U4D1 MCR3UI

UNIT TEST: Thurs. April 18, 2019
QUIZ: Monday, April 8/19

Exponent Laws

Integral Exponents

a^n

Positive Integral Exponent
$a^n =$
Negative Integral Exponent
$a^{-n} =$
Zero Exponent
$a^0 =$

An expression of the form a^n is called a _________.

Law for...	General Form	Example
Multiplication of powers	$x^m \cdot x^n =$	$5^4 \cdot 5^7 =$
Division of Powers	$\frac{x^m}{x^n} =$	$\frac{4^6}{4^2} =$
Power of a Power	$(x^m)^n =$	$(6^5)^2 =$
Power of a Product	$(xy)^n =$	$(3y)^3 =$
Power of a Quotient	$\left(\frac{x}{y}\right)^n =$	$\left(\frac{3}{2}\right)^4 =$

Example 1. Simplify. Express your answer with positive exponents

a) $x^{-3} \cdot x^{-5}$

b) $m^2 \div m^{-3}$

c) $\frac{a^5b^3}{a^2b^2}$

d) $(-2c^3d^{-5}e)^2$

e) $(4x^3y^2)(7x^2y^4)$

f) $\left(\frac{3x^2}{z^3}\right)^2$

g) $\frac{(2x^{-2}y)^3}{10x^{-4}y^{-3}}$

h) $\frac{(-2x^{-3}y)(-12x^{-4}y^{-2})}{6xy^{-3}}$

Example 2. Evaluate. Answers should be left as reduced fractions (decimal answers are not acceptable). Do not use a calculator!!!

a) $\left(\frac{3}{4}\right)^{-2}$

b) $\frac{(-6)^0}{2^{-3}}$

c) $\frac{2^{-4}+2^{-6}}{2^{-3}}$

d) $\frac{3^{-5}}{3^{-4}+3^{-3}}$

Pg. 9 #2-9 (every other one for each questions) #11, 12a

U4D2 MCR3UI **Rational Exponents**

$$a^{\frac{m}{n}} =$$

Think of ______ as the ______________ and _____ as the _______.

To Evaluate:

Either:

- Take the 'nth' ________ of 'a' and then raise the answer to the _____________ 'm'

  OR

- Raise 'a' to the _______________ 'm' and then take the 'nth' ____________ of the answer

Remember all exponent laws apply when simplifying rational exponents.

Example 1: Evaluate....do not use a calculator!

a) $25^{\frac{3}{2}}$

b) $(-27)^{-\frac{1}{3}}$

c) $-9^{2.5}$

d) $4^{\frac{3}{2}} \div 16^{\frac{1}{4}}$

Example: Write using exponents, in fully simplified form.

a) $\sqrt[3]{\sqrt{2x^4}}$

b) $\left(\sqrt[3]{a^2b^4}\right)^5$

Pg. 16 #1-5 (every other one for each questions), 6agm
#10, 11a
U4D2 Worksheet Extra Practice

Simplifying Expressions Using Exponent Laws

1. Simplify

   a) $5a^{-3}\times 8a^{-9}$　　b) $-24c^5d^3\div 4c^8d^{-3}$　　c) $m^2n^5\times m^3n^{-7}$

   d) $\left(\dfrac{24c^8d^5}{-8c^2d}\right)\left(\dfrac{15c^3d^9}{18cd^5}\right)$　　e) $\dfrac{12m^5n^{-2}\times 5m^{-11}n^6}{15m^3n^{-4}}$　　f) $(xy^{\frac{2}{3}})^6\div(x^{\frac{1}{2}}y^{\frac{1}{4}})^8$

2. Write in radical form, then evaluate.

   a) $81^{\frac{3}{4}}$　　b) $16^{\frac{-3}{4}}$　　c) $625^{0.75}$　　d) $4^{-\frac{3}{2}}$　　e) $8^{\frac{4}{3}}$

3. Evaluate. Do not convert fraction answers to decimals.

   a) $\left(\dfrac{1}{9}\right)^{-\frac{3}{2}}$　　b) $\left(-\dfrac{1}{32}\right)^{0.8}$　　c) $\left(\dfrac{49}{25}\right)^{\frac{1}{2}}$　　d) $\left(-\dfrac{27}{125}\right)^{\frac{4}{3}}$　　e) $\left(\dfrac{625}{343}\right)^{0}$

4. Evaluate.

   a) $32^{\frac{2}{5}}\times 243^{\frac{2}{5}}$　　b) $64^{\frac{2}{3}}\times 125^{\frac{1}{3}}$　　c) $4^{\frac{5}{2}}\times 81^{\frac{3}{4}}$

5. Simplify.

   a) $a^{\frac{1}{2}}\times a^{-\frac{1}{2}}$　　b) $\left(n^{\frac{1}{2}}\right)^{-6}$　　c) $x^{\frac{-3}{2}}\div x^{-\frac{1}{4}}$　　d) $(9a^4b^{-2}\times 4a^2b^{-6})^{\frac{1}{2}}$　　e) $8m^{\frac{1}{3}}n^{\frac{-3}{2}}(-2m^{\frac{-2}{3}}n^{\frac{1}{3}})^{-4}$

