MCR3UI Unit 4 Lesson 1

Exponent Laws

1. **Multiplication Law:** $x^m \times x^n = x^{m+n}$

When multiplying powers with the same base, keep the base the same and add the exponents.

Ex 1. $x^3 \times x^2$ (Note: $x^3 \times x^2 = x \cdot x \cdot x \cdot x \cdot x$)

$= x^{3+2}$

$= x^5$

Ex. 2 $2^3 \times 2^4$

$= 2^{3+4}$

$= 2^7$

$= 128$

2. **Division Law:** $x^m \div x^n = x^{m-n}$

When dividing powers with the same base, keep the base the same and subtract the exponents.

Ex. 1 $x^5 \div x^2$

$= x^{5-2}$

$= x^3$

Ex. 2 $2^4 \div 2^3$

$= 2^{4-3}$

$= 2^1$

Note: $2^4 \div 2^3$

$= \frac{2^4}{2^3}$

$= \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$

$= 2$

3. **Power of a Power Law:** $(x^m)^n = x^{m \times n}$

If a power is raised to an exponent, **multiply the exponents.**

Ex. $(x^3)^2 = x^{3\times 2}$

$= x^6$

NOTE: $(x^3)^2 = (x^3)(x^3)$

$= x^{3+3}$

$= x^{3\times 2}$

$= x^6$

4. **Power of a Product Law:** $(x \cdot y)^m = x^m y^m$

If a Product is raised to an exponent, distribute the exponent to each factor in the base.

NOTE: This rule does NOT apply to the power of a sum or difference!

Ex. 1 $(x \cdot y)^5 = x^5 y^5$

Ex. 2 $(3x^5y^3)^2$

$= (3)^2 (x^5)^2 (y^3)^2$

$= 9x^{10}y^6$

NOTE: There is no "Sum of a Power Rule" or "Difference of a Power Rule". $(a + b)^n \neq a^2 + b^2$

(note: the does not equal sign above!)

5. Power of a Quotient Law: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

If a Quotient is raised to an exponent, distribute the exponent to every factor in the numerator and denominator.

Ex. 1 $\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$

$= \frac{x^2}{y^2}$

Ex. 2 $\left(\frac{2}{3}\right)^2$

$= \frac{2^2}{3^2}$

$= \frac{4}{9}$

Ex. 3 $\left(\frac{2x^3}{3y^2}\right)^3 = \frac{(2)^3(x^3)^3}{(3)^3(y^2)^3}$

$= \left(\frac{8x^9}{27y^6}\right)$

6. Zero Exponents: $x^0 = 1$

Any power with an exponent of zero is equal to one.

Ex. 1 $(-2)^0 = 1$

Ex. 2 $-2^0 = -(2^0)$

$= -1$

Ex. 3 $(-237x^3y^7)^0 = 1$

<u>**Proof**</u>:

$3^2 \div 3^2$	$3^2 \div 3^2$
$= \frac{3 \times 3}{3 \times 3}$	$= 3^{2-2}$
$= \frac{9}{9}$	$= 3^0$
$= 1$	So, $3^0 = 1$

7. Negative Exponents: $x^{-m} = \frac{1}{x^m}$

A negative in the exponent of a power means to 'flip the base' or 'take the reciprocal'. A negative exponent has nothing to do with the sign of the number.

Ex. 1 $x^{-2} = \frac{1}{x^2}$

Ex. 2 $4^{-2} = \frac{1}{4^2}$

Ex. 3 $\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$

$= \frac{5^3}{4^3}$

$= \frac{125}{64}$

Ex. 4 $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2$

$= 3^2$

$= 9$

Ex. 4 Simplify first, then evaluate using x = 2.

$(x^{-3})(x^2)(x^5)$

$= x^{-3+2+5}$

$= x^4$

When x = 2,

$= 2^4$

$= 16$

Example 1: Simplify. Leave answers with only positive exponents.

