

U3D7_T Transformations WITH STRETCHES

Monday, March 18, 2019 6:45 PM



U3D7_T
Transfor...

U3D7 MCR 3UI

Transformations Including Stretches and Compressions

Recall: State the characteristics of $y = -3(x - 2)^2 + 4$.

Vertex $(2, 4)$

Domain $\{x \in \mathbb{R}\}$

Direction of opening down

Axis of symmetry $x = 2$

Range $\{y \leq 4\}$

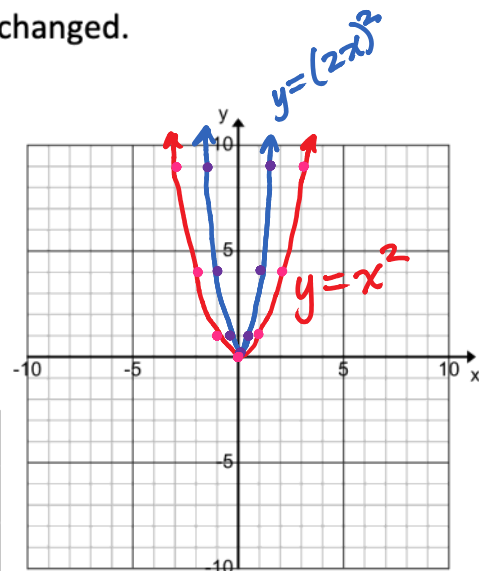
What would the graph of $y = (2x)^2$ look like?

Using algebra, it simplifies to $y = 2^2x^2$ or $y = 4x^2$... this horizontal change was simplified to look like a vertical stretch factor 4.

Let's look at a table of values to see how x changed.

x	$y=x^2$
0	0
1	1
2	4
3	9

x	$y=(2x)^2$	$y=4x^2$
0	0	0
$\frac{1}{2}$	1	$4\left(\frac{1}{2}\right)^2 = 1$
1	4	$4(1)^2 = 4$
$\frac{3}{2}$	9	$4\left(\frac{3}{2}\right)^2 = 4\left(\frac{9}{4}\right) = 9$



Notice: to get the same y-values, x is half as much when there is a two in front of the x.

Try $y = \left(\frac{1}{3}x\right)^2$

x	$y=x^2$
0	0
1	1
2	4
3	9

x	$y=\left(\frac{1}{3}x\right)^2$
0	0
3	1
6	4
9	9

Notice: to get the same y-values, x is three times as much when there is a one-third in front of the x.

In General: $y = a f[b(x - h)] + k$

a is: a reflection in the x-axis when $a < 0$

a vertical stretch when $|a| > 1$,

a vertical compression when $0 < |a| < 1$

b is: a reflection in the y-axis when $b < 0$

a horizontal stretch factor $\frac{1}{b}$ when $0 < |b| < 1$

a horizontal compression factor $\frac{1}{b}$ when $|b| > 1$

(or a horizontal compression by b when $|b| > 1$)

$\frac{1}{b}$ is the reciprocal of b.

Horizontal is opposite to what it looks like...

When $b = 3$, it is a horizontal compression by 3 or a horizontal compression factor $\frac{1}{3}$ (divide by 3 or multiply by a third).

When $b = \frac{1}{3}$, it is a horizontal stretch factor 3.

Applying the transformations you have learned to the **Root Function**.