6. Simplify.

   a) $\dfrac{36x^{-2}y^3z^{-4}}{12xy^{-2}z^{-2}}$　　b) $\sqrt{\dfrac{32x^{-5}y^2\times 18x^2y}{4xy^{-3}}}$　　c) $\left(\dfrac{3x^{-2}y^3}{12xy^{-1}}\right)\left(\dfrac{10x^4y^{-2}}{5x^{-1}y^2}\right)$

   d) $\dfrac{8^{1-2x}\times 4^{2x+3}}{16^{2-3x}}$　　e) $\dfrac{16^{2m-n}\times 9^{m+3n}}{27^{m+n}\times 8^{m-n}}$　　f) $\dfrac{5^{-200}-5^{-198}}{5^{-199}+5^{-200}}$

7. Simplify.

   a) $\dfrac{(c^{a+b})(c^{a-b})}{c^2}$　　b) $\dfrac{(x^a)^2(x^b)^2}{(x^{a+b})(x^{a-b})}$　　c) $\dfrac{x^{2a-b}\cdot x^{a-3b}}{(x^{3a+b})^{-2}}$　　d) $\dfrac{(m^{x-1})(m^{2x+5})}{m^{3x-1}}$　　e) $\dfrac{3^{-6a}+3^{-5a}}{3^{-6a}+3^{-7a}}$

8. Evaluate.

   a) $(5^{\frac{1}{2}}+2^{\frac{1}{2}})(5^{\frac{1}{2}}-2^{\frac{1}{2}})$　　b) $(8^{\frac{2}{3}}-5^{\frac{1}{2}})(8^{\frac{2}{3}}+5^{\frac{1}{2}})$

9. Simplify.

   a) $\left(\sqrt{49y^{\frac{2}{m}}}\right)^{\frac{-1}{n}}$　　b) $\sqrt[3]{\dfrac{m^{\frac{1}{2}}\sqrt{mn}}{\frac{1}{\sqrt{n}}}}$　　c) $\left(\dfrac{\sqrt[4]{a^{2n-1}}\times\sqrt[4]{a}}{\sqrt{a}}\right)^2$

ANSWERS:

1a) $\dfrac{40}{a^{12}}$　b) $\dfrac{-6d^6}{c^3}$　c) $\dfrac{m^5}{n^2}$　d) $\dfrac{-5c^8d^8}{2}$　e) $\dfrac{4n^8}{m^9}$　f) x^2y^2

2a) 27　b) $\dfrac{1}{8}$　c) 125　d) $\dfrac{1}{8}$　e) 16

3a) 27　b) $\dfrac{1}{16}$　c) $\dfrac{7}{5}$　d) $\dfrac{81}{625}$　e) 1

4a) 36　b) 80　c) 864

5a) 1　b) $\dfrac{1}{n^3}$　c) $\dfrac{1}{x^{\frac{5}{4}}}$　d) $\dfrac{6a^3}{b^4}$　e) $\dfrac{m^3}{2n^{\frac{17}{6}}}$

6a) $\dfrac{3y^5}{x^3z^2}$　b) $\dfrac{12y^3}{x^2}$　c) $\dfrac{x^2}{2}$　d) 2^{10x+1}　e) $2^{5m-n}3^{3n-m}$　f) -4

7a) c^{2a-2}　b) x^{2b}　c) x^{9a-2b}　d) m^5　e) 3^a

8 a) 3　b) 11

9a) $\dfrac{1}{7^{\frac{1}{n}}y^{\frac{1}{mn}}}$　b) $m^{\frac{1}{3}}n^{\frac{1}{3}}$　c) a^{n-1}

U4D3 MCR3UI **Solving Exponential Equations**

Warm Up:

Simplify.

a) $3^x \cdot 3^4$

b) $\sqrt{\sqrt[5]{x^4y^6}}$

Solving Exponential Equations

Method 1: Using a common base If there is a ________________ base, you can ________________ the exponents. This gives a linear equation that you can ________________.

a) $4^x = 4^5$

b) $2^{x+3} = 2^{2x-1}$

Method 1 con't: If the bases are NOT the ____________, you can either make them the same OR Method 2: you can use a ______________________________ to figure out the value of the unknown (trial and error).

c) $3^x = 27$

d) $4^{3k} = 64$

Method 1:

Method 2:

e) $4^x = 8^{x-1}$

Method 1: Method 2:

Examples Involving Rationals:

a) $3^{3x-1} = \frac{1}{81}$ b) $27(3^{3x+1}) = 9$ c) $2(5^{k+1}) = 1250$

Example Involving Common Factor:

$3^{x+2} - 3^x = 216$

Pg. 23 #1-6 (every other one for each questions), 9abe, 10abf

Warm Up: a) $(2a^2bc^3)(-6a^4bc)^{-2}$ b) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

U4D4__MCR3UI **<u>Exploring Properties of Exponential Functions</u>**

Investigation:

1. Complete the following tables.

i)

x	y=x
0	
1	
2	
3	
4	
5	
6	

ii)

x	y=2x
0	
1	
2	
3	
4	
5	
6	

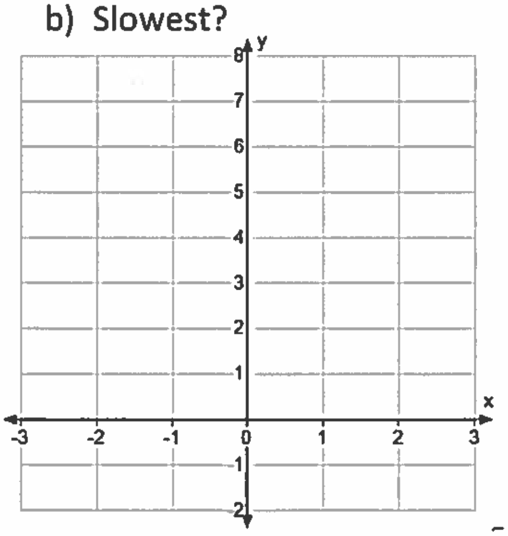
iii)

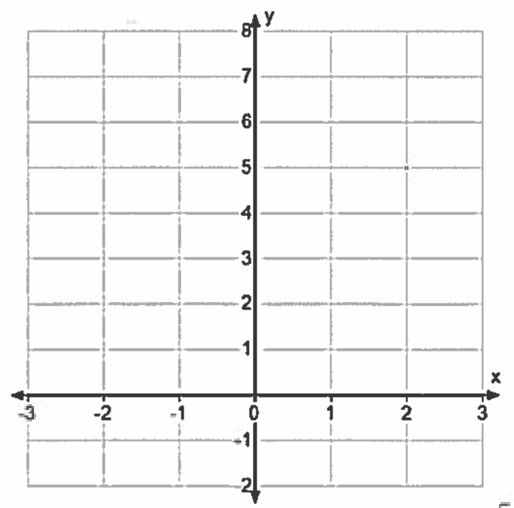
x	$y=x^2$
0	
1	
2	
3	
4	
5	
6	

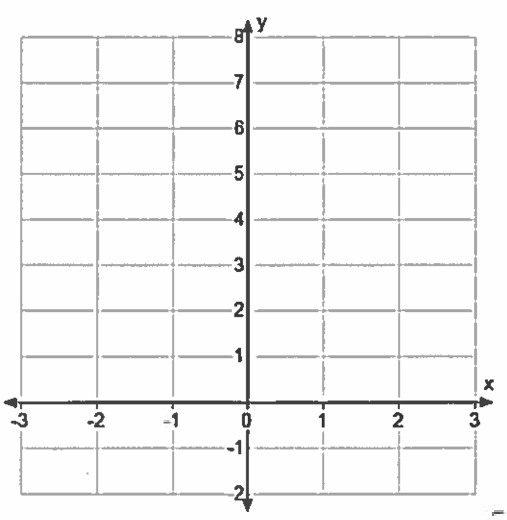
iv)

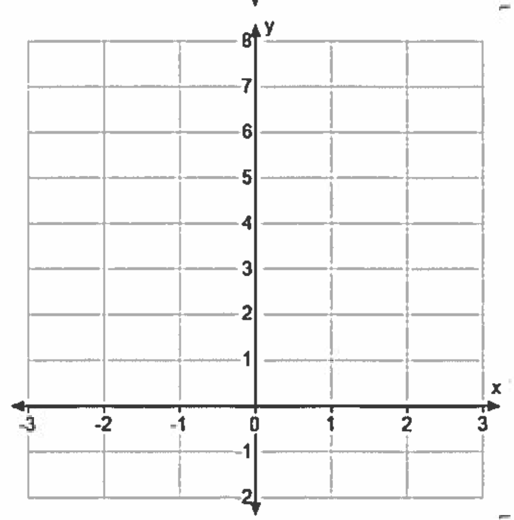
x	$y=2^x$
0	
1	
2	
3	
4	
5	
6	

2. Which pattern is growing:
   a) Fastest?
   b) Slowest?









3. Complete the First and second differences.

x	y=x	First Differences	Second Differences
0	0		
1	1		
2	2		
3	3		
4	4		
5	5		
6	6		

x	y=2x	First Differences	Second Differences
0	0		
1	2		
2	4		
3	6		
4	8		
5	10		
6	12		

x	$y=x^2$	First Differences	Second Differences
0	0		
1	1		
2	4		
3	9		
4	16		
5	25		
6	36		

x	$y=2^x$	First Differences	Second Differences
0	1		
1	2		
2	4		
3	8		
4	16		
5	32		
6	64		

4. What do you notice about the finite differences?

5. Complete the following tables.

i)

x	$y=3^x$	First Differences	Second Differences
0			
1			
2			
3			
4			

ii)

x	$y=0.5^x$	First Differences	Second Differences
0			
1			
2			
3			
4			

6. How do $y = 3^x$ and $y = 0.5^x$ compare with $y=2^x$?

7. Complete the following chart.

	$y=2^x$	$y=3^x$	$y=0.5^x$
Domain			
Range			
x-intercepts?			
y-intercept			
Interval of increase			
Interval of decrease			
Description of graph			
Sketch of graph			
Asymptotes ?			

8. Sam's mom told him that if he consistently does all of his chores, each day she will give him double the amount that was given the previous day. She gives him $0.50 the first day.

(a) Assuming Sam does his chores consistently, how much money will his mom give him on the fourth day?

(b) Sam is saving up to buy a new $300 graphics card for his computer. On what day can he buy his graphics card?

