U2D9_T Zeros and the Discriminant MCR 3UI

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U2D9_T
Zeros and...

U2D9 MCR 3UI **ZEROS**

Warm Up Solve the following:

a) $3x^2 - 5x + 2 = 0$

$(3x-2)(x-1)=0$

$x=\frac{2}{3}$ or $x=1$

b) $3x^2 - 5x + 2 \leq 0$

zeros: $x=\frac{2}{3}$ or $x=1$

below x-axis

$\{\frac{2}{3} \leq x \leq 1\}$

c) $3x^2 - 5x + 2 > 0$

$\{x < \frac{2}{3}$ or $x > 1\}$

$\{x < \frac{2}{3}$ or $1 < x\}$

Zeros of a Quadratic Function

1. **Complete the Chart.**

Equation ($y=a(x-h)^2+k$)	Vertex (h,k)	Direction of Opening	Sketch	Number of Roots
$y=-6x^2+9$ $y=-6(x-0)^2+9$	(0, 9)	down	∧	2
$y=\frac{3}{2}x^2-5$	(0, -5)	up	∪	2
$y=-(x-3)^2+17$	(3, 17)	down	∧	2
$y=5(x+2)^2+4$	(-2, 4)	up	∪	0

2. **Determine the number of roots for the following:**

a) $y=6x^2-3x$

$y=3x(2x-1)$

∴ 2 roots

(2 real & distinct roots)

b) $y=2x^2-16x+32$

$y=2(x^2-8x+16)$

$y=2(x-4)^2$

∴ 1 root

(roots are real & equal)

c) $y=-4x^2+49$

$y=-(4x^2-49)$

$y=-(2x-7)(2x+7)$

∴ 2 roots.

(2 real & distinct roots.)

d) $y=x^2-3x+8$

$x=\frac{3\pm\sqrt{9-4(1)(8)}}{2}$

$x=\frac{3\pm\sqrt{-23}}{2}$ no real answer

∴ no roots

(no real roots)

The Quadratic Formula and the Discriminant

The quantity $b^2 - 4ac$ is called the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$. From it we can determine the **nature of the roots** of the equation. It can also be used in **establishing conditions so that the roots have desired properties**.

The equation $ax^2 + bx + c = 0$ has **two roots** if $b^2 - 4ac$ is positive

The equation $ax^2 + bx + c = 0$ has **one root** if $b^2 - 4ac$ is zero

The equation $ax^2 + bx + c = 0$ has **no roots** if $b^2 - 4ac$ is negative

Ex 1: Determine the **nature of the roots** for each of the following quadratic equations:

a) $x^2 + 4x + 5 = 0$

$a=1, b=4, c=5$

$D = b^2 - 4ac$

$= 16 - 4(1)(5)$

$= -4$

< 0

$\therefore$ no roots.

b) $3x^2 - 2x - 1 = 0$

$D = 4 - 4(3)(-1)$

> 0

$\therefore$ two roots.

Ex 2: Find the value(s) of *k* if:

a) $kx^2 + 3x - 1 = 0$ has 2 distinct roots

$a = k, b = 3, c = -1$

$D > 0$

$b^2 - 4ac > 0$

$9 - 4(k)(-1) > 0$

$9 + 4k > 0$

$4k > -9$

$k > \frac{-9}{4}$

b) $x^2 + kx + 25 = 0$ has 1 root

$a = 1, b = k, c = 25$

$D = 0$

$b^2 - 4ac = 0$

$k^2 - 4(1)(25) = 0$

$k^2 - 100 = 0$

$(k+10)(k-10) = 0$

$\therefore k \in \{-10, 10\}$

$k^2 - 100 = 0$

$(k+10)(k-10) = 0$

$k = -10$ or $k = 10$

c) $x^2 + kx + 25 = 0$ has 2 roots

$D > 0$

$b^2 - 4ac > 0$

$k^2 - 100 > 0$

$(k-10)(k+10) > 0$

zeros: $-10, 10$

$\{k < -10$ or $10 < k\}$

$\{k < -10$ or $k > 10\}$

d) $kx^2 + 12x + k = 0$ has no roots

$D < 0$

$a = k,\ b = 12,\ c = k$

$b^2 - 4ac < 0$

$144 - 4(k)(k) < 0$

$144 - 4k^2 < 0$

zeros: $144 - 4k^2 = 0$

$\div -4$

$k^2 - 36 = 0$

$(k+6)(k-6) = 0$

$k = -6$ or 6

OR

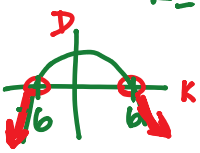
$(12+2k)(12-2k) = 0$

$k = -6$ OR $k = 6$

OR

$k^2 - 36 > 0$

$k = \pm 6$



$\{k < -6$ or $6 < k\}$

$\{k < -6$ or $k > 6\}$

U2D9 Workshee...

U2D9 MCR3UI Worksheet **Zeros & The Discriminant**

1. Determine the vertex and the direction of opening for each quadratic function. Then state the number of zeros.
   a) $f(x) = 3x^2 - 5$ b) $f(x) = -4x^2 + 7$ c) $f(x) = 5(x+2)^2$ d) $f(x) = 0.5(x-4)^2 - 2$
2. Factor each quadratic to determine the number of zeros.
   a) $f(x) = x^2 - 6x - 16$ b) $f(x) = 2x^2 - 6x$ c) $f(x) = 4x^2 - 1$ d) $f(x) = 9x^2 + 6x + 1$
3. Calculate the value of $b^2 - 4ac$ to determine the number of zeros.
   a) $f(x) = 2x^2 - 6x - 7$ b) $f(x) = 3x^2 + 2x + 7$ c) $f(x) = x^2 + 8x + 16$ d) $f(x) = 9x^2 - 14.4x + 5.76$
4. Determine the number of zeros.
   a) $f(x) = -3(x-2)^2 + 4$ b) $f(x) = 5(x-3)(x+4)$ c) $f(x) = 4x^2 - 2x$ d) $f(x) = 3x^2 - x + 5$
5. For each profit function, determine whether the company can break even. If the company can break even. If the company can break even, determine in how many ways it can do so.
   a) $P(x) = -2.1x^2 + 9.06x - 5.4$ b) $P(x) = -0.3x^2 + 2x - 7.8$
   c) $P(x) = -2x^2 + 6.4x - 5.12$ d) $P(x) = -2.4x^2 + x - 1.2$
6. For what value(s) of k will the function $f(x) = 3x^2 - 4x + k$ have one x-intercept?
7. For what value(s) of k will the function $f(x) = kx^2 - 4x + k$ have no zeros?
8. For what value(s) of k will the function $f(x) = 3x^2 + 4x + k$ have no zeros? one zero? two zeros?
9. The graph of the function $f(x) = x^2 - kx + k + 8$ touches the x-axis at one point.
   What are the possible values of k?
10. Determine the nature of the roots for each equation.
    a) $4x^2 + 7x - 2 = 0$ b) $2x^2 - 7x - 15 = 0$ c) $3x^2 - 8x + 7 = 0$
    d) $7x^2 + 10x - 3 = 0$ e) $16x^2 + 8x + 1 = 0$ f) $12x^2 - 9x + 5 = 0$

