MCR3UI Unit 4 Lesson 1

Exponent Laws

1. Multiplication Law: $x^m \times x^n = x^{m+n}$

When multiplying powers with the same base, keep the base the same and add the exponents.

Ex 1.
$$x^3 \times x^2$$
 (Note: $x^3 \times x^2 = x \cdot x \cdot x \cdot x \cdot x$) Ex. 2 $2^3 \times 2^4$
= x^{3+2} = 2^{3+4}
= x^5 = 2^7 = 128

2. Division Law: $x^m \div x^n = x^{m-n}$

When dividing powers with the same base, keep the base the same and subtract the exponents.

Ex. 1
$$x^5 \div x^2$$
 Ex. 2 $2^4 \div 2^3$ Note: $2^4 \div 2^3$
 $= x^{5-2}$ $= 2^{4-3}$ $= \frac{2^4}{2^3}$
 $= x^3$ $= 2^1$ $= \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$
 $= 2$

3. Power of a Power Law: $(x^m)^n = x^{m \times n}$

If a power is raised to an exponent, multiply the exponents.

Ex.
$$(x^3)^2 = x^{3\times 2}$$

 $= x^6$
A. Power of a Product Law: $(x \cdot y)^m = x^m y^m$

NOTE: This rule does NOT apply to the power of a sum or difference!

Ex. 1
$$(x \cdot y)^5 = x^5 y^5$$

= $(3x^5 y^3)^2$
= $(3)^2 (x^5)^2 (y^3)^2$
= $9x^{10} y^6$

NOTE: There is no "Sum of a Power Rule" or "Difference of a Power Rule". $(a + b)^n \neq a^2 + b^2$ (note: the <u>does not equal</u> sign above!)

5. Power of a Quotient Law: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

If a Quotient is raised to an exponent, distribute the exponent to every factor in the numerator and denominator.

Ex. 1
$$\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$$
 Ex. 2 $\left(\frac{2}{3}\right)^2$ Ex. 3 $\left(\frac{2x^3}{3y^2}\right)^3 = \frac{(2)^3(x^3)^3}{(3)^3(y^2)^3}$
 $= \frac{x^2}{y^2}$ $= \frac{2^2}{3^2}$ $= \left(\frac{8x^9}{27y^6}\right)$
 $= \frac{4}{9}$

6. Zero Exponents: $x^0 = 1$

Any power with an exponent of zero is equal to one.

Ex. 1
$$(-2)^0 = 1$$
 Ex. 2 $-2^0 = -(2^0)$ Ex. 3 $(-237x^3y^7)^0 = 1$
= -1

Proof:

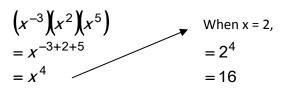
7. Negative Exponents: $x^{-m} = \frac{1}{x^m}$	
=1 So	$3^0 = 1$
= $\frac{9}{9}$ =	3 ⁰
$=\frac{3\times3}{3\times3}$	3 ^{2–2}
	$b^2 \div 3^2$

A negative in the exponent of a power means to 'flip the base' or 'take the reciprocal'. A negative exponent has nothing to do with the sign of the number.

Ex. 1
$$x^{-2} = \frac{1}{x^2}$$
 Ex. 2 $4^{-2} = \frac{1}{4^2}$ Ex. 3 $\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$ Ex. 4 $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2$
= $\frac{5^3}{4^3}$ = 3^2
Ex. 4 Simplify first, then evaluate using x = 2. = $\frac{125}{64}$ = 9

= 9

Ex. 4 Simplify first, then evaluate using x = 2.



Example 1: Simplify. Leave answers with only positive exponents.

a)
$$a^{5} \times a^{2} \times a$$
 b) $(n^{2})^{3}$ c) $\left(\frac{y^{2}}{y^{3}}\right) \times y^{-6} \times y^{0}$ d) $2a^{-3}$ e) $(4a)^{-2}$
= $a^{5^{+2+1}}$ = n^{6} = y^{-1-6+0} = $\frac{2}{a^{3}}$ = $\frac{1}{(4)^{2}(a)^{2}}$
= a^{8} = $\sqrt{7}$ = $\frac{1}{16a^{2}}$
= $\frac{1}{y^{7}}$
f) $\frac{y^{0}}{2a^{-3}}$ g) $(4a^{3}c^{2})^{3}(-3ac^{-4})^{2}$ h) $\frac{x^{7y+1}}{x^{7y-6}}$ i) $\frac{3^{-1}}{x^{0}+2^{3}}$, $x \neq 0$ j) $\frac{2^{-5}+2^{-3}}{2^{-4}}$
= $\frac{1}{2a^{-3}}$ = $(4)^{3}(a^{3})^{3}(c^{2})^{3}(-3)^{2}(a)^{2}(c^{-4})^{2}$ = $x^{(7y+1)-(7y-6)}$ = $\frac{2^{-5}}{2^{-4}} + \frac{2^{-3}}{2^{-4}}$
= $\frac{1}{2} \times \frac{a^{3}}{1}$ = $64 \times 9a^{9}a^{2}c^{6}c^{-8}$ = x^{1+6} = $2^{-5+4} + 2^{-3+4}$
= $\frac{a^{3}}{2}$ = $576a^{11}c^{-2}$ = x^{7} = $2^{-1} + 2$
= $\frac{576a^{11}}{c^{2}}$ = $2\frac{1}{2}$
OR $\frac{2^{-5}+2^{-3}}{2^{-4}} \times \frac{2^{5}}{2^{5}}$
= $\frac{2^{0}+2^{2}}{2}$
= $\frac{1+4}{2}$
= $\frac{5}{2}$
There are still other ways