Properties of Exponential Functions:

- As the independent variable increases by a constant amount, the dependent variable increases by a ________________ ______________.
  (As the independent variable increases by one, the dependent variable increases by a ______________ _________________ equal to the ____________ of the exponential function.)
- The ________________ of consecutive finite differences is a constant.
- For bases ___________________ than 1, the graph ___________________ at a constant rate (the slope of the graph gets steeper as x increases)
- For bases ___________________ 0 and 1, the graph ___________________ at a constant rate (the slope of the graph gets less steep as x increases)
- $b^0 = 1$, for all $b \in R$, $b \neq 0$

Booklet day 1
#1-4, **5,6**

U4D5 MCR3UI

Graphing Exponential Functions and Determining Exponential Equations of the form y=a(b)^x

Warm Up:
Simplify.

a) $\left(\frac{2x^2}{yz^3}\right)^2\left(\frac{y^2z^3}{2x^4}\right)^3$

b) $81^{-\frac{1}{2}} \div 27^{-\frac{2}{3}}$

c) $\frac{(y^{x-1})(y^{2x+5})}{y^{3x-1}}$

d) $y = \frac{1}{8}(2)^{n-1}$

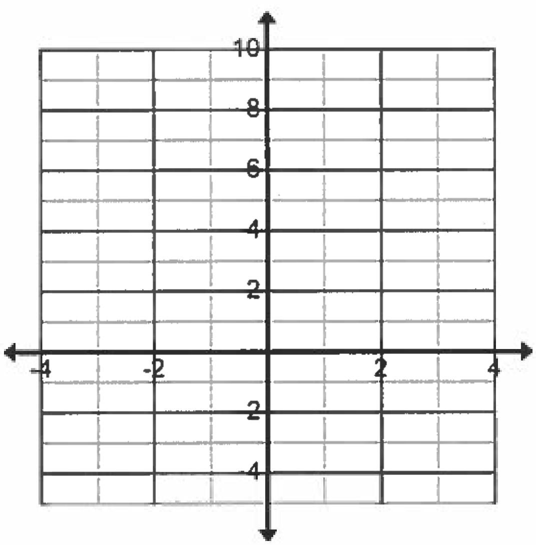
e) $y = 12(3)^{n+2}$

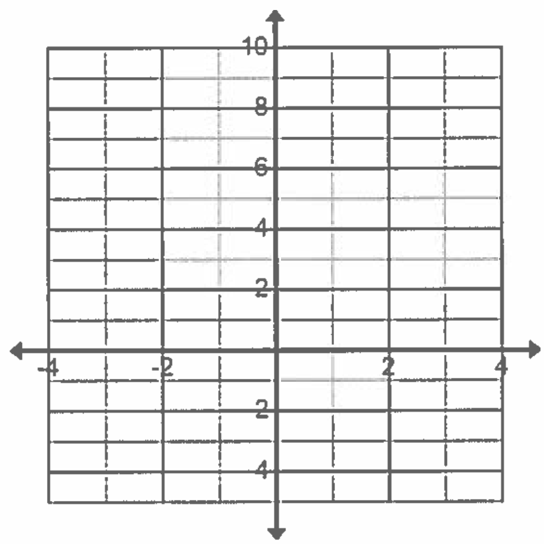
f) $y = \frac{(2)^{n-1}(4)^n}{(8)^{n-4}}$

Graphing Base Exponential Functions

$f(x) = 2^x$

$g(x) = \left(\frac{1}{3}\right)^x$



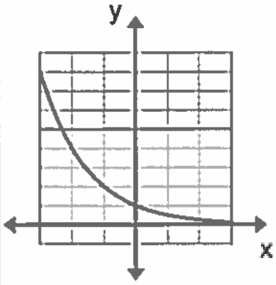
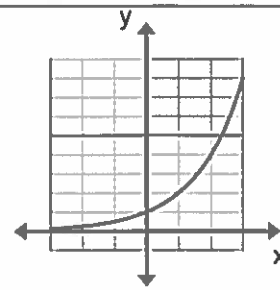
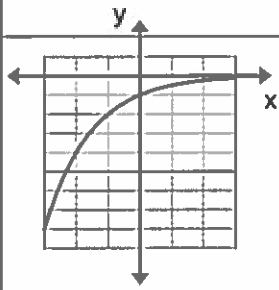
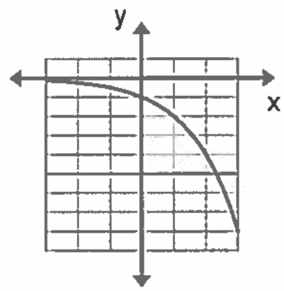


Determining the Equation of an Exponential Function

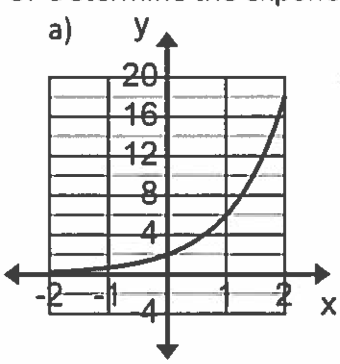
1. Complete the chart to compare the effect of changing the value of a in y=a(2^x).