a: $y = a f(x)$

or $y = a\sqrt{x}$

b: $y = a f(bx)$

or $y = a\sqrt{bx}$

h: $y = a f[b(x-h)]$

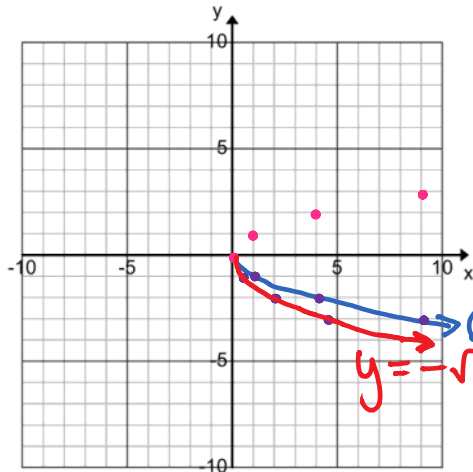
or $y = a\sqrt{b(x-h)}$

k: $y = a f[b(x-h)] + k$

or $y = a\sqrt{b(x-h)} + k$

Describe the transformations to the Root function and apply them as necessary to graph the following equations. State the domain and range. ***Remember: When applying transformations, stretches and reflections must always be done before shifts.***

1. $y = -\sqrt{2x}$



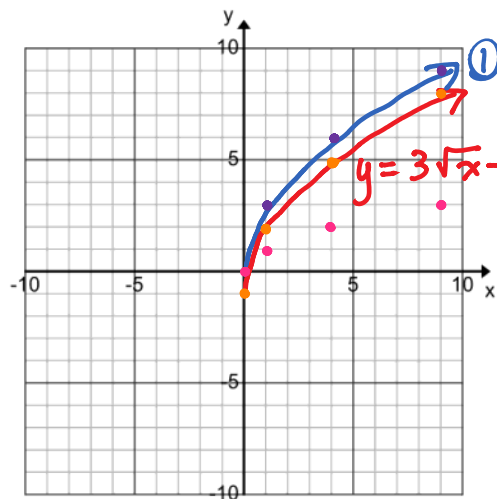
① reflection in x-axis

2. H. Comp. factor $\frac{1}{2}$

D: { $x \geq 0$ }

R: { $y \leq 0$ }

2. $f(x) = 3\sqrt{x} - 1$



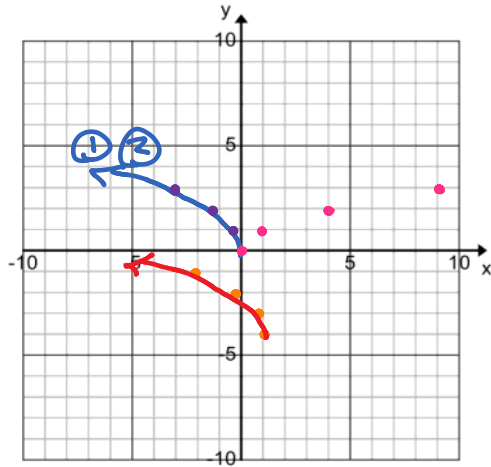
1. V. Stretch factor 3 $y \text{ times } 3$

2. Shift down 1 $3y - 1$

D: { $x \geq 0$ }

R: { $y \geq -1$ }

3. $y = -4 + \sqrt{3 - 3x}$



$y = \sqrt{-3x+3} - 4$ * must have b factored out.
 $y = \sqrt{-3(x-1)} - 4$

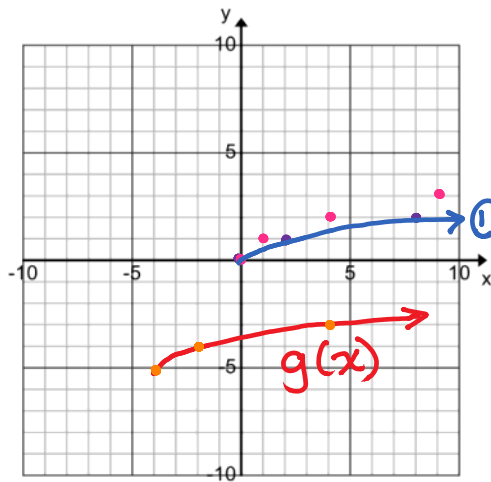
- ① reflection in y-axis
- ② H. Comp. factor $\frac{1}{3}$

