U2D9 MCR3UI Worksheet **Zeros & The Discriminant**

1. Determine the vertex and the direction of opening for each quadratic function. Then state the number of zeros.
   a) $f(x) = 3x^2 - 5$ b) $f(x) = -4x^2 + 7$ c) $f(x) = 5(x+2)^2$ d) $f(x) = 0.5(x-4)^2 - 2$
2. Factor each quadratic to determine the number of zeros.
   a) $f(x) = x^2 - 6x - 16$ b) $f(x) = 2x^2 - 6x$ c) $f(x) = 4x^2 - 1$ d) $f(x) = 9x^2 + 6x + 1$
3. Calculate the value of $b^2 - 4ac$ to determine the number of zeros.
   a) $f(x) = 2x^2 - 6x - 7$ b) $f(x) = 3x^2 + 2x + 7$ c) $f(x) = x^2 + 8x + 16$ d) $f(x) = 9x^2 - 14.4x + 5.76$
4. Determine the number of zeros.
   a) $f(x) = -3(x-2)^2 + 4$ b) $f(x) = 5(x-3)(x+4)$ c) $f(x) = 4x^2 - 2x$ d) $f(x) = 3x^2 - x + 5$
5. For each profit function, determine whether the company can break even. If the company can break even. If the company can break even, determine in how many ways it can do so.
   a) $P(x) = -2.1x^2 + 9.06x - 5.4$ b) $P(x) = -0.3x^2 + 2x - 7.8$
   c) $P(x) = -2x^2 + 6.4x - 5.12$ d) $P(x) = -2.4x^2 + x - 1.2$
6. For what value(s) of k will the function $f(x) = 3x^2 - 4x + k$ have one x-intercept?
7. For what value(s) of k will the function $f(x) = kx^2 - 4x + k$ have no zeros?
8. For what value(s) of k will the function $f(x) = 3x^2 + 4x + k$ have no zeros? one zero? two zeros?
9. The graph of the function $f(x) = x^2 - kx + k + 8$ touches the x-axis at one point. What are the possible values of k?
10. Determine the nature of the roots for each equation.
    a) $4x^2 + 7x - 2 = 0$ b) $2x^2 - 7x - 15 = 0$ c) $3x^2 - 8x + 7 = 0$
    d) $7x^2 + 10x - 3 = 0$ e) $16x^2 + 8x + 1 = 0$ f) $12x^2 - 9x + 5 = 0$

11 & 12 extra practice

11. Solve the following for $x \in \mathbb{R}$
    a) $5x^2 + 4x - 1 = 0$ b) $2x^2 - 8x + 5 = 0$ c) $5x(x+3) = (3x+2)(x-1)$
    d) $(2x+5)(x-3) = (4x+7)(3x-1)$ e) $(x+2)(5x+1) = 5x - 2(2x+1)(x+1)$
    f) $(2x+7)(x+4) = (3x+5)(x-2)$
12. Solve the following for $x \in \mathbb{R}$
    a) $\frac{x^2+5}{3} - \frac{7}{2} = \frac{x+8}{2}$ b) $\frac{8}{x} + \frac{5}{x+2} = 1$ c) $\frac{3}{2x+1} - \frac{x+2}{3x-1} = \frac{x-3}{2x+1}$ d) $\sqrt{3x+1} = x - 3$ e) $\sqrt{2x^2 - 2} - x = 1$
13. For what value(s) of k does each equation have two *equal real roots*?
    a) $3x^2 - kx + 8 = 0$ b) $5x^2 + 8x - 2k = 0$ c) $kx^2 + 9 = 18x$ d) $(3k+1)x^2 + kx + 1 = 0$
14. For what value(s) of m does each equation have two *distinct real roots*?
    a) $2x^2 + mx + 8 = 0$ b) $5mx^2 + 6x + 2 = 0$ c) $3(x^2 - 2m) = 9x$ d) $4x^2 - 2mx + 3 = 0$
15. Using the Discriminant, determine the following.
    a) For what values of k does $5kx^2 + 6x + 2 = 0$ have 2 real roots?
    b) For what values of k does $2x^2 + kx + 9 = 0$ have no real roots?
    c) For what values of k does $4x^2 - 2kx + 3 = 0$ have 2 real roots?