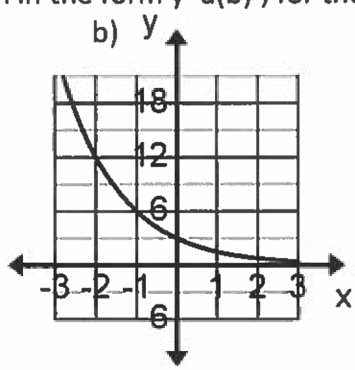
	f(x)=2^x	y=3(2)^x	y=0.5(2)^x	y=-(2)^x	y=-3(2)^x
Domain					
Range					
y-intercept					
asymptote					
Inc./dec.					

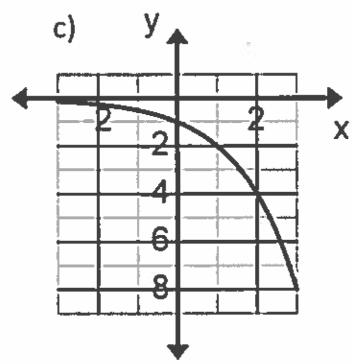
2. Summary: <u>Exponential Equations of the form $y=a(b)^x$</u>

	$0 < b < 1$	$b > 1$
$a > 0$ (i.e. a is positive)	 ______________ on $x \epsilon R$ D={ } R={ } Horizontal Asymptote: ______ y-intercept: y=___	 ______________ on $x \epsilon R$ D={ } R={ } Horizontal Asymptote: ______ y-intercept: y= ____
$a < 0$ (i.e. a is negative)	 ______________ on $x \epsilon R$ D={ } R={ } Horizontal Asymptote: ______ y-intercept: y= ____	 ______________ on $x \epsilon R$ D={ } R={ } Horizontal Asymptote: ______ y-intercept: y= ____

3. Determine the exponential equation in the form $y=a(b)^x$, for the given graphs.

a)


b)


c)


4. Write an Exponential Function given the properties within each situation below (solution on back):

i) A bacteria colony doubles every hour. The initial population contained 5 bacteria. Write a function to relate the population of bacteria to the time, in hours.

ii) A radioactive sample has a half-life of 3 days. The initial sample is 200 mg. Write a function to relate the amount remaining, in milligrams, to the time, in days. Then, determine the <u>range</u> for the radioactive sample.

Booklet day 2
#1-7, **8,9**

U4D6 MCR3UI **TRANSLATIONS AND RELFECTIONS OF EXPONENTIAL FUNCTIONS**

Warm Up:

Describe the transformations that have occurred to f(x) to obtain the following function:

$$y = -f(x+3) - 7$$

If $f(x) = \frac{1}{x}$, what would be the horizontal and vertical asymptote equations for the transformed function above?

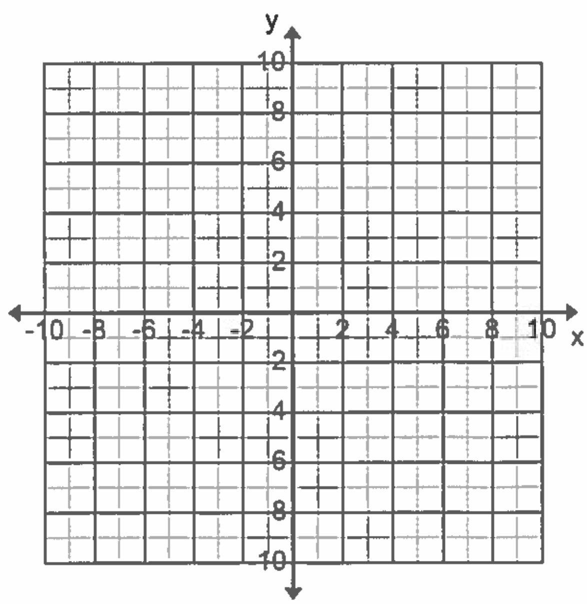
Translations and Reflections of Exponential Functions

1. Each function given below is a translation and/or reflection of the exponential function $f(x)=3^x$. For each of these transformations, write the equation as a transformation of $f(x)=3^x$ in function notation. Then, describe how $f(x)=3^x$ should be shifted and/or reflected to obtain the new graph of the transformed function.

Function	$y=3^x+1$	$y=3^{x-2}$	$y=3^{x+4}$
Function Notation			
Description of Transformation			

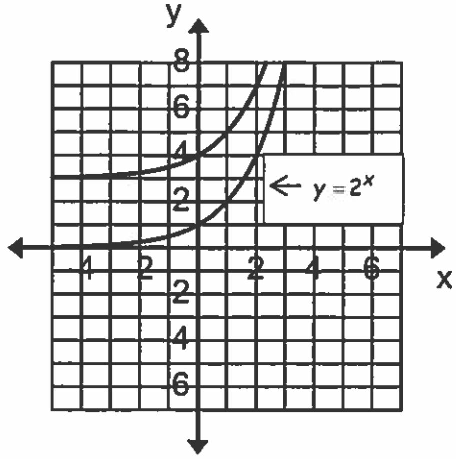
Function	$y=-3^x$	$y=3^{-x}$	$y=-3^{x+3}-1$
Function Notation			
Description of Transformation			

2. Draw the graph of $f(x)=\left(\frac{1}{2}\right)^x$ and the transformation $y=-f(x+3)-5$. What is the equation of the transformed function?



3. Given the original graph $y = 2^x$ and each of the following four transformations, Describe each of the transformations and write the new equation.

