1. **Multiplication Law:** \( x^m \times x^n = x^{m+n} \)
   When multiplying powers with the same base, keep the base the same and add the exponents.
   - Ex. 1: \( x^3 \times x^2 \) (Note: \( x^3 \times x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x \))
     \[ = x^{3+2} = x^5 \]
   - Ex. 2: \( 2^3 \times 2^4 \)
     \[ = 2^{3+4} = 2^7 = 128 \]

2. **Division Law:** \( x^m \div x^n = x^{m-n} \)
   When dividing powers with the same base, keep the base the same and subtract the exponents.
   - Ex. 1: \( x^5 \div x^2 \)
     \[ = x^{5-2} = x^3 \]
   - Ex. 2: \( 2^4 \div 2^3 \)
     \[ = 2^{4-3} = 2 \]

3. **Power of a Power Law:** \( (x^m)^n = x^{m \times n} \)
   If a power is raised to an exponent, multiply the exponents.
   - Ex. \( (x^3)^2 = x^{3 \times 2} \)
     \[ = x^6 \]
   - NOTE: \( (x^3)^2 = (x^3)(x^3) \)
     \[ = x^{3+3} = x^6 \]

4. **Power of a Product Law:** \( (x \cdot y)^m = x^m y^m \)
   If a Product is raised to an exponent, distribute the exponent to each factor in the base.
   NOTE: This rule does NOT apply to the power of a sum or difference!
   - Ex. 1: \( (x \cdot y)^5 = x^5 y^5 \)
   - Ex. 2: \( (3x^5 y^3)^2 \)
     \[ = (3)^2(x^5)^2(y^3)^2 \]
     \[ = 9x^{10}y^6 \]

**NOTE:** There is no “Sum of a Power Rule” or “Difference of a Power Rule”. \((a + b)^n \neq a^n + b^n\)
(note: the does not equal sign above!)
5. **Power of a Quotient Law:**  \( \left( \frac{x}{y} \right)^m = \frac{x^m}{y^m} \)

If a Quotient is raised to an exponent, distribute the exponent to every factor in the numerator and denominator.

**Example 1** \( \left( \frac{x}{y} \right)^2 = \frac{x^2}{y^2} \)

**Example 2** \( \left( \frac{2}{3} \right)^2 = \frac{2^2}{3^2} = \frac{4}{9} \)

**Example 3** \( \left( \frac{2x^3}{3y^2} \right)^3 = \frac{(2x^3)^3}{(3y^2)^3} = \left( \frac{8x^9}{27y^6} \right) \)

6. **Zero Exponents:**  \( x^0 = 1 \)

Any power with an exponent of zero is equal to one.

**Example 1** \( (-2)^0 = 1 \)

**Example 2** \( -2^0 = -(-2^0) = -1 \)

**Example 3** \( (-237x^3y^7)^0 = 1 \)

**Proof:**

\[
3^2 + 3^2 = 9 + 9 = 18
\]

\[
\frac{3 \times 3}{3 \times 3} = \frac{9}{9} = 1
\]

So, \( 3^0 = 1 \)

7. **Negative Exponents:**  \( x^{-m} = \frac{1}{x^m} \)

A negative in the exponent of a power means to ‘flip the base’ or ‘take the reciprocal’. A negative exponent has nothing to do with the sign of the number.

**Example 1** \( x^{-2} = \frac{1}{x^2} \)

**Example 2** \( 4^{-2} = \frac{1}{4^2} \)

**Example 3** \( \left( \frac{4}{5} \right)^{-3} = \left( \frac{5}{4} \right)^3 \)

**Example 4** \( \left( \frac{1}{3} \right)^{-2} = \left( \frac{3}{1} \right)^2 \)

\[
= \frac{5^3}{4^3} = 3^2
\]

Ex. 4 Simplify first, then evaluate using \( x = 2 \).

\[
\left( x^{-3} \right) \left( x^2 \right) \left( x^5 \right) = x^{-3+2+5} = x^4
\]

When \( x = 2 \),

\[
= 2^4 = 16
\]
Example 1: Simplify. Leave answers with only positive exponents.

a) \( a^5 \times a^2 \times a \)  
\[ = a^{5+2+1} = a^8 \]

b) \( (n^2)^3 \)  
\[ = n^6 \]

c) \( \left(\frac{y^2}{y^3}\right) \times y^{-6} \times y^0 \)  
\[ = y^{-1-6+0} = \frac{2}{a^3} = \frac{1}{4} \]

d) \( 2a^{-3} \)  
\[ = \frac{1}{16a^2} \]

e) \( (4a)^{-2} \)  
\[ = \frac{1}{y^7} \]

f) \( \frac{y^0}{2a^{-3}} \)  
\[ = \frac{1}{2a^{-3}} \]

g) \( (4a^3c^2)^3(-3ac^4)^2 \)  
\[ = (4)^3(a^3)^3(c^2)^3(-3)(a)^2(c^4)^2 = 64 \times 9a^9a^2c^6c^8 = 576a^{11}c^{-2} = \frac{576a^{11}}{c^2} \]

h) \( \frac{x^7y^1}{x^7y^6} \)  
\[ = x^{7y+1} = x^7 \]

i) \( \frac{3^{-1}}{x^{0+2}} \)  
\[ = x^{-1} + 6 = x^{1+6} = x^7 = 2^{-1} + 2 = 2^{1/2} \]

\[ \text{OR} \quad \frac{2^{-5} + 2^{-3}}{2^4} = \frac{2^{0+2^2}}{2^4} = \frac{5}{2} \]

There are still other ways
to correctly evaluate
this question using exponent laws

Example 2: What is the volume of the cube with side lengths of \( x^2y \)?