- 3. shift right 1
- 4. shift down 4

D: $\{ x \leq 1 \}$

R: $\{ y \geq -4 \}$

4. $g(x) = \sqrt{\frac{1}{2}(x+4)} - 5$



- 1. H. stretch factor 2 ($2 \cdot x$)
- 2. shift left 4
- 3. shift down 5

D: $\{ x \geq -4 \}$

R: $\{ y \geq -5 \}$

State the domain and range for the following without graphing.

$$1. y = \sqrt{\frac{1}{4}x} + 2$$

\uparrow \uparrow
 $4x$ $y+2$

$D: \{x \geq 0\}$
 $R: \{y \geq 2\}$

$$2. g(x) = 3 - \sqrt{x-2}$$

$$g(x) = -\sqrt{x-2} + 3$$

\uparrow \uparrow
 $-y+3$ $x+2$

$D: \{x \geq 2\}$
 $R: \{y \leq 3\}$

$$3. h(x) = \sqrt{3x-6}$$

$$h(x) = \sqrt{3(x-2)}$$

\uparrow \uparrow
 $\frac{1}{3}x + 2$

$D: \{x \geq 2\}$
 $R: \{y \geq 0\}$

Applying the transformations you have learned to the **Reciprocal**

Function $f(x) = \frac{1}{x}$

a: $y = a f(x)$ or $y = a\left(\frac{1}{x}\right) = \frac{a}{x}$

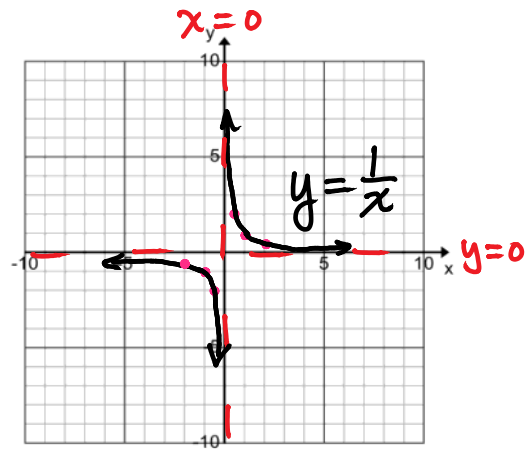
b: $y = a f(bx)$ or $y = \frac{a}{bx}$ *note: same as*
 $y = \left(\frac{a}{b}\right)\left(\frac{1}{x}\right)$

h: $y = a f[b(x-h)]$ or $y = \frac{a}{b(x-h)}$

k: $y = a f[b(x-h)] + k$ or $y = \frac{a}{b(x-h)} + k$

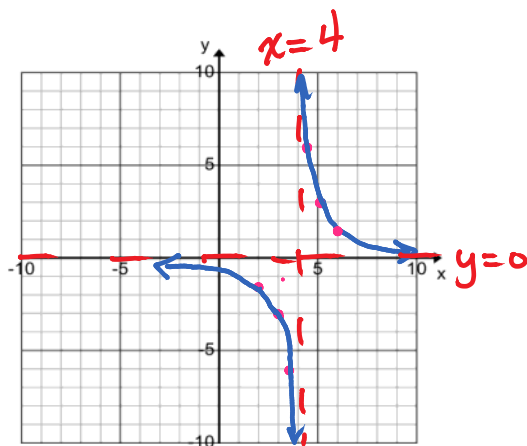
Remember the graph of

$$y = \frac{1}{x}$$



Describe the transformations to the Reciprocal function and apply them as necessary to graph the following equations. State the domain and range.