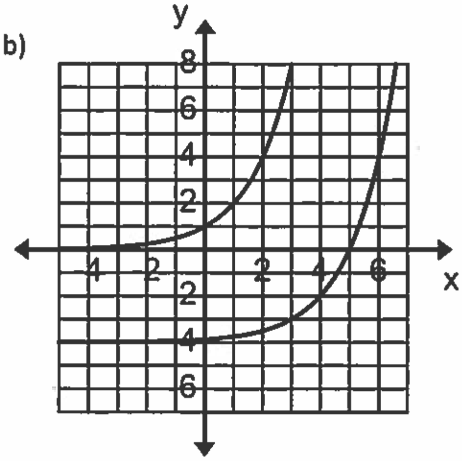
a)



Description:

New Equation:

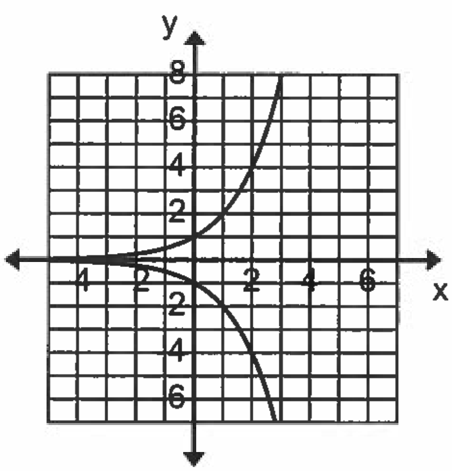
b)



Description:

New Equation:

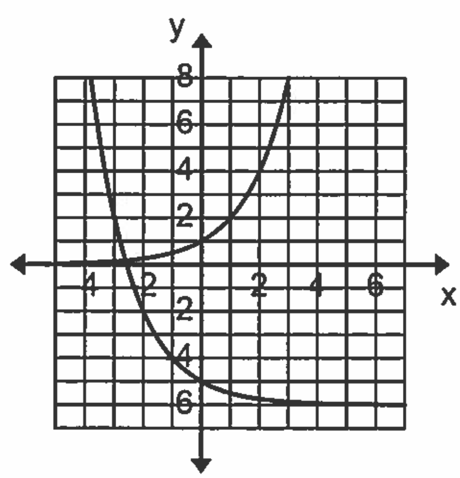
c)



Description:

New Equation:

d)



Description:

New Equation:

General Equation of Exponential Functions:

$$y = ab^{k(x-d)} + c$$

$a < 0 \Rightarrow$ reflection in x-axis

$k < 0 \Rightarrow$ reflection in y-axis

$d \Rightarrow$ shift right/left ($d > 0 \Rightarrow$ right, $d < 0 \Rightarrow$ left)

$c \Rightarrow$ shift up/down ($c > 0 \Rightarrow$ up, $c < 0 \Rightarrow$ down)

Booklet day 3
1-3

Transformations of Exponential Functions

General Equation of an Exponential Function:

$a < 0 \Rightarrow$ reflection in ____-axis

$k < 0 \Rightarrow$ reflection in ____-axis

$0 < |a| < 1 \Rightarrow$ Vertical Compression factor

$|a| > 1 \Rightarrow$ Vertical Stretch factor

$0 < |k| < 1 \Rightarrow$ Horizontal Stretch factor

$|k| > 1 \Rightarrow$ Horizontal Compression factor

$d \Rightarrow$ shift right/left ($d > 0 \Rightarrow$ ________, $d < 0 \Rightarrow$ ________)

$c \Rightarrow$ shift up/down ($c > 0 \Rightarrow$ ________, $c < 0 \Rightarrow$ ________)

1. Match each transformation with the corresponding equation, using $f(x) = 10^x$ as the base. Not all transformations will match an equation.

Transformation	Equation
a) Horizontal stretch factor 3	A $y = 10^x + 3$
b) Shift 3 units up	B $y = 10^{x+3}$
c) Shift 3 units left	C $y = -10^x$
d) Vertical compression factor $\frac{1}{3}$	D $y = 10^x - 3$
e) Vertical stretch factor 3	E $y = 10^{3x}$
f) Shift 3 units right	F $y = 10^{-x}$
g) Reflect in x-axis	G $y = \left(\frac{1}{3}\right)10^x$
h) Shift 3 units down	
i) Horizontal compression factor $\frac{1}{3}$	

2. Given the function defined by the equation : $y = 2(3)^{4(x-2)} + 7$

a) State the base/parent function.

b) Is this function increasing or decreasing?