11 & 12 extra practice

11. Solve the following for $x \in \mathbb{R}$
    a) $5x^2 + 4x - 1 = 0$ b) $2x^2 - 8x + 5 = 0$ c) $5x(x+3) = (3x+2)(x-1)$
    d) $(2x+5)(x-3) = (4x+7)(3x-1)$ e) $(x+2)(5x+1) = 5x - 2(2x+1)(x+1)$
    f) $(2x+7)(x+4) = (3x+5)(x-2)$
12. Solve the following for $x \in \mathbb{R}$
    a) $\frac{x^2+5}{3} - \frac{7}{2} = \frac{x+8}{2}$ b) $\frac{8}{x} + \frac{5}{x+2} = 1$ c) $\frac{3}{2x+1} - \frac{x+2}{3x-1} = \frac{x-3}{2x+1}$ d) $\sqrt{3x+1} = x - 3$ e) $\sqrt{2x^2-2} - x = 1$
13. For what value(s) of k does each equation have two *equal real roots*?
    a) $3x^2 - kx + 8 = 0$ b) $5x^2 + 8x - 2k = 0$ c) $kx^2 + 9 = 18x$ d) $(3k+1)x^2 + kx + 1 = 0$
14. For what value(s) of m does each equation have two *distinct real roots*?
    a) $2x^2 + mx + 8 = 0$ b) $5mx^2 + 6x + 2 = 0$ c) $3(x^2 - 2m) = 9x$ d) $4x^2 - 2mx + 3 = 0$
15. Using the Discriminant, determine the following.
    a) For what values of k does $5kx^2 + 6x + 2 = 0$ have 2 real roots?
    b) For what values of k does $2x^2 + kx + 9 = 0$ have no real roots?
    c) For what values of k does $4x^2 - 2kx + 3 = 0$ have 2 real roots?

Answers:

1. a) V(0,-5); up; 2 b) V(0, 7); down; 2 c) V(-2, 0); up; 1 d) V(4, -2); up; 2
2. a) $(x-8)(x+2)$; 2 b) $(2x)(x-3)$; 2 c) $(2x+1)(2x-1)$; 2 d) $(3x+1)^2$; 1
3. a) $D = 92$; 2 b) $D = -80$; 0 c) $D = 0$; 1 d) $D = 0$; 1
4. a) 2 b) 2 c) 2 d) 0
5. a) yes, 2 ways b) cannot break even c) yes, one way d) cannot break even
6. $\left\{k = \frac{4}{3}\right\}$
7. $\{k < -2 \text{ or } k > 2\}$
8. No zeros -- $\left\{k > \frac{4}{3}\right\}$ One zero -- $\left\{k = \frac{4}{3}\right\}$ Two zeros -- $\left\{k < \frac{4}{3}\right\}$
9. $k \in \{-4, 8\}$
10. a) 2 real & distinct b) 2 real & distinct c) no real roots d) 2 real & distinct e) one root (real & equal) f) no real roots
11. a) $x \in \{-1, \frac{1}{5}\}$ b) $x \in \left\{\frac{4 \pm \sqrt{6}}{2}\right\}$ c) $x \in \{-4 \pm \sqrt{15}\}$ d) $x \in \{-1, -\frac{4}{5}\}$ e) $x \in \left\{-\frac{2}{3}\right\}$ f) $x \in \{8 \pm \sqrt{102}\}$
12. a) $x \in \{-\frac{7}{2}, 5\}$ b) $x \in \left\{\frac{11 \pm \sqrt{185}}{2}\right\}$ c) $x \in \left\{\frac{4}{5}, 2\right\}$ d) $x \in \{1, 8\}$ e) $x \in \{-1, 3\}$
13. a) $k \in \{\pm 4\sqrt{6}\}$ b) $k \in \left\{-\frac{8}{5}\right\}$ c) $k \in \{9\}$ d) $k \in \{6 \pm 2\sqrt{10}\}$
14. a) $\{m < -8 \text{ or } m > 8\}$ b) $\left\{m < \frac{9}{10}\right\}$ c) $\left\{m > -\frac{9}{8}\right\}$ d) $\{m < -2\sqrt{3} \text{ or } m > 2\sqrt{3}\}$
15. a) $\left\{k < \frac{9}{10}\right\}$ b) $\{-6\sqrt{2} < k < 6\sqrt{2}\}$ c) $\{k > 2\sqrt{3} \text{ or } k < -2\sqrt{3}\}$