Write solution on reverse

## The Ambiguous Case:

If you are using the sine law and you are looking for an angle...
If there is ANY possibility that the angle you are looking for is obtuse then you MUST check for the ambiguous case (where two triangles are possible)...
To consider the ambiguous case given $a, b, A . .$.

- Solve for $B_{1}$ using the sine law

Then use ASTT to solve for $\mathrm{C}_{1}$

- The second case is:
$\mathrm{B}_{2}=180^{\circ}-\mathrm{B}_{1}$
$\mathrm{C}_{2}=180^{\circ}-\mathrm{A}-\mathrm{B}_{2}$ (ASTT)

If this gives you a positive $\mathrm{C}_{2}$ value

$$
\left(A+B_{2}<180^{\circ} \text { and } C_{2}>0^{\circ}\right)
$$

Then this IS the ambiguous case and there are two possible triangles: $\mathrm{A}, \mathrm{B}_{1}, \mathrm{C}_{1}$ and $A, B_{2}, C_{2}$
( if the $C_{2}$ value is negative then there is only one possible triangle: $A, B_{1}, C_{1}$.

For homework number \#19. Bearing is measured clockwise from North. So a bearing of $240^{\circ}$ is the same as $660^{\circ} \mathrm{W}$.