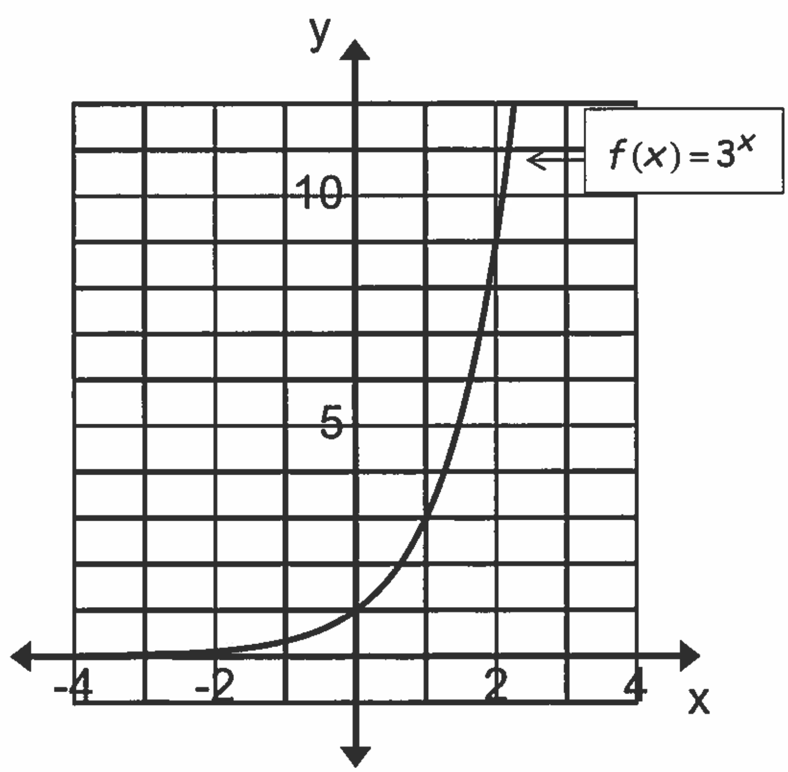
c) Describe the transformations (in order) to the exponential function compared to the parent function. Use the technical vocabulary you have learned this year.

d) State the y-intercept.

e) State the equation of the asymptote.

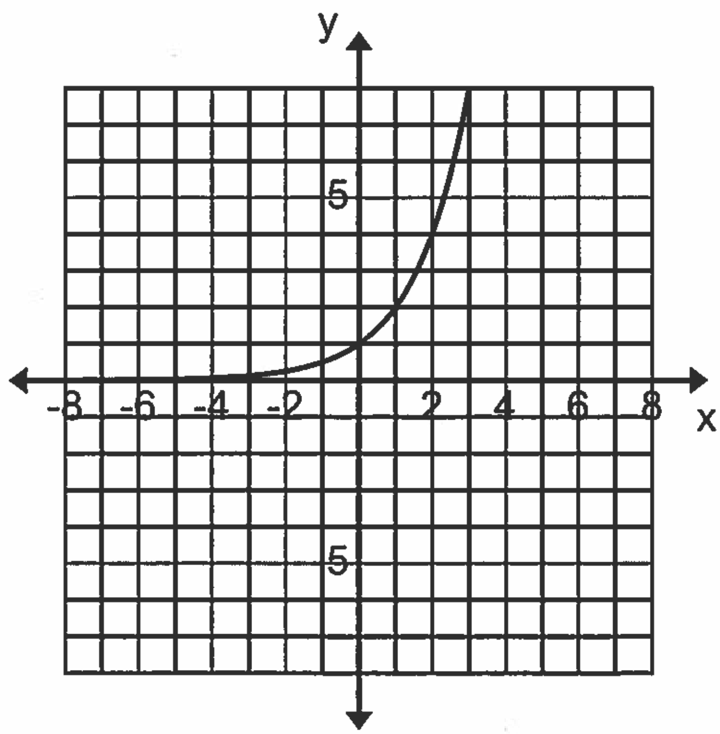
f) State the domain and range of this function.

3. Given $f(x) = 3^x$, graph $y = \frac{1}{2}(3)^{\frac{1}{2}x}$ and describe the transformations.



4. a) Identify the transformations of $f(x) = 2^x$ that will produce the graph of $y = -f(-2x + 6) + 5$, and determine the new equation.

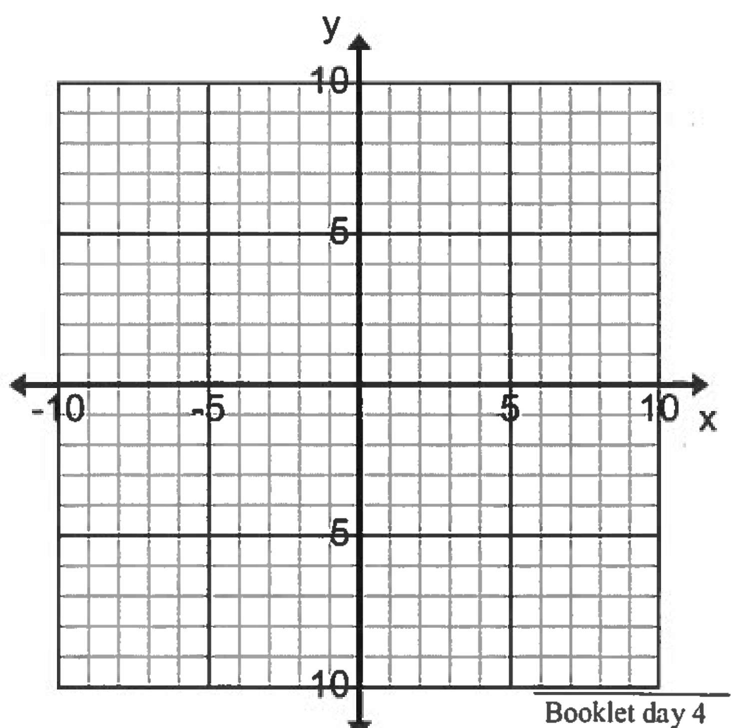
y =



b) Graph the transformation. Label the final graph with its equation.