Answers:

1. a) V(0,-5); up; 2 b) V(0, 7); down; 2 c) V(-2, 0); up; 1 d) V(4, -2); up; 2
2. a) $(x-8)(x+2)$; 2 b) $(2x)(x-3)$; 2 c) $(2x+1)(2x-1)$; 2 d) $(3x+1)^2$; 1
3. a) $D = 92$; 2 b) $D = -80$; 0 c) $D = 0$; 1 d) $D = 0$; 1
4. a) 2 b) 2 c) 2 d) 0
5. a) yes, 2 ways b) cannot break even c) yes, one way d) cannot break even
6. $\left\{k = \frac{4}{3}\right\}$
7. $\{k < -2 \text{ or } k > 2\}$
8. No zeros -- $\left\{k > \frac{4}{3}\right\}$ One zero -- $\left\{k = \frac{4}{3}\right\}$ Two zeros -- $\left\{k < \frac{4}{3}\right\}$
9. $k \in \{-4, 8\}$
10. a) 2 real & distinct b) 2 real & distinct c) no real roots d) 2 real & distinct e) one root (real & equal) f) no real roots
11. a) $x \in \{-1, \frac{1}{5}\}$ b) $x \in \left\{\frac{4 \pm \sqrt{6}}{2}\right\}$ c) $x \in \{-4 \pm \sqrt{15}\}$ d) $x \in \{-1, -\frac{4}{5}\}$ e) $x \in \left\{-\frac{2}{3}\right\}$ f) $x \in \{8 \pm \sqrt{102}\}$
12. a) $x \in \{-\frac{7}{2}, 5\}$ b) $x \in \left\{\frac{11 \pm \sqrt{185}}{2}\right\}$ c) $x \in \left\{\frac{4}{5}, 2\right\}$ d) $x \in \{1, 8\}$ e) $x \in \{-1, 3\}$
13. a) $k \in \{\pm 4\sqrt{6}\}$ b) $k \in \left\{-\frac{8}{5}\right\}$ c) $k \in \{9\}$ d) $k \in \{6 \pm 2\sqrt{10}\}$
14. a) $\{m < -8 \text{ or } m > 8\}$ b) $\left\{m < \frac{9}{10}\right\}$ c) $\left\{m > -\frac{9}{8}\right\}$ d) $\{m < -2\sqrt{3} \text{ or } m > 2\sqrt{3}\}$
15. a) $\left\{k < \frac{9}{10}\right\}$ b) $\{-6\sqrt{2} < k < 6\sqrt{2}\}$ c) $\{k > 2\sqrt{3} \text{ or } k < -2\sqrt{3}\}$

1.

quadratic	Vertex	Direction of Opening		Number of zeros.
a) $f(x)=3x^2-5$	$(0,-5)$	up		two
b) $f(x)=-4x^2+7$	$(0,7)$	down		two
c) $f(x)=5(x+2)^2$	$(-2,0)$	up		one
d) $f(x)=0.5(x-4)^2-2$	$(4,-2)$	up.		two

2.

Quadratic	Factored Form	Number of Zeros.
a) $f(x)=x^2-6x-16$	$f(x)=(x-8)(x+2)$	two
b) $f(x)=2x^2-6x$	$f(x)=2x(x-3)$	two
c) $f(x)=4x^2-1$	$f(x)=(2x-1)(2x+1)$	two
d) $f(x)=9x^2+6x+1$	$f(x)=(3x+1)^2$	one

3.

Function	$D=b^2-4ac$	Number of Zeros.
a) $f(x)=2x^2-6x-7$	$36-4(2)(-7)$ $=92$	two
b) $f(x)=3x^2+2x+7$	$4-4(3)(7)$ $=-80$	none
c) $f(x)=x^2+8x+16$	$64-64$ $=0$	one
d) $f(x)=9x^2-14.4x+5.76$	$207.36-36(5.76)$ $=0$	one.

4. function | work | number of zeros.

a) $f(x)=-3(x-2)^2+4$ $V(2,4)$ $\cap$    two

b) $f(x)=5(x-3)(x+4)$ two

c) $f(x)=4x^2-2x$ $=2x(2x-1)$ two

d) $f(x)=3x^2-x+5$ b^2-4ac
$= 1-4(3)(5)$
<0 $\therefore$ none

5. a) $p(x)=-2.1x^2+9.06x-5.4$
b^2-4ac
$=36.7236$
$\therefore$ two solutions
Company can break even in two ways.

b) $P(x)=-0.3x^2+2x-7.8$
b^2-4ac
$=1-4(-2.4)(-1.2)$
<0 no solution
$\therefore$ Company cannot break even.