1. $f(x) = \frac{3}{x-4}$

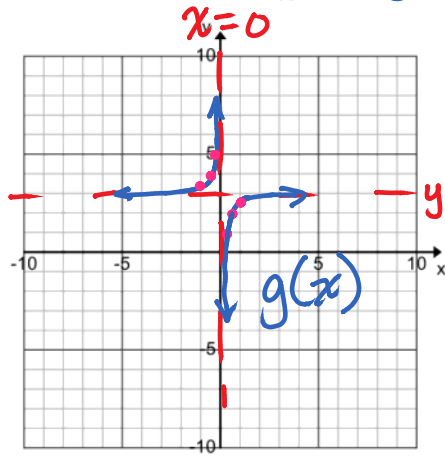


1. Vertical Stretch factor 3
2. Shift right 4

D: { $x \neq 4$ }

R: { $y \neq 0$ }

$$2. \quad g(x) = 3 - \frac{1}{2x} \quad g(x) = \frac{-1}{2x} + 3$$



1. reflection in x -axis
2. H. Compression factor $\frac{1}{2}$
3. shift up 3

$$D: \{ x \neq 0 \}$$

$$R: \{ y \neq 3 \}$$

State the domain and range for the following without graphing.
 (Remember: asymptotes only move with shifts (L/R, U/D))

1. $y = \frac{1}{x+3} + 8$

D: $\{x \neq -3\}$

R: $\{y \neq 8\}$

2. $f(x) = \frac{5}{x-9} - 11$

D: $\{x \neq 9\}$

R: $\{y \neq -11\}$

3. $y = \frac{2}{5-3x} - 7$

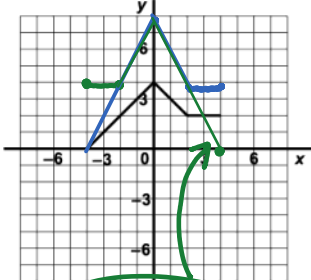
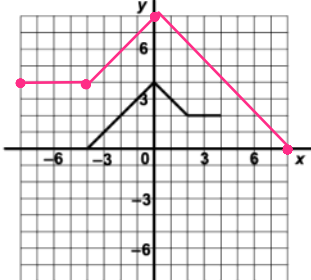
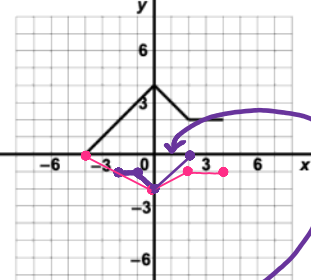
D: $\{x \neq \frac{5}{3}\}$

R: $\{y \neq -7\}$

$$\begin{aligned} 5-3x &\neq 0 \\ -3x &\neq -5 \\ x &\neq \frac{5}{3} \end{aligned}$$

The function given in each graph below is $f(x)$. Sketch the graph of the indicated new function. **REMEMBER** — Stretch and reflect **FIRST**, then slide **LAST**.

<p>$y = \frac{3}{4} f(x)$</p> <p>V. Comp. factor $\frac{3}{4}$</p>	<p>$y = f\left(\frac{1}{2}x\right)$</p> <p>H. Stretch factor 2</p>	<p>$y = f(2x)$</p> <p>H. Comp. factor $\frac{1}{2}$</p> <p>OR</p> <p>H. Comp. by 2</p>

 <p style="text-align: center;">$y = 2f(-x)$</p> <p style="text-align: center;"> ↙ V. Stretch factor 2 ↘ reflect in y-axis </p>	 <p style="text-align: center;">$y = 2f(-\frac{1}{2}x)$</p> <p style="text-align: center;"> ⇒ same as final graph to left stretched horizontally factor 2. </p>	 <p style="text-align: center;">$y = -\frac{1}{2}f(-2x)$</p> <p style="text-align: center;"> ↙ reflect in x-axis, vert. comp. factor $\frac{1}{2}$ ↘ reflect in y-axis, hor. comp. factor $\frac{1}{2}$ </p> <p style="text-align: center;"> $(x, y) \Rightarrow (-\frac{1}{2}x, -\frac{1}{2}y)$ </p>
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U3D7 Practice: p. 229 #3, 4ii, 5 (odds), 6 (odds), 7, 11 (odds)