Recall:
How do we know if a relation is a function?
- Each independent value (i.e., x-value) has only one dependent value (i.e. y value)
- The graph passes the **Vertical Line Test** - a vertical line will only ever cross the graph once.

Every function has a domain and range.

**Domain:** The set of all possible *x-values* in the relation

**Range:** The set of all possible *y-values* in the relation

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**Real Numbers**

<table>
<thead>
<tr>
<th>I</th>
<th>Q</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrational</td>
<td>Rational</td>
<td>Natural</td>
</tr>
<tr>
<td>Non-terminating decimal with no repeating pattern</td>
<td>Can be represented as a fraction (ratio)</td>
<td>The counting numbers</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\frac{1}{4}$</td>
<td>1, 2, 3, ...</td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td>$0.999999...$</td>
<td>2.121212...</td>
</tr>
<tr>
<td>$e$</td>
<td>3.57</td>
<td>...</td>
</tr>
</tbody>
</table>

Z **Integers**
- All whole #s and their opposites
  - ... -2, -1, 0, 1, 2, ...

W **Whole**
- All Natural #s and zero
  - 0, 1, 2, ...

\[ \text{N Natural} \]
Example 1:

a)

Function? Yes or No

D: \( \{ x \in \mathbb{R}^2 \} \)
R: \( \{ y \in \mathbb{R}^2 \} \)

b)

Function? Yes or No

D: \( \{ x \leq 1 \} \)
R: \( \{ y \in \mathbb{R}^2 \} \)

c)

Function? Yes or No

D: \( \{ x \in \mathbb{R}^2 \} \)
R: \( \{ y \geq -3 \} \)

d)

Function? Yes or No

D: \( \{ -3 \leq x \leq 3 \} \)
R: \( \{ -2 \leq y \leq 2 \} \)
e) \{(1,2), (2, -3), (4, 5)\}

Function? Yes or No
D: \{1, 2, 4\}
R: \{-3, 2, 5\}

f) \(g(x) = x^2\)

Function? Yes or No
D: \(\{x \in \mathbb{R}\}\)
R: \(\{y \geq 0\}\)

g) [Graph of a function]

Function? Yes or No
D: \{-4, -3, -2, -1, 0, 1, 2\}
R: \{-3, -2, -1, 0, 1, 2\}

h) [Graph of a function]

Function? Yes or No
D: \(\{x \in \mathbb{R}\}\)
R: \(\{y \geq 0\}\)
i) Function? Yes or No
D: \( \{ x \neq -3 \} \)
R: \( \{ y \leq 4 \} \)

j) 3x - 5y = 12
Function? Yes or No
D: \( \{ x \in \mathbb{R} \} \)
R: \( \{ y \in \mathbb{R} \} \)

k) f(x) = -3x^2 + 5
Function? Yes or No
D: \( \{ x \in \mathbb{R} \} \)
R: \( \{ y \leq 5 \} \)
FUNCTION NOTATION

Function notation is an alternative way to write a function. A common function notation used is \( f(x) \), which is read as “f of x”. This just means that a function’s final value, \( f(x) \), is dependent on the value of \( x \).

Note: \( f(x) \) is exactly the same thing as \( y \) in our equations. Also note, that it does not mean “f times x”!

The variable used to represent the function can also change so that you can decipher multiple different functions within any given problem (i.e., \( g(x) \), \( h(x) \), etc).

Function notation helps give a little bit more information about the function. For example, if you see \( f(4) = -6 \), this means that the function has a value of -6 when \( x = 4 \), thus a point on the graph would be (4, -6).

Example 2:

a) If \( f(x) = 3x^2 - 7 \), evaluate \( f(-3) \)

\[
\begin{align*}
    f(-3) &= 3(-3)^2 - 7 \\
    &= 27 - 7 \\
    &= 20
\end{align*}
\]

b) If \( f(x) = 1 - 3x \), find and simplify \( 3f(m+2) \)

\[
\begin{align*}
    3\left[1 - 3(m+2)\right] &= 3\left[1 - 3m - 6\right] \\
    &= 3(-3m - 5) \\
    &= -9m - 15
\end{align*}
\]

\[
\therefore 3f(m+2) = -9m - 15
\]
c) If \( f(x) = x^2 - 5x \), find the values of \( x \) if the value of \( f(x) \) is 6.