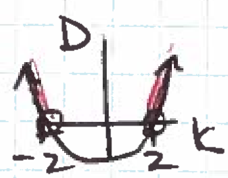
c) $P(x)=-2x^2+6.4x-5.12$
b^2-4ac
$=6.4^2-4(-2)(-5.12)$
$=0$
$\therefore$ there is one way to break even.

d) $P(x)=-2.4x^2+x-1.2$
b^2-4ac
$=1-4(-2.4)(-1.2)$
<0
$\therefore$ cannot break even.

6. $f(x)=3x^2-4x+k$
one intercept $\Rightarrow$ $D=0$
$b^2-4ac=0$
$16-4(3)(k)=0$
$16-12k=0$
$12k=16$
$k=\frac{4}{3}$

$\{k=\frac{4}{3}\}$

7. $f(x)=kx^2-4x+k$
$D<0$
$b^2-4ac<0$
$16-4(k)(k)<0$
$16-4k^2<0$
$4k^2-16>0$
$k^2-4>0$
$(k+2)(k-2)>0$



$x\leq -2$ $2\leq x$

$\{k\leq -2 \text{ or } k\geq 2\}$

8. $f(x) = 3x^2 + 4x + k$.

$D = b^2 - 4ac$

$D = 16 - 4(3)(k)$

$D = 16 - 12k$

a) no zeros

$D < 0$

$16 - 12k < 0$

$-4(-4 + 3k) < 0$

$3k - 4 > 0$

$\therefore \{k > \frac{4}{3}\}$

b) one zero

$D = 0$

$\therefore \{k = \frac{4}{3}\}$

c) two zeros

$D > 0$

$-12k + 16 > 0$

$-12k > -16$

$\therefore \{k < \frac{4}{3}\}$

9. $f(x) = x^2 - kx + k + 8$ has one zero.

$a = 1 \quad b = -k \quad c = k + 8$

$b^2 - 4ac = 0$

$k^2 - 4(1)(k+8) = 0$

$k^2 - 4k - 32 = 0$

$(k-8)(k+4) = 0$

$k = 8$ OR $k = -4$

$\therefore \{k = 8$ OR $k = -4\}$

10. a) $4x^2 + 7x - 2 = 0$

$b^2 - 4ac$

$= 49 - 4(4)(-2)$

> 0

$\therefore$ roots are real and distinct (2 roots)

b) $2x^2 - 7x - 15 = 0$

$b^2 - 4ac$

$= 49 - 4(2)(-15)$

$= 49 + 120$

> 0

$\therefore$ roots are real and distinct (2 roots).

10c) $3x^2-8x+7=0$

b^2-4ac

$= 64-4(3)(7)$

$= 64-84$

< 0

$\therefore$ no real roots

d) $7x^2+10x-3=0$

b^2-4ac

$= 100-4(7)(-3)$

> 0

$\therefore$ two real and distinct roots.

e) $16x^2+8x+1=0$

b^2-4ac

$= 64-4(16)(1)$

$= 0$

$\therefore$ one root (roots are real & equal).

... it means $16x^2+8x+1$ is a perfect square... note it factors to $(4x+1)^2$

f) $12x^2-9x+5=0$

b^2-4ac

$= 81-4(12)(5)$

< 0

$\therefore$ no real roots.

