

Simplifying Expressions Using Exponent Laws

1. Simplify
 - a) $5a^{-3} \times 8a^{-9}$
 - b) $-24c^5d^3 \div 4c^8d^{-3}$
 - c) $m^2n^5 \times m^3n^{-7}$
 - d) $\left(\frac{24c^8d^5}{-8c^2d}\right)\left(\frac{15c^3d^9}{18cd^5}\right)$
 - e) $\frac{12m^5n^{-2} \times 5m^{-11}n^6}{15m^3n^{-4}}$
 - f) $(xy^3)^6 \div (x^2y^4)^8$
2. Write in radical form, then evaluate.
 - a) $81^{\frac{3}{4}}$
 - b) $16^{\frac{-3}{4}}$
 - c) $625^{0.75}$
 - d) $4^{\frac{3}{2}}$
 - e) $8^{\frac{4}{3}}$
3. Evaluate. Do not convert fraction answers to decimals.
 - a) $\left(\frac{1}{9}\right)^{\frac{3}{2}}$
 - b) $\left(-\frac{1}{32}\right)^{0.8}$
 - c) $\left(\frac{49}{25}\right)^{\frac{1}{2}}$
 - d) $\left(-\frac{27}{125}\right)^{\frac{4}{3}}$
 - e) $\left(\frac{625}{343}\right)^0$
4. Evaluate.
 - a) $32^{\frac{2}{5}} \times 243^{\frac{2}{5}}$
 - b) $64^{\frac{2}{3}} \times 125^{\frac{1}{3}}$
 - c) $4^{\frac{5}{2}} \times 81^{\frac{3}{4}}$
5. Simplify.
 - a) $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$
 - b) $\left(n^{\frac{1}{2}}\right)^{-6}$
 - c) $x^{\frac{-3}{2}} \div x^{\frac{1}{4}}$
 - d) $(9a^4b^{-2} \times 4a^2b^{-6})^{\frac{1}{2}}$
 - e) $8m^{\frac{1}{3}}n^{\frac{-3}{2}}(-2m^{\frac{-2}{3}}n^{\frac{1}{3}})^{-4}$
6. Simplify.
 - a) $\frac{36x^{-2}y^3z^{-4}}{12xy^{-2}z^{-2}}$
 - b) $\sqrt{\frac{32x^{-5}y^2 \times 18x^2y}{4xy^{-3}}}$
 - c) $\left(\frac{3x^{-2}y^3}{12xy^{-1}}\right)\left(\frac{10x^4y^{-2}}{5x^{-1}y^2}\right)$
 - d) $\frac{8^{1-2x} \times 4^{2x+3}}{16^{2-3x}}$
 - e) $\frac{16^{2m-n} \times 9^{m+3n}}{27^{m+n} \times 8^{m-n}}$
 - f) $\frac{5^{-200} - 5^{-198}}{5^{-199} + 5^{-200}}$
7. Simplify.
 - a) $\frac{(c^{a+b})(c^{a-b})}{c^2}$
 - b) $\frac{(x^a)^2(x^b)^2}{(x^{a+b})(x^{a-b})}$
 - c) $\frac{x^{2a-b} \cdot x^{a-3b}}{(x^{3a+b})^{-2}}$
 - d) $\frac{(m^{x-1})(m^{2x+5})}{m^{3x-1}}$
 - e) $\frac{3^{-6a+3-5a}}{3^{-6a+3-7a}}$
8. Evaluate.
 - a) $(5^{\frac{1}{2}} + 2^{\frac{1}{2}})(5^{\frac{1}{2}} - 2^{\frac{1}{2}})$
 - b) $(8^{\frac{2}{3}} - 5^{\frac{1}{2}})(8^{\frac{2}{3}} + 5^{\frac{1}{2}})$
9. Simplify.
 - a) $\left(\sqrt{49y^m}\right)^{\frac{-1}{n}}$
 - b) $\sqrt[3]{\frac{m^{\frac{1}{2}}\sqrt{mn}}{\frac{1}{\sqrt{n}}}}$
 - c) $\left(\frac{\sqrt[4]{a^{2n-1}} \times \sqrt[4]{a}}{\sqrt{a}}\right)^2$

ANSWERS:

- 1a) $\frac{40}{a^{12}}$ b) $\frac{-6d^6}{c^3}$ c) $\frac{m^5}{n^2}$ d) $\frac{-5c^8d^8}{2}$ e) $\frac{4n^8}{m^9}$ f) x^2y^2 2a) 27 b) $\frac{1}{8}$ c) 125 d) $\frac{1}{8}$ e) 16
- 3a) 27 b) $\frac{1}{16}$ c) $\frac{7}{5}$ d) $\frac{81}{625}$ e) 1 4a) 36 b) 80 c) 864
- 5a) 1 b) $\frac{1}{n^3}$ c) $\frac{1}{x^4}$ d) $\frac{6a^3}{b^4}$ e) $\frac{m^3}{2n^{\frac{17}{6}}}$ 6a) $\frac{3y^5}{x^3z^2}$ b) $\frac{12y^3}{x^2}$ c) $\frac{x^2}{2}$ d) 2^{10x+1} e) $2^{5m-n}3^{3n-m}$ f) -4
- 7a) c^{2a-2} b) x^{2b} c) x^{9a-2b} d) m^5 e) 3^a 8 a) 3 b) 11 9a) $\frac{1}{7^n y^{mn}}$ b) $m^{\frac{1}{3}}n^{\frac{1}{3}}$ c) a^{n-1}

$$\begin{array}{lll}
 \text{1a)} & 5a^{-3} \times 8a^{-9} & \text{b)} -24c^5d^3 \div 4c^8d^{-3} \\
 & = 40a^{-12} & = -6c^{-8}d^{3+3} \\
 & = \frac{40}{a^{12}} & = -\frac{6d^6}{c^8} \\
 & & \text{c)} m^2n^5 \times m^3n^{-7} \\
 & & = m^5n^{-2} \\
 & & = \frac{m^5}{n^2}
 \end{array}$$

$$\text{d)} \left(\frac{24c^8d^5}{-8c^2d} \right) \left(\frac{15c^3d^9}{18cd^5} \right)$$

$$= \left(\frac{-5}{2} \right) (c^6d^4) (c^2d^4)$$

$$= -\frac{5c^8d^8}{2}$$

$$\text{e)} \frac{12m^5n^{-2} \times 5m^{-11}n^6}{15m^3n^{-4}}$$

$$= \frac{4m^{-6}n^4}{m^3n^{-4}}$$

$$= 4m^{-9}n^{4+4}$$

$$= \frac{4n^8}{m^9}$$

$$\text{f)} (xy^{\frac{2}{3}})^6 \div (x^{\frac{1}{2}}y^{\frac{1}{4}})^8$$

$$= \frac{(x)^6 (y^{\frac{2}{3}})^6}{(x^{\frac{1}{2}})^8 (y^{\frac{1}{4}})^8}$$

$$= \frac{x^6 y^4}{x^4 y^2}$$

$$= x^2 y^2$$

$$\text{2a)} 81^{\frac{3}{4}}$$

$$= (\sqrt[4]{81})^3$$

$$= 3^3$$

$$= 27$$

$$\text{b)} 16^{-\frac{3}{4}}$$

$$= \frac{1}{(\sqrt[4]{16})^3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