to correctly evaluate

this question using exponent laws

Example 2: What is the volume of the cube with side lengths of $x^{-2}y$?

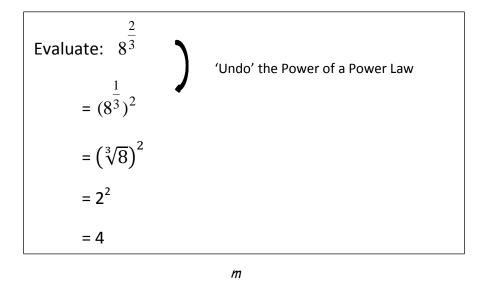
V = s³, where s is the side length of the cube. V = $(x^{-2}y^{3})^{3}$ = $x^{-6}y^{9}$ = $\frac{y^{9}}{x^{6}}$ Use your calculator to evaluate: a) $9^{\frac{1}{2}}$ b) $49^{\frac{1}{2}}$ c) $625^{\frac{1}{4}}$ = 3
= 7
= 5 Note: $\sqrt{9} = 3$ $\sqrt{49} = 7$ $\sqrt[4]{625} = 5$ $\therefore 9^{\frac{1}{2}} = \sqrt{9}$ and $625^{\frac{1}{4}} = \sqrt[4]{625}$; so the denominator of the exponent determines the "index" of the root. $a^{\frac{1}{n}} = \sqrt[n]{a}$, where "n" is called the "index", and "a" is called the "radicand". This is read as the "nth root of a". "What number to the exponent n will equal a?"

8. Powers of the form: χ^{n} The exponent $\frac{1}{n}$ means to take the nth root. i.e. $\mathbf{X}^{\frac{1}{n}} = \sqrt[n]{\mathbf{X}}$ Ex. 2 $x^{\frac{1}{3}}$ Ex. 3 $x^{\frac{1}{12}}$ $x^{\frac{1}{2}}$ 81² Ex. 4 Ex 1. $=\sqrt[2]{X}$ $=\sqrt[3]{x}$ $= \sqrt[12]{x}$ $=\sqrt{81}$ $=\sqrt{X}$ = 9Ex. 6 $(-64)^{\overline{4}}$ Ex. 7 $(64)^{\overline{3}}$ Ex. 5 $(-27)^3$ Ex. 8 (64)6 $=\sqrt[4]{-64}$ $=\sqrt[3]{-27}$ $=\sqrt[3]{64}$ $=\sqrt[6]{64}$ = -3 =not possible = 4 = 2 (You may take the odd root of a negative number but

you may **<u>not</u>** take the even root of a negative number)

MCR 3UI Unit 4 Lesson 2

Laws for Rational Exponents



9. Powers of the form: X^n

The exponent $\frac{m}{n}$ means to take the nth root and raise the answer to an exponent m.

i.e., $x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{(x^m)}^*$ the denominator of the exponent is the index of the root. Ex 1. $x^{\frac{3}{4}}$ Ex. 2 $x^{\frac{2}{3}}$ Ex. 3 $81^{\frac{3}{4}}$ Ex. 4 $(-125)^{\frac{2}{3}}$ $= \sqrt[4]{x^3}$ $= \sqrt[3]{x^2}$ $= (\sqrt[4]{81})^3$ $= (\sqrt[3]{-125})^2$ Or $= (\sqrt[4]{x})^3$ Or $= (\sqrt[3]{x})^2$ $= (3)^3$ $= (-5)^2$ = 27 = 25

Ex. 5
$$9^{\frac{3}{2}}$$
 Ex. 6 $9^{-2.5}$ Ex. 7 $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$ Ex. 8 $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$
 $= (\sqrt{9})^3 = 9^{-\frac{5}{2}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{25}{4}\right)^{\frac{3}{2}}$
 $= 2^3 = \frac{1}{(\sqrt{9})^5} = \frac{(\sqrt{25})^3}{(\sqrt{27})^2} = \frac{(\sqrt{25})^3}{(\sqrt{4})^3}$
 $= 8 = \frac{1}{3^5} = \frac{2^2}{3^2} = \frac{5^3}{2^3}$
 $= \frac{4}{9} = \frac{125}{8}$