\( V = s^3 \), where \( s \) is the side length of the cube.
\[ V = (x^2y^3)^3 \]
\[ = x^6y^9 \]
\[ = \frac{y^9}{x^6} \]
Use your calculator to evaluate:

\[ a) \ 9^{\frac{1}{2}} \quad b) \ 49^{\frac{1}{2}} \quad c) \ 625^{\frac{1}{4}} \]

\[ = 3 \quad = 7 \quad = 5 \]

Note: \( \sqrt{9} = 3 \quad \sqrt[4]{49} = 7 \quad \sqrt[4]{625} = 5 \)

\[ \therefore \ 9^\frac{1}{2} = \sqrt{9} \text{ and } 625^{\frac{1}{4}} = \sqrt[4]{625} ; \text{ so the denominator of the exponent determines the} \]

"index" of the root.

\[ a^n = \sqrt[n]{a} \text{, where "n" is called the "index", and "a" is called the "radicand".} \]

This is read as the "n\text{th} root of a". "What number to the exponent n will equal a?"

8. **Powers of the form: \( x^n \)**

The exponent \( \frac{1}{n} \) means to take the \( n\text{th} \) root. i.e. \( x^\frac{1}{n} = \sqrt[n]{x} \)

Ex 1. \( x^\frac{1}{2} \quad \text{Ex. 2} \quad x^\frac{1}{3} \quad \text{Ex. 3} \quad x^\frac{1}{12} \quad \text{Ex. 4} \quad 81^\frac{1}{2} \)

\[ = \sqrt{x} \quad = \sqrt[3]{x} \quad = \sqrt[12]{x} \quad = \sqrt{81} \quad = 9 \]

Ex. 5 \( (-27)^\frac{1}{3} \quad \text{Ex. 6} \quad (-64)^\frac{1}{4} \quad \text{Ex. 7} \quad (64)^\frac{1}{3} \quad \text{Ex. 8} \quad (64)^\frac{1}{6} \)

\[ = \sqrt[3]{-27} \quad = \sqrt[4]{-64} \quad = \sqrt[3]{64} \quad = \sqrt[6]{64} \]

\[ = -3 \quad = \text{not possible} \quad = 4 \quad = 2 \]

(You may take the odd root of a negative number but you may **not** take the even root of a negative number)
### Laws for Rational Exponents

Evaluate: \[ \frac{2}{8^3} \]

`Undo` the Power of a Power Law

\[ = (8^3)^2 \]
\[ = \left(\frac{3}{\sqrt[3]{8}}\right)^2 \]
\[ = 2^2 \]
\[ = 4 \]

9. **Powers of the form:** \( x^n \)

The exponent \( \frac{m}{n} \) means to take the \( n \)th root and raise the answer to an exponent \( m \).

i.e., \( x^n = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m} \) *the denominator of the exponent is the index of the root.*

<table>
<thead>
<tr>
<th>Ex. 1</th>
<th>x⁴</th>
<th>Ex. 2</th>
<th>x³</th>
<th>Ex. 3</th>
<th>8¹⁴</th>
<th>Ex. 4</th>
<th>(-125)³³</th>
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</thead>
<tbody>
<tr>
<td>( = \frac{3}{\sqrt[3]{x^3}} )</td>
<td>( = \frac{3}{\sqrt[2]{x^2}} )</td>
<td>( = (\sqrt[3]{8})^3 )</td>
<td>( = (\sqrt[3]{125})^3 )</td>
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<td>Or ( = \left(\frac{3}{\sqrt[3]{x}}\right)^3 )</td>
<td>Or ( = \left(\frac{3}{\sqrt[2]{x}}\right)^2 )</td>
<td>( = (3)^3 )</td>
<td>( = (-5)^2 )</td>
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<th>Ex. 5</th>
<th>9²</th>
<th>Ex. 6</th>
<th>9⁻².₅</th>
<th>Ex. 7</th>
<th>( \left(\frac{27}{8}\right)^{-\frac{2}{3}} )</th>
<th>Ex. 8</th>
<th>( \left(\frac{4}{25}\right)^{-\frac{3}{2}} )</th>
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<tbody>
<tr>
<td>( = (\sqrt{9})^3 )</td>
<td>( = 9^{-\frac{5}{2}} )</td>
<td>( = \left(\frac{8}{27}\right)^{\frac{2}{3}} )</td>
<td>( = \left(\frac{25}{4}\right)^{\frac{3}{2}} )</td>
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<tr>
<td>= 2³</td>
<td>= ( \frac{1}{(\sqrt{9})^5} )</td>
<td>( = \frac{(\sqrt[3]{8})^2}{(\sqrt[3]{27})^2} )</td>
<td>( = \frac{(\sqrt[4]{25})^3}{(\sqrt[4]{4})^3} )</td>
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<td>( = \frac{5^3}{2^3} )</td>
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<td>= ( \frac{1}{243} )</td>
<td>= ( \frac{4}{9} )</td>
<td>= ( \frac{125}{8} )</td>
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