$$\text{2c)} 625^{0.75}$$

$$= 625^{\frac{3}{4}}$$

$$= (\sqrt[4]{625})^3$$

$$= 5^3$$

$$= 125$$

$$\text{2d)} 4^{-\frac{3}{2}}$$

$$= \frac{1}{(\sqrt{4})^3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

$$\text{e)} 8^{\frac{4}{3}}$$

$$= (\sqrt[3]{8})^4$$

$$= 2^4$$

$$= 16$$

$$3a) \left(\frac{1}{9}\right)^{-3/2}$$

$$= 9^{3/2}$$

$$= (\sqrt{9})^3$$

$$= 3^3$$

$$= 27$$

$$b) \left(-\frac{1}{32}\right)^{0.8}$$

$$= \left(-\frac{1}{32}\right)^{4/5}$$

$$= \frac{1}{(\sqrt[5]{-32})^4}$$

$$= \frac{1}{(-2)^4}$$

$$= \frac{1}{16}$$

$$c) \left(\frac{49}{25}\right)^{1/2}$$

$$= \frac{\sqrt{49}}{\sqrt{25}}$$

$$= \frac{7}{5}$$

$$d) \left(-\frac{27}{125}\right)^{4/3}$$

$$= \frac{(\sqrt[3]{-27})^4}{(\sqrt[3]{125})^4}$$

$$= \frac{(-3)^4}{5^4}$$

$$= \frac{81}{625}$$

$$3e) \left(\frac{625}{343}\right)^0$$

$$= 1$$

$$4a) 32^{2/5} \times 243^{2/5}$$

$$= (\sqrt[5]{32})^2 \times (\sqrt[5]{243})^2$$

$$= 2^2 \times 3^2$$

$$= 4 \times 9$$

$$= 36$$

$$b) 64^{2/3} \times 125^{1/3}$$

$$= (\sqrt[3]{64})^2 \times \sqrt[3]{125}$$

$$= 4^2 \times 5$$

$$= 16 \times 5$$

$$= 80$$

$$4c) 4^{5/2} \times 81^{3/4}$$

$$= (\sqrt{4})^5 \times (\sqrt[4]{81})^3$$

$$= 2^5 \times 3^3$$

$$= 32 \times 27$$

$$= 864$$

$$5a) a^{1/2} \times a^{-1/2}$$

$$= a^0$$

$$= 1$$

$$b) (n^{1/2})^{-6}$$

$$= n^{-3}$$

$$= \frac{1}{n^3}$$

$$c) x^{-3/2} \div x^{-1/4}$$

$$= x^{-3/2 - (-1/4)}$$

$$= x^{-6/4 + 1/4}$$

$$= x^{-5/4}$$

$$= \frac{1}{(\sqrt[4]{x})^5}$$

$$\textcircled{R} \frac{1}{x^{5/4}}$$

$$\begin{aligned}
 5d) & (9a^4b^{-2} \times 4a^2b^{-6})^{\frac{1}{2}} \\
 &= (36a^6b^{-8})^{\frac{1}{2}} \\
 &= (\sqrt{36})(a^6)^{\frac{1}{2}}(b^{-8})^{\frac{1}{2}} \\
 &= 6a^3b^{-4} \\
 &= \frac{6a^3}{b^4}
 \end{aligned}$$

$$\begin{aligned}
 e) & 8m^{\frac{1}{3}}n^{-\frac{3}{2}}(-2m^{-\frac{2}{3}}n^{\frac{1}{3}})^{-4} \\
 &= \frac{8m^{\frac{1}{3}}n^{-\frac{3}{2}}}{(-2)^4(m^{-\frac{2}{3}})^4(n^{\frac{1}{3}})^4} \\
 &= \frac{8m^{\frac{1}{3}}n^{-\frac{3}{2}}}{16m^{-\frac{8}{3}}n^{\frac{4}{3}}} \\
 &= \frac{1}{2}m^{\frac{1}{3}+\frac{8}{3}}n^{-\frac{3}{2}-\frac{4}{3}} \\
 &= \frac{1}{2}m^{\frac{9}{3}}n^{-\frac{9}{6}-\frac{8}{6}} \\
 &= \frac{m^3}{2n^{\frac{17}{6}}}
 \end{aligned}$$

$$\begin{aligned}
 6a) & \frac{36x^{-2}y^3z^{-4}}{12xy^{-2}z^{-2}} \\
 &= 3x^{-3}y^5z^{-2} \\
 &= \frac{3y^5}{x^3z^2}
 \end{aligned}$$

$$\begin{aligned}
 b) & \sqrt{\frac{32x^5y^2 \times 18x^2y}{4xy^{-3}}} \\
 &= \sqrt{\frac{144x^{-3}y^3}{xy^{-3}}} \\
 &= \sqrt{144(x^{-4})^{\frac{1}{2}}(y^6)^{\frac{1}{2}}} \\
 &= 12x^{-2}y^3 \\
 &= \frac{12y^3}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 c) & \left(\frac{3x^{-2}y^3}{12xy^{-1}}\right)\left(\frac{10x^4y^{-2}}{5x^{-1}y^2}\right) \\
 &= \frac{1}{4}x^{-3}y^4(2)(x^5)(y^{-4}) \\
 &= \frac{x^2}{2}
 \end{aligned}$$

$$\begin{aligned}
 6d) & \frac{8^{1-2x} \times 4^{2x+3}}{16^{2-3x}} \\
 &= \frac{(2^3)^{1-2x} \times (2^2)^{2x+3}}{(2^4)^{2-3x}} \\
 &= \frac{2^{3-6x+4x+6}}{2^{8-12x}} \\
 &= 2^{9-2x-(8-12x)}
 \end{aligned}$$

$$\begin{aligned}
 &= 2^{1+10x} \quad (6e) \quad \frac{16^{2m-n} \times 9^{m+3n}}{27^{m+n} \times 8^{m-n}} \\
 &= 2^{10x+1} = \frac{(2^4)^{2m-n} \times (3^2)^{m+3n}}{(3^3)^{m+n} \times (2^3)^{m-n}} \\
 &= 2^{8m-4n-(3m-3n)} \times 3^{2m+6n-(3m+3n)} \\
 &= 2^{5m-n} \times 3^{-m+3n} \\
 &= 2^{5m-n} 3^{3n-m}
 \end{aligned}$$

$$\begin{aligned}
 \text{6f)} \quad & \frac{5^{-200} - 5^{-198}}{5^{-199} + 5^{-200}} \times \frac{5^{200}}{5^{200}} \\
 &= \frac{5^0 - 5^2}{5^1 + 5^0} \\
 &= \frac{1 - 25}{5 + 1} \\
 &= \frac{-24}{6} \\
 &= -4
 \end{aligned}$$