5. Apply the appropriate transformations to the exponential function to graph the following and state the domain and range.

$$y = -2\left(\frac{1}{2}\right)^{3-x} + 6$$



D: {   }

R: {   }

Booklet day 4
1-6

U4D8 Warm Up:

A bacteria colony doubles every minute. If there are 10 bacteria in the colony initially, how many are there in 9 minutes?

In general, for exponential growth / decay problems:

where,

$f(x)$ is the ______________________ value

a is the ______________________ value

b is the __________________ (if b > 1) OR

the ________________ (if 0 < b < 1)

x is the number of ___________________ or ________________ periods

Important Notes:

If a growth rate is given (as a percent), then the base of the power in the equation (b) can be obtained by

____

ex. A growth rate of 18% involves

Also, the units for the growth and decay rate and for the number of growth and decay periods

____

ex. *monthly* interest rate of 0.05%,

Growth Problem

1. Maryville had a population of about 7500 people in 2009. It is expected that the town's population will increase 5% each year.
   a. What is the initial population?

   b. What is the growth rate, r?

   c. Write the algebraic model for this situation using the above information. Include let statements.

   d. Use the model to predict the population in 2018.

   e. In approximately what year will Maryville double its current population, assuming it continues to grow at this rate? Predict to the nearest tenth of a year.

Decay Problems

2. A 200g sample of radioactive polonium-210 has a half-life of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount.

    a. What is the rate of decay?
    b. Determine an equation to model this situation. Include let statements.

    c. Determine the mass that remains after 5 years.

    d. How much polonium-210 was there 414 days ago?

    e. Use your model to predict how long it would take for this 200g sample to decay to 110g.

3. A new car costs $24,000. It *loses* 18% of its value each **year** after it is purchased. This is called depreciation.

   a. Write an equation that models the decay/decline of the investment. Include let statements.

   b. Use the equation to determine the value of the automobile after 30 ***months***.

   c. If the car was purchased June 3, 2015, during what **month** would the cars value first fall below $10 000?

Booklet – Growth Worksheet # 1 – 4;
Decay Worksheet # 1 - 4
Pg. 24 #13, 14, 16, 19, 20a

U4D9 Review: Exponents and Exponential Functions

A: Exponent Laws & Exponential Expressions

1) Evaluate.

$$\left(\frac{5}{7}\right)^{-2}$$

2) Rewrite in radical form and then evaluate.

$$(-64)^{-\frac{2}{3}}$$

3) Simplify and rewrite using positive exponents.

$$\frac{(2x^{-5}y^{3})^{2}(-6x^{4}y^{-1})}{3xy^{-7}}$$

4) Rewrite in radical form and simplify.

$$\left(\sqrt[6]{27a^{3}b^{4}}\right)^{2}$$

5) Solve.

$$3^{2k} = 243$$

B: Exponential Functions

1. List the transformations in the order they must be applied.

$$f(x) = -\left(\frac{1}{3}\right)^{\left(\frac{1}{4}x+1\right)} - 1$$

2. Identify each table of values as linear, quadratic, or exponential. Show calculations to help explain/support your answer. For the exponential function(s) state whether it is growth or decay AND determine the equation.

x	y
--2	5.75
-1	5.3
0	4.85
1	4.4
2	3.95

x	y
-2	5.0625
-1	5.25
0	6
1	9
2	21

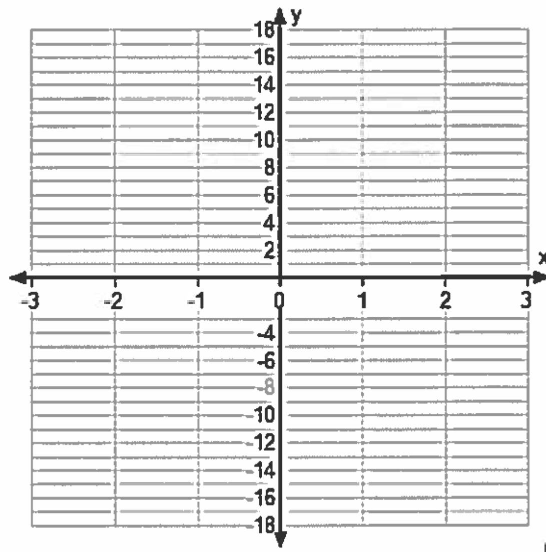
3. For $g(x) = \frac{1}{2}(4)^{-x} + 2$

State the base/parent function

State the transformations in the order that they that must be applied

State the x and y-intercepts, and the equation of the asymptote

Graph the new function



State the domain and range

D:

R:

Is the function increasing or decreasing?

4. The town of Vanessa is growing exponentially at a rate of 4.5% each year.

a) If the population of Vanessa is now 15 000, how many people will be living there in 42 months?

b) How many years would it take for the population to quadruple? (accurate to nearest tenth of a year)

5. A 500g sample of plutonium-243 has a half-life of 12 days.

a) Determine an equation to model this situation.

b) Determine how many grams of plutonium-243 remain after 6 weeks.

c) Determine how long it would take for only one-quarter of the original sample to remain. (accurate to the nearest day)

Pg. 85 -86 # 1-8, 9abcf, 10abdf, 12
Pg. 90-91 #1-3, 5, 6abc, 12
Booklet - day 7 #1-5
Booklet - Growth Worksheet (#5 – 8)
Decay Worksheet (#5 – 7)