Vertical and Horizontal Translations of Functions

Vertical Translations
How do the graphs of \( f(x) = x^2 \) and \( y = x^2 + 3 \) compare? (Sketch, state domain and range):

\[
\begin{array}{c|c|c|c|c}
\text{ } & x & f(x) & x & y \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
D: \{ & \}
\end{array}
\]

\[
\begin{array}{c|c}
R: \{ & \}
\end{array}
\]

The second graph is a ______________ translation of 3 units __________ from the first graph. (All y-values in \( f(x) \) have been translated __________).
To write the second graph in function notation, we write____________________

2. Describe the graph of \( y = \sqrt{x} - 3 \).

The graph of \( y = \sqrt{x} - 3 \) is a ______________ translation of 3 units __________ from the graph of \( g(x) = \_______ \). \( y = \sqrt{x} - 3 \) is ______________ in function notation.

State the Domain and Range of each graph in the description above:

\[
\begin{array}{c|c}
\text{ } & \{ & \}
\end{array}
\]

\[
\begin{array}{c|c}
\text{ } & \{ & \}
\end{array}
\]

The graph of \( y = f(x) + k \) is congruent to \( y = f(x) \).
If \( k > 0 \), translate the graph of \( f(x) \) \( k \)-units up.
If \( k < 0 \), translate the graph of \( f(x) \) \( k \)-units down.
**U3D3 MCR3UI**

**Horizontal Translations**

How do the graphs of \( h(x) = \sqrt{x} \) and \( y = \sqrt{x + 2} \) compare? (Sketch, state domain and range)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( h(x) )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>

\[ \text{D: } \{ \} \quad \text{D: } \{ \} \]
\[ \text{R: } \{ \} \quad \text{R: } \{ \} \]

The graph of \( y = \sqrt{x + 2} \) is a _______________ translation of 2 units to the _________ of the graph \( h(x) = \sqrt{x} \). In function notation, \( y = \)

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The graph of \( y = f(x-h) \) is congruent to the graph of \( y = f(x) \).

If \( h > 0 \), translate the graph of \( f(x) \) to the right \( h \)-units.

If \( h < 0 \), translate the graph of \( f(x) \) to the left \( h \)-units.

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**Note:** Remember, for horizontal shifts, it is opposite of what you see in the brackets.

**Examples:**

1. Describe the graph of \( y = (x + 4)^2 - 5 \).

2. For the function shown, \( f(x) \),
   i) describe how the graph of \( y = f(x - 2) + 3 \) can be obtained from the graph of \( y = f(x) \)
   ii) graph \( y = f(x - 2) + 3 \)
3. Given \( j(x) = \frac{1}{x} \). Determine the equation of \( y = j(x - 5) + 3 \). Describe the graph of the second function.

4. Given \( h(x) = \sqrt{x} \).
   a) Use function notation to describe the graph of \( h(x) \), shifted left 11 units and up 5 units.
   b) Write the equation of the translated function described in part (a).

5. Given \( m(x) = \frac{1}{x+3} \).
   a) Write the image equation for the transformation \( y = m(x - 7) + 2 \).
   b) State the Domain and Range of each function.
   c) Graph both functions on the same grid.