$$\begin{aligned}
 \text{7a)} \quad & \frac{(c^{a+b})(c^{a-b})}{c^2} \\
 &= c^{2a-2}
 \end{aligned}$$

$$\begin{aligned}
 \text{7b)} \quad & \frac{(x^a)^2 (x^b)^2}{(x^{a+b})(x^{a-b})} \\
 &= \frac{x^{2a} x^{2b}}{x^{2a}} \\
 &= x^{2b}
 \end{aligned}$$

$$\begin{aligned}
 \text{7c)} \quad & \frac{x^{2a-b} x^{a-3b}}{(x^{3a+b})^{-2}} \\
 &= \frac{x^{3a-4b}}{x^{-6a-2b}} \\
 &= x^{9a-2b}
 \end{aligned}$$

$$\begin{aligned}
 \text{7d)} \quad & \frac{m^{x-1} (m^{2x+5})}{m^{3x-1}} \\
 &= m^{x+2x-3x-1+5+1} \\
 &= m^5
 \end{aligned}$$

$$\begin{aligned}
 \text{7e)} \quad & \frac{3^{-6a} + 3^{-5a}}{3^{-6a} + 3^{-7a}} \\
 &= \frac{3^{-6a} (3^0 + 3^a)}{3^{-7a} (3^a + 3^0)} \\
 &= 3^{-6a+7a} \frac{(3^a + 1)}{(3^a + 1)} \\
 &= 3^a
 \end{aligned}$$

OR

$$\begin{aligned}
 & \frac{3^{-6a} + 3^{-5a}}{3^{-6a} + 3^{-7a}} \times \frac{3^{7a}}{3^{7a}} \\
 &= \frac{3^a + 3^{2a}}{3^a + 3^0} \\
 &= \frac{3^a (1 + 3^a)}{(1 + 3^a)} \\
 &= 3^a
 \end{aligned}$$

$$\begin{aligned}
 8a) & (5^{\frac{1}{2}} + 2^{\frac{1}{2}})(5^{\frac{1}{2}} - 2^{\frac{1}{2}}) \quad (\text{difference of squares}) \\
 &= (5^{\frac{1}{2}})(5^{\frac{1}{2}}) - (2^{\frac{1}{2}})(2^{\frac{1}{2}}) \\
 &= (5^{\frac{1}{2}})^2 - (2^{\frac{1}{2}})^2 \\
 &= 5 - 2 \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 8b) & (8^{\frac{2}{3}} - 5^{\frac{1}{2}})(8^{\frac{2}{3}} + 5^{\frac{1}{2}}) \\
 &= (8^{\frac{2}{3}})^2 - (5^{\frac{1}{2}})^2 \\
 &= 8^{\frac{4}{3}} - 5^1 \\
 &= (\sqrt[3]{8})^4 - 5 \\
 &= 2^4 - 5 \\
 &= 16 - 5 \\
 &= 11
 \end{aligned}$$

$$\begin{aligned}
 9a) & \left(\sqrt[4]{49 y^{\frac{2}{m}}} \right)^{-\frac{1}{n}} \\
 &= (\sqrt{49})^{-\frac{1}{n}} \left[\left(y^{\frac{2}{m}} \right)^{\frac{1}{2}} \right]^{-\frac{1}{n}} \\
 &= 7^{-\frac{1}{n}} y^{-\frac{1}{mn}} \\
 &= \frac{1}{7^{\frac{1}{n}} y^{\frac{1}{mn}}}
 \end{aligned}$$

$$\begin{aligned}
 9b) & \sqrt[3]{\frac{m^{\frac{1}{2}} \sqrt{mn}}{\frac{1}{\sqrt{n}}}} \\
 &= \left[\frac{m^{\frac{1}{2}} (m)^{\frac{1}{2}} (n)^{\frac{1}{2}}}{(n)^{-\frac{1}{2}}} \right]^{\frac{1}{3}} \\
 &= (mn)^{\frac{1}{3}} \\
 &= m^{\frac{1}{3}} n^{\frac{1}{3}}
 \end{aligned}$$

$$\begin{aligned}
 9c) & \left(\frac{\sqrt[4]{a^{2n-1}} \times \sqrt[4]{a}}{\sqrt{a}} \right)^2 \\
 &= \left(\frac{(a^{2n-1})^{\frac{1}{4}} \times a^{\frac{1}{4}}}{a^{\frac{1}{2}}} \right)^2 \\
 &= \frac{(a^{\frac{2n-1}{4}})^2 \times (a^{\frac{1}{4}})^2}{(a^{\frac{1}{2}})^2} \\
 &= \frac{(a^{\frac{2n-1}{2}})(a^{\frac{1}{2}})}{a^1} \\
 &= a^{n-\frac{1}{2}+\frac{1}{2}-1} \\
 &= a^{n-1}
 \end{aligned}$$