

Reflections of Functions

Example 1: Given $f(x) = \sqrt{x}$, graph $y = f(x)$ and $y = -f(x)$ and compare the two.

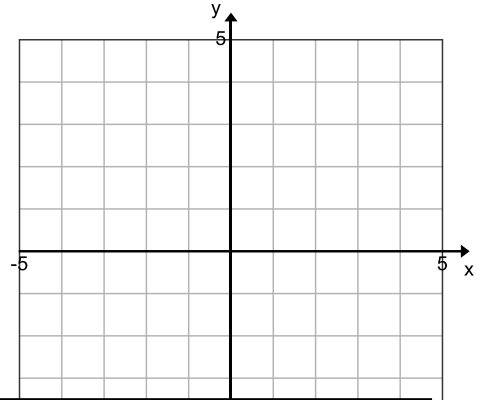
$f(x) = \sqrt{x}$

| x | y |
|---|---|
| 0 | |
| 1 | |
| 4 | |

→
Multiply the y's by negative 1

$-f(x) = \underline{\hspace{2cm}}$

| x | y |
|---|---|
| | |
| | |
| | |



How do the graphs compare?

The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis.
 $(x, y) \implies (x, -y)$

Points that are unaltered by a transformation are said to be invariant.

Example 2: Given $f(x) = \sqrt{x}$, graph $y = f(x)$ and $y = f(-x)$ and compare the two.

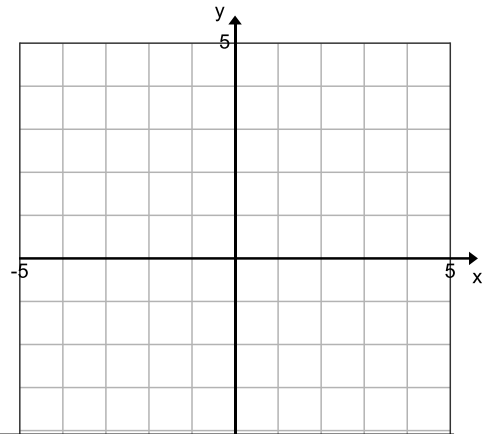
$f(x) = \sqrt{x}$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |

→
Multiply the x's by negative 1

$y = f(-x) = \underline{\hspace{2cm}}$

| x | y |
|---|---|
| | |
| | |
| | |



How do the graphs compare?

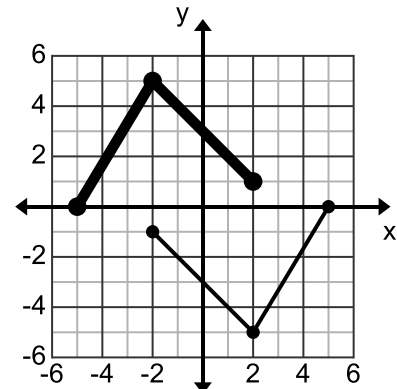
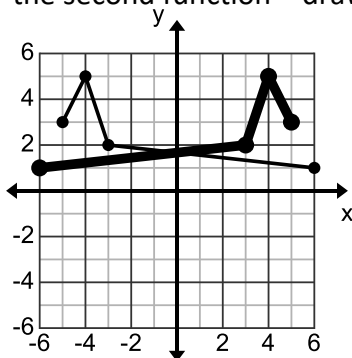
The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis.
 $(x, y) \implies (-x, y)$

Example 3: If $f(x) = 3x + 2$, state the equation of the function that is:

(a) reflection in the y-axis

(b) reflection in the x-axis

Example 4: In each graph below, $f(x)$ is indicated and drawn with a thin line. Give the correct function notation for the second function – drawn with the thicker line.



Example 5:

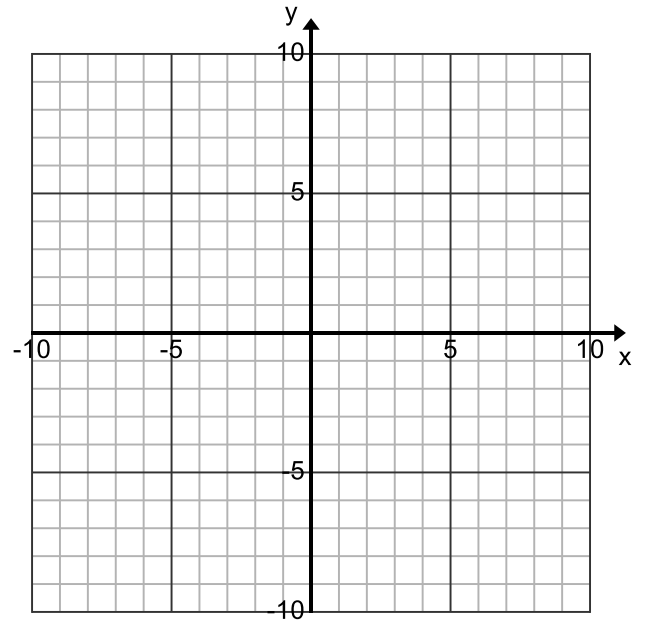
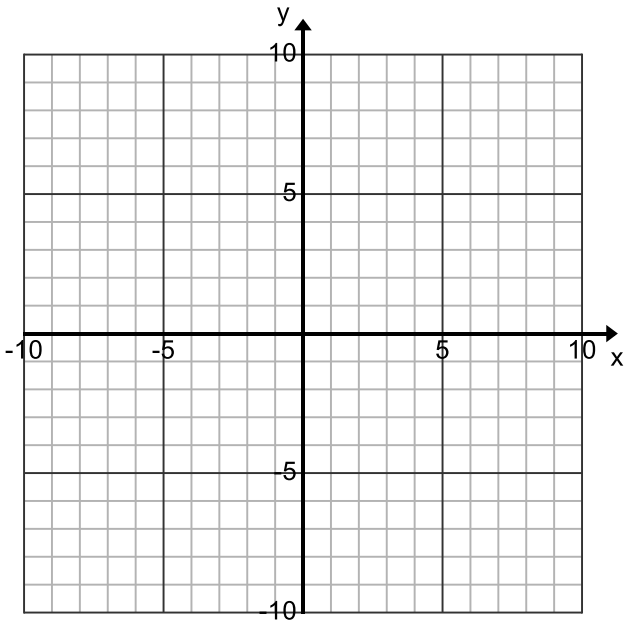
If $f(x) = \sqrt{x-3}$, compare its graph, domain and range to $f(-x)$ and $-f(x)$.

| | | | |
|---|---|---|-------------|
| $f(x) = \sqrt{x-3}$ | $-f(x) = \underline{\hspace{2cm}}$ | $f(-x) = \underline{\hspace{2cm}}$ | Invariants? |
| D: { } | D: { } | D: { } | |
| R: { } | R: { } | R: { } | |

| | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|--|---|---|---|--|---|--|---|--|--|---|---|---|--|---|--|---|--|---|---|---|----|--|----|--|----|--|--|
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="width: 50px;">x</td><td style="width: 50px;">y</td></tr> <tr><td style="text-align: center;">3</td><td></td></tr> <tr><td style="text-align: center;">4</td><td></td></tr> <tr><td style="text-align: center;">7</td><td></td></tr> </table> | x | y | 3 | | 4 | | 7 | | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="width: 50px;">x</td><td style="width: 50px;">y</td></tr> <tr><td style="text-align: center;">3</td><td></td></tr> <tr><td style="text-align: center;">4</td><td></td></tr> <tr><td style="text-align: center;">7</td><td></td></tr> </table> | x | y | 3 | | 4 | | 7 | | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="width: 50px;">x</td><td style="width: 50px;">y</td></tr> <tr><td style="text-align: center;">-3</td><td></td></tr> <tr><td style="text-align: center;">-4</td><td></td></tr> <tr><td style="text-align: center;">-7</td><td></td></tr> </table> | x | y | -3 | | -4 | | -7 | | |
| x | y | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| x | y | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| x | y | | | | | | | | | | | | | | | | | | | | | | | | | | |
| -3 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| -4 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| -7 | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Example 6: If $f(x) = \frac{1}{x}$, write an equation for $-f(x)$ and $f(-x)$. Describe and sketch the three graphs on the same axes.

$y = -f(x)$ $y = f(-x)$



Example 7: Given $f(x) = x^2 + 4x$. Compare the vertex and x-intercepts of $-f(x)$ and $f(-x)$.

| | f(x) | -f(x) | f(-x) |
|--------------|------|-------|-------|
| Vertex | | | |
| x-intercepts | | | |