\[
6 = x^2 - 5x
\]
\[
x^2 - 5x = 6
\]
\[
x^2 - 5x - 6 = 0
\]
\[
(x - 6)(x + 1) = 0
\]
\[
x = 6 \text{ or } x = -1
\]
U3D1 WORKSHEET Part A Functions, Relations, Domain & Range

1. Sketch a relation with the following properties.
   a) It is a function with \( D: \{ x \in \mathbb{R} \}, R: \{ y \in \mathbb{R}, y \leq 5 \} \)
   b) It is not a function and \( D: \{ x \in \mathbb{R}, -3 \leq x \leq 3 \}, R: \{ y \in \mathbb{R}, -3 \leq y \leq 3 \} \)

2. Is each relation a function? Explain. (Sketch a graph of each, if you like.)
   a) \( y = x - 5 \)  
   b) \( x = y^2 - 3 \)  
   c) \( y = 2(x - 1)^2 - 2 \)  
   d) \( x^2 + y^2 = 4 \)

3. Is each relation a function? Justify your answer. If the relation is a function, state the independent variable and the dependent variable.
   a) The amount of money taken in for the fundraiser is related to the number of raffle tickets a hockey team sells.
   b) The age of students is related to their grade level.
   c) The time it takes Matteo to walk to school is related to the speed at which he walks.

4. For each given domain and range, draw one relation that is a function and one that is not. Use the same set of axes for each part.
   a) Domain: \( \{ x \in \mathbb{R} \} \), Range: \( \{ y \in \mathbb{R} \} \)
   b) Domain: \( \{ x \in \mathbb{R} \} \), Range: \( \{ y \in \mathbb{R}, y \leq -1 \} \)
   c) Domain: \( \{ x \in \mathbb{R}, x \leq 2 \} \), Range: \( \{ y \in \mathbb{R}, y \geq -2 \} \)

5. Avery has 90 m of fencing to enclose an area in a petting zoo with two dividers to separate three types of young animals.
   The three pens are to have the same area.
   a) Express the area function for the three pens together, in terms of \( x \), where \( x \) is the length of each pen.
   b) Determine the domain and range for the area function.

6. Determine the domain and range of each relation. Use a graph to help you if necessary.
   a) \( y = -x + 3 \)  
   b) \( y = (x + 1)^2 - 4 \)  
   c) \( y = -3x^2 + 1 \)
   d) \( x^2 + y^2 = 9 \)  
   e) \( y = \frac{1}{x+3} \)  
   f) \( y = \sqrt{2x + 1} \)

7. For each function, determine \( f(4), f(-5) \), and \( f\left(-\frac{2}{3}\right) \).
   a) \( f(x) = \frac{2}{5}x + 11 \)  
   b) \( f(x) = -6c \)  
   c) \( f(x) = \frac{1}{x} \)  
   d) \( f(x) = \sqrt{x + 5} \)

8. State the domain and the range of each relation. Is each relation a function? Explain.
   a) \( \{(5,5), (6,6), (7,7), (8,8), (9,9)\} \)  
   b) \( \{(3,1), (4,1), (5,1), (6,1)\} \)
   c) \( \{(1,6), (1,-14), (1,11), (1,-8), (1,0)\} \)  
   d) \( \{(1,5), (4,11), (3,9), (5,1), (11,4)\} \)
   e) \( \{(3,2), (2,1), (1,0), (2,-1), (3,-2)\} \)

9. The domain and range of some relations are given. Each relation consists of five points. Is each a function? Explain.
   a) \( D: \{1,2,3,4,5\} \)  
   b) \( D: \{-3,-1,1,3,5\} \)  
   c) \( D: \{2,3,6\} \)  
   d) \( D: \{-2\} \)

10. Describe the graph of a relation that has
   a) one entry in the domain and one entry in the range.
   b) one entry in the domain and many entries in the range.
   c) many entries in the domain and one entry in the range.

Challenge: Given \( f(x) + 2g(x) = 12x^2 + 3x + 8 \) and \( 2f(x) + 3g(x) = 18x^2 + 6x + 13 \), find the value of \( f(2) + g(3) \).
U3D1 Worksheet Part B Functions, Relations, Domain & Range

1. State the Domain and Range of each of the given relations in the space provided. Assume the graphs drawn to the edge of the grid continue on infinitely.

- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }
- D: { }, R: { }