U3D4_T Reflections of Functions

Monday, March 18, 2019   6:45 PM
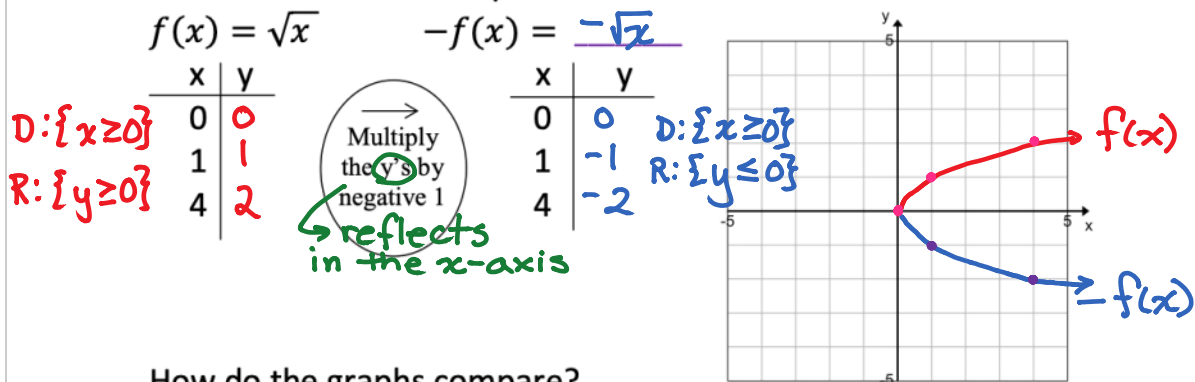
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U3D4_T
Reflection...

U3D4 MCR3UI

Reflections of Functions

<u>Example 1</u>: Given f(x) = $\sqrt{x}$, graph $y = f(x)$ and $y = -f(x)$
and compare the two.

$f(x) = \sqrt{x}$ $-f(x) = \underline{-\sqrt{x}}$

x	y
0	0
1	1
4	2

D:{x≥0}
R:{y≥0}

Multiply
the y's by
negative 1

↳reflects
in the x-axis

x	y
0	0
1	-1
4	-2

D:{x≥0}
R:{y≤0}

f(x)

-f(x)

How do the graphs compare?

The x-axis (y=0) is
the axis of symmetry

(0,0) is an
invariant point.

**Points that are <u>unaltered</u> by a transformation
are said to be <u>invariant</u>.** Points on
the axis of symmetry are invariant.

**The graph of y = -f(x) is a <u>reflection</u> of the
graph of y = f(x) in the <u>x-axis</u>.**
(x, y) ⟹ (x, -y)

Example 2: Given $f(x) = \sqrt{x}$, graph $y = f(x)$ and $y = f(-x)$ and compare the two.

$$y = f(-x)$$
$$= \sqrt{-x}$$

$f(x) = \sqrt{x}$

D:$\{x \geq 0\}$
R:$\{y \geq 0\}$

x	y
0	0
1	1
4	2

Multiply the x's by negative 1

reflects in y-axis.

x	y
0	0
-1	1
-4	2

D:$\{x \leq 0\}$
R:$\{y \geq 0\}$

$f(-x)$ $f(x)$

(0,0) is invariant because it is on the axis of symmetry $(x = 0)$

How do the graphs compare?

The y-axis $(x=0)$ is the axis of symmetry.

The graph of y = f(-x) is a **reflection** of the graph of y = f(x) in the **y-axis**.

(x, y) $\implies$ (-x, y)

Example 3: If $f(x) = \boxed{3x + 2,}$ state the equation of the function that is:
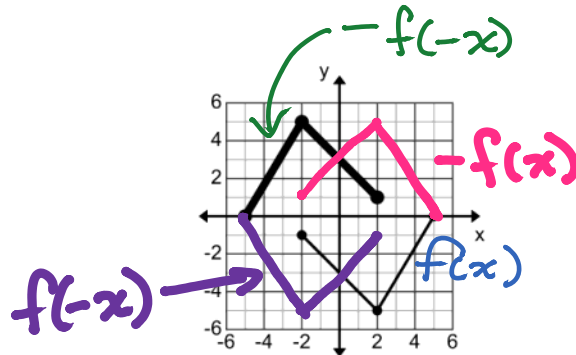
(a) reflection in the y-axis
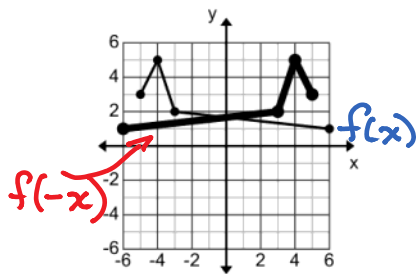
$$f(-x) = 3(-x) + 2$$
$$f(-x) = -3x + 2$$

(b) reflection in the x-axis

$$-f(x) = -(3x + 2)$$
$$-f(x) = -3x - 2$$

Example 4: In each graph below, $f(x)$ is indicated and drawn with a thin line. Give the correct function notation for the second function – drawn with the thicker line.

