U3D8_T Combining Transformations

Warm Up:
Describe the transformations that have occurred to \( h(x) \) to obtain the function \( y = -5h \left( \frac{2}{3}x \right) \):
1. reflection in x-axis
2. vertical stretch factor 5
3. horizontal stretch factor 1.5

If \((-1, 4)\) is a point on the function \( h(x) \), what would be the new point on the transformed function?

\[
( x, y ) \rightarrow ( 1.5x, -5y )
\]

\[
( -1, 4 ) \rightarrow ( 1.5(-1), -5(4) )
\]

\[
( -1, 4 ) \rightarrow ( -1.5, -20 )
\]
Combining Transformations

When a function has a combination of transformations, apply them in order left to right when in the form:

\[ y = \pm af[\pm b(x - h)] + k \]

1. Reflection in x-axis with Vertical Stretches and Compressions.
2. Reflection in y-axis with Horizontal Stretches and Compressions.
3. Translations (Horizontal and Vertical Shifts)
Example 1:

a) Describe, in order, how the graph of \( y = -2f(4(x - 2)) \) can be obtained from the graph of \( y = f(x) \). 
   \[ b = \frac{1}{4} \]
   1. reflection in x-axis
   2. vertical stretch factor 2.
   3. horizontal compression factor \( \frac{1}{4} \)
      (or horizontal compression by 4)
   4. shift right 2.

b) If \((x, y)\) was a point on \( f(x) \), what would the value of the coordinates be after the 4 transformations?

\[(x, y) \rightarrow \left( \frac{1}{4} x + 2, -2y \right)\]
Example 2:

Given \( f(x) = x^2 + 3 \)

a) Describe how the graph of \( y = \frac{1}{2} f(2x + 6) - 2 \) can be obtained from \( f(x) \).

*First factor out the coefficient on \( x \) if there is a Horizontal Stretch/Comp. and a Left/Right Shift.

\[
y = -\frac{1}{2} f \left( 2(x + 3) \right) - 2
\]

1. reflection in \( x \)-axis.
2. Vertical Compression factor \( \frac{1}{2} \)
3. Horizontal Compression factor \( \frac{1}{2} \)
4. Shift left 3
5. Shift down 2.

b) What is the new “image equation”?

\[
y = -\frac{1}{2} \left[ \left( 2(x + 3) \right)^2 + 3 \right] - 2
\]

\[
y = -\frac{1}{2} \left[ \left( 2^2 \right)(x + 3)^2 + 3 \right] - 2
\]

\[
y = -\frac{1}{2} \left( 4 \right)(x + 3)^2 - \frac{3}{2} - \frac{4}{2}
\]

\[
y = -2(x + 3)^2 - \frac{7}{2}
\]
c) Graph.

\[ f(x) = x^2 + 3 \]

\[ y = -\frac{1}{2} [2(x+3)] - 2 \]

\[ y = -2 (x+3)^2 - \frac{7}{2} \]

U3D8 Practice: p. 240 #7(odd), 8-9(odd, sketch one from each), 14