a) $a^5 \times a^2 \times a$

$= a^{5+2+1}$

$= a^8$

b) $(n^2)^3$

$= n^6$

c) $\left(\frac{y^2}{y^3}\right) \times y^{-6} \times y^0$

$= y^{-1-6+0}$

$= y^{-7}$

$= \frac{1}{y^7}$

d) $2a^{-3}$

$= \frac{2}{a^3}$

e) $(4a)^{-2}$

$= \frac{1}{(4)^2(a)^2}$

$= \frac{1}{16a^2}$

f) $\frac{y^0}{2a^{-3}}$

$= \frac{1}{2a^{-3}}$

$= \frac{1}{2} \times \frac{a^3}{1}$

$= \frac{a^3}{2}$

g) $(4a^3c^2)^3(-3ac^{-4})^2$

$= (4)^3(a^3)^3(c^2)^3(-3)^2(a)^2(c^{-4})^2$

$= 64 \times 9a^9a^2c^6c^{-8}$

$= 576a^{11}c^{-2}$

$= \frac{576a^{11}}{c^2}$

h) $\frac{x^{7y+1}}{x^{7y-6}}$

$= x^{(7y+1)-(7y-6)}$

$= x^{1+6}$

$= x^7$

i) $\frac{3^{-1}}{x^0+2^3}$, $x \neq 0$

j) $\frac{2^{-5}+2^{-3}}{2^{-4}}$

$= \frac{2^{-5}}{2^{-4}} + \frac{2^{-3}}{2^{-4}}$

$= 2^{-5+4} + 2^{-3+4}$

$= 2^{-1} + 2$

$= 2\frac{1}{2}$

OR $\frac{2^{-5}+2^{-3}}{2^{-4}} \times \frac{2^5}{2^5}$

$= \frac{2^0+2^2}{2}$

$= \frac{1+4}{2}$

$= \frac{5}{2}$

There are still other ways
to correctly evaluate
this question using exponent laws

Example 2: What is the volume of the cube with side lengths of $x^{-2}y$?

$V = s^3$, where s is the side length of the cube.

$V = (x^{-2}y^3)^3$

$= x^{-6}y^9$

$= \frac{y^9}{x^6}$

Unit 4 Lesson 2 **Laws for Rational Exponents**

Use your calculator to evaluate:

a) $9^{\frac{1}{2}}$
= 3

b) $49^{\frac{1}{2}}$
= 7

c) $625^{\frac{1}{4}}$
= 5

Note: $\sqrt{9} = 3$ $\sqrt{49} = 7$ $\sqrt[4]{625} = 5$

$\therefore 9^{\frac{1}{2}} = \sqrt{9}$ *and* $625^{\frac{1}{4}} = \sqrt[4]{625}$; so the denominator of the exponent determines the "index" of the root.

$a^{\frac{1}{n}} = \sqrt[n]{a}$, where "n" is called the "index", and "a" is called the "radicand".

This is read as the "n^{th} root of a". "What number to the exponent n will equal a?"

8. Powers of the form: $x^{\frac{1}{n}}$

The exponent $\frac{1}{n}$ means to take the n^{th} root. i.e. $x^{\frac{1}{n}} = \sqrt[n]{x}$

Ex 1. $x^{\frac{1}{2}}$
$= \sqrt[2]{x}$
$= \sqrt{x}$

Ex. 2 $x^{\frac{1}{3}}$
$= \sqrt[3]{x}$

Ex. 3 $x^{\frac{1}{12}}$
$= \sqrt[12]{x}$

Ex. 4 $81^{\frac{1}{2}}$
$= \sqrt{81}$
$= 9$

Ex. 5 $(-27)^{\frac{1}{3}}$
$= \sqrt[3]{-27}$
= -3

Ex. 6 $(-64)^{\frac{1}{4}}$
$= \sqrt[4]{-64}$
= not possible

Ex. 7 $(64)^{\frac{1}{3}}$
$= \sqrt[3]{64}$
= 4

Ex. 8 $(64)^{\frac{1}{6}}$
$= \sqrt[6]{64}$
= 2

(You may take the odd root of a negative number but
you may **not** take the even root of a negative number)

MCR 3UI Unit 4 Lesson 2

Laws for Rational Exponents

Evaluate: $8^{\frac{2}{3}}$

$= (8^{\frac{1}{3}})^2$

$= (\sqrt[3]{8})^2$

$= 2^2$

$= 4$

'Undo' the Power of a Power Law

9. Powers of the form: $x^{\frac{m}{n}}$

The exponent $\frac{m}{n}$ means to take the n^{th} root and raise the answer to an exponent m.

i.e., $x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{(x^m)}$ * the denominator of the exponent is the index of the root.

Ex 1. $x^{\frac{3}{4}}$

$= \sqrt[4]{x^3}$

Or $= (\sqrt[4]{x})^3$

Ex. 2 $x^{\frac{2}{3}}$

$= \sqrt[3]{x^2}$

Or $= (\sqrt[3]{x})^2$

Ex. 3 $81^{\frac{3}{4}}$

$= (\sqrt[4]{81})^3$

$= (3)^3$

$= 27$

Ex. 4 $(-125)^{\frac{2}{3}}$

$= (\sqrt[3]{-125})^2$

$= (-5)^2$

$= 25$

Ex. 5 $9^{\frac{3}{2}}$

$= (\sqrt{9})^3$

$= 2^3$

$= 8$

Ex. 6 $9^{-2.5}$

$= 9^{-\frac{5}{2}}$

$= \frac{1}{(\sqrt{9})^5}$

$= \frac{1}{3^5}$

$= \frac{1}{243}$

Ex. 7 $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

$= \left(\frac{8}{27}\right)^{\frac{2}{3}}$

$= \frac{(\sqrt[3]{8})^2}{(\sqrt[3]{27})^2}$

$= \frac{2^2}{3^2}$

$= \frac{4}{9}$

Ex. 8 $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$

$= \left(\frac{25}{4}\right)^{\frac{3}{2}}$

$= \frac{(\sqrt{25})^3}{(\sqrt{4})^3}$

$= \frac{5^3}{2^3}$

$= \frac{125}{8}$