$f(x)$

$f(-x)$

$-f(-x)$

$-f(x)$

$f(-x)$

$f(x)$

Example 5:

$\cancel{x} \;\; \cancel{+} \;\; \boxed{\cancel{+} \; c^{\sqrt{x-3}}}$

If $f(x) = \sqrt{x-3}$, compare its graph, domain and range to $f(-x)$ and $-f(x)$.

$f(x) = \sqrt{x-3}$ $-f(x) = -\sqrt{x-3}$ $f(-x) = \sqrt{-x-3}$

D: { $x \geq 3$ }   D: { $x \geq 3$ }   D: { $x \leq -3$ }

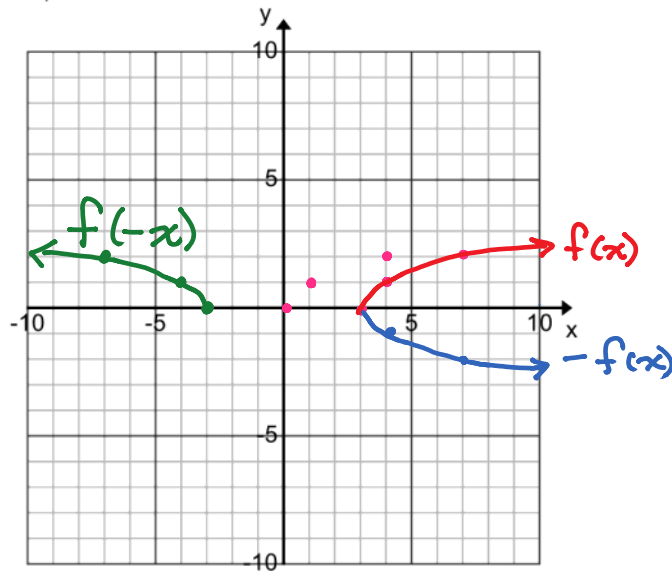R: { $y \geq 0$ }   R: { $y \leq 0$ }   R: { $y \geq 0$ }

x	y
3	0
4	1
7	2

x	y
3	0
4	-1
7	-2

invariant point

x	y
-3	0
-4	1
-7	2

no invariant point

Invariants?

$f(-x)$ $f(x)$

$-f(x)$

Example 6: If f(x) = $\dfrac{1}{x}$, write an equation

for $-f(x)$ and $f(-x)$. Describe and sketch the
three graphs on the same axes.

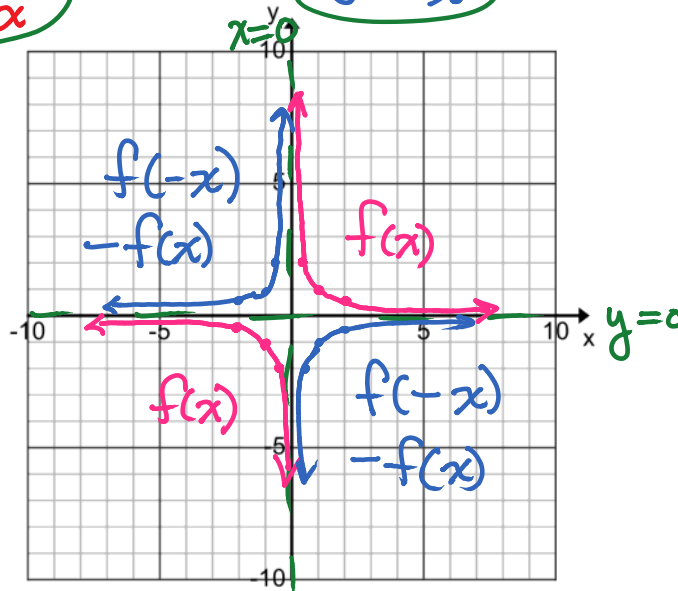$y = -f(x)$ $\qquad\qquad\qquad y = f(-x)$

$$y = -\left(\dfrac{1}{x}\right) \qquad\qquad y = \dfrac{1}{-x}$$

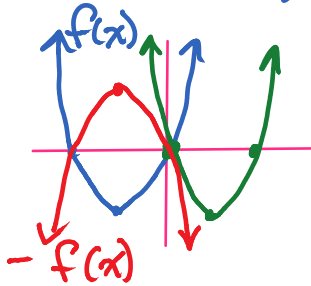$$y = \dfrac{-1}{x} \quad \leftarrow \text{Same} \rightarrow \quad y = \dfrac{-1}{x}$$

Example 7: Given $f(x) = x^2 + 4x$.

Compare the vertex and x-intercepts of $-f(x)$ and $f(-x)$.

$f(x) = x(x+4)$

zeros:

$x = 0, -4$

A of S:

$x = \dfrac{0 + (-4)}{2}$

$x = -2$

$f(-2) = (-2)^2 + 4(-2)$

$f(-2) = 4 - 8$

$f(-2) = -4$

$V(-2, -4)$

	$f(x)$	$-f(x)$	$f(-x)$
Vertex	$(-2, -4)$	$(-2, 4)$	$(2, -4)$
x-intercepts	$(0,0)\ (-4,0)$	$(0,0)\ (-4,0)$	$(0,0)\ (4,0)$

p. 203 #1c, 2abc, 3, 5, 7, 10