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Vertical and Horizontal Translations of Functions Vertical Translations How do the graphs of $f(x)=x^{2}$ and $y=x^{2}+3$ compare?
(Sketch, state domain and range):

| $f(x)=x^{2}$ |  | $y=x^{2}+3$ |  |
| :---: | :---: | :---: | :---: |
|  |  |  | $f(x)+3$ |
| $x$ | $f(\mathrm{x})$ | x | y |
| -3 | 9 | -3 | 12 |
| -2 | 4 | -2 | 7 |
| -1 | 1 | -1 | 4 |
| 0 | 0 | 0 | 3 |
| 1 | 1 | 1 | 4 |
| 2 | 4 | 2 | 7 |
|  | 9 |  | 12 |


$D:\{x \in \mathbb{R}\}$
$D:\{x \in \mathbb{R}\}$
$R:\{\quad y \geq 0$
\}
$R:\{y \geq 3\}$

The second graph is a vertical translation of 3 units up from the first graph. (All $y$-values in $f(x)$ have been translated up 3).
To write the second graph in function notation, we write $y=f(x)+3$

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2. Describe the graph of $y=\sqrt{x}-3$.

The graph of $y=\sqrt{x}-3$ is a vertical translation of 3 units down from the graph of $g(x)=\sqrt{x}$. $y=\sqrt{x}-3$ is $y=g(x)-3$ in function notation.
State the Domain and Range of each graph in the description above:

$$
g(x)=\sqrt{x} \xrightarrow{\rightleftarrows}=\sqrt{x}-3
$$

D: $\{\quad x \geq 0$
\}
D: $\{\quad x \geq 0$
$\mathrm{R}:\{\quad y \geq 0$
\} $\mathrm{R}:\{$
$y \geq-3$
\}

The graph of $y=f(x)+k$ is congruent to $y=f(x)$.
If $k>0$, translate the graph of $f(x) k$-units up. If $\boldsymbol{k}<\mathbf{0}$, translate the graph of $\boldsymbol{f}(\boldsymbol{x}) \boldsymbol{k}$-units down.

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$$
\begin{array}{rr}
y=x^{2} & y=(x+2)^{2} \\
V(0,0) & V(-2,0)
\end{array}
$$

Horizontal Translations
How do the graphs of $\mathrm{b}(\mathrm{x})=\sqrt{\mathrm{x}}$ and $y=\sqrt{x+2}$ compare?
(Sketch, state domain and range)

$$
\mathrm{b}(\mathrm{x})=\sqrt{\mathrm{x}} \quad y=\sqrt{x+2}
$$

| $x$ | $b(x)$ | $x$ | $y$ |
| :---: | :--- | :--- | :--- |
| 0 | 0 |  | -2 |
| 1 | 0 |  |  |
| 4 | 2 |  | -1 |
| 1 | 2 |  |  |
| 9 | 3 | 7 | 3 |
| 16 | 4 | 14 | 4 |


$\mathrm{D}:\{x \geq 0 \quad\} \quad \mathrm{D}:\{\quad x \geq-2 \quad\}$
$R:\{\quad y \geq 0 \quad R:\{\quad y \geq 0 \quad\}$

The graph of $y=\sqrt{x+2}$ is a horizontal translation of 2 units to the left_ of the graph $\mathrm{b}(\mathrm{X})=\sqrt{\mathrm{X}}$. In function notation, $y=b(x+2)$

The graph of $y=f(x-h)$ is congruent to the graph of $y=f(x)$.
If $\boldsymbol{h} \boldsymbol{>} \mathbf{0}$, translate the graph of $\boldsymbol{f}(\boldsymbol{x})$ to the right $\boldsymbol{h}$-units. If $\boldsymbol{h}<\boldsymbol{0}$, translate the graph of $\boldsymbol{f}(\boldsymbol{x})$ to the left $\boldsymbol{h}$-units.

Note: Remember, for horizontal shifts, it is opposite of what you see in the brackets.
Examples:

1. Describe the graph of $y=(x+4)^{2}-5$.
parent graph $\rightarrow$ qua ratic function $\left(y=x^{2}\right)$ horizontal translation left 4 units (shift $\mathbf{L}^{4}$ ) vertical translation down 5 units (shift D5)
2. For the function shown, $f(x)$,
i) describe how the graph of $y=f(x-2)+3$ can be obtained from the graph of $y=f(x)$ shift right 2 shift up 3
ii) graph $y=f(x-2)+3$

3. Given $j(x)=\frac{1}{x}$. Determine the equation of $y=j(x-5)+3$. Describe the graph of the second function.

$$
y=\frac{1}{(x-5)}+3
$$

parent graph $\rightarrow$ reciprocal function Shift Right 5 units shift Up 3 units.
4. Given $h(x)=\sqrt{x}$.
a) Use function notation to describe the graph of $h(x)$, shifted left 11 units and up 5 units.

$$
y=h(x+11)+5
$$

b) Write the equation of the translated function described in part (a).

$$
y=\sqrt{x+11}+5
$$

5. Given $m(x)=\frac{1}{x+3}$.
a) Write the image equation for the transformation

$$
\begin{aligned}
& y=m(x-7)+2 \\
& y=\frac{1}{(x-7)+3}+2
\end{aligned} \quad y=\frac{1}{x-4}+2
$$

b) State the Domain and Range of each function.

$$
\begin{array}{ccc}
m(x) & D:\{x \neq-3\} & R:\{y \neq 0\} \\
y & D:\{x \neq 4\} & R:\{y \neq 2\}
\end{array}
$$

c) Graph both functions on the same grid.