11. a) $5x^2+4x-1=0$

$(5x-1)(x+1)=0$

$x=\frac{1}{5}$ OR $x=-1$

b) $2x^2-8x+5=0$

$x = \frac{8 \pm \sqrt{64-4(2)(5)}}{4}$

$x = \frac{8 \pm \sqrt{24}}{4}$

$x = \frac{8 \pm 2\sqrt{6}}{4}$

$x = \frac{2(4 \pm \sqrt{6})}{4}$

$x = \frac{4 \pm \sqrt{6}}{2}$

11c) $5x(x+3)=(3x+2)(x-1)$

$5x^2+15x = 3x^2-3x+2x-2$

$2x^2+16x+2=0$

$x^2+8x+1=0$

$x = \frac{-8 \pm \sqrt{64-4(1)(1)}}{2}$

$x = \frac{-8 \pm \sqrt{60}}{2}$

$x = \frac{-8 \pm 2\sqrt{15}}{2}$

$x = \frac{2(-4 \pm \sqrt{15})}{2}$

$x = -4 \pm \sqrt{15}$

~~d) $5x(x+3)=(3x+2)(x-1)$~~

~~$5x^2+15x = 3x^2-3x+2x-2$~~

$2x^2$

d) $(2x+5)(x-3)=(4x+7)(3x-1)$

$2x^2-6x+5x-15 = 12x^2-4x+21x-7$

$-10x^2-x-15-17x+7=0$

$-10x^2-18x-8=0$

$5x^2+9x+4=0$

$(5x+4)(x+1)=0$

$x = -\frac{4}{5}$ or $x=-1$

11e) $(x+2)(5x+1) = 5x - 2(2x+1)(x+1)$

$5x^2 + x + 10x + 2 = 5x - 2(2x^2 + 3x + 1)$

$5x^2 + 11x + 2 - 5x + 4x^2 + 6x + 2 = 0$

$9x^2 + 12x + 4 = 0$

$(3x+2)^2 = 0$

$x = -\frac{2}{3}$

11f) $(2x+7)(x+4) - (3x+5)(x-2) = 0$

$2x^2 + 15x + 28 - (3x^2 - x - 10) = 0$

$-x^2 + 16x + 38 = 0$

$x^2 - 16x - 38 = 0$

$x = \frac{16 \pm \sqrt{256 - 4(1)(-38)}}{2}$

$x = \frac{16 \pm \sqrt{408}}{2}$

$4 \,|\, 408$
$2 \,|\, 102$
$3 \,|\, 51$
17

$x = \frac{16 \pm 2\sqrt{102}}{2}$

$x = \frac{2(8 \pm \sqrt{102})}{2}$

$x = 8 \pm \sqrt{102}$

12. a) $\frac{x^2+5}{3} - \frac{7}{2} = \frac{x+8}{2}$

$\frac{6}{3}(x^2+5) - \frac{6}{2}(7) = \frac{6}{2}(x+8)$

$2x^2 + 10 - 21 = 3x + 24$

$2x^2 - 3x - 35 = 0$

$(2x+7)(x-5) = 0$

$x = -\frac{7}{2}$ OR $x = 5$

b) $\frac{8}{x} + \frac{5}{x+2} = 1$

$\frac{8(x+2) + 5(x) - 1(x)(x+2)}{x(x+2)} = 0$

$D: \{x \mid x \in \mathbb{R}, x \neq 0, -2\}$

$8x + 16 + 5x - x^2 - 2x = 0$

$-x^2 + 11x + 16 = 0$

$x^2 - 11x - 16 = 0$

$x = \frac{11 \pm \sqrt{121 - 4(-16)}}{2}$

$x = \frac{11 \pm \sqrt{121 + 64}}{2}$

$x = \frac{11 \pm \sqrt{185}}{2}$

12c) $\dfrac{3}{2x+1} - \dfrac{x+2}{3x-1} = \dfrac{x-3}{2x+1}$

LCD $(2x+1)(3x-1)$

$$\frac{3(3x-1)-(x+2)(2x+1)-(x-3)(3x-1)}{(2x+1)(3x-1)}=0$$

$D: \{x \mid x \in \mathbb{R},\ x \neq -\frac{1}{2}, \frac{1}{3}\}$

$$9x-3-(2x^2+5x+2)-(3x^2-10x+3)=0$$
$$9x-3-2x^2-5x-2-3x^2+10x-3=0$$
$$-5x^2+14x-8=0$$
$$5x^2-14x+8=0$$
$$(5x-4)(x-2)=0$$
$$x=\frac{4}{5} \text{ OR } x=2$$

$\therefore$ solution is $x \in \{\frac{4}{5}, 2\}$

12d) $\sqrt{3x+1} = x-3$, $D: \{x \mid x \in \mathbb{R},\ x \geq -\frac{1}{3}\}$

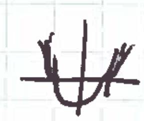
$$3x+1 \geq 0$$
$$3x \geq -1$$
$$x \geq -\frac{1}{3}$$

$$3x+1 = (x-3)^2$$
$$x^2-6x+9-3x-1=0$$
$$x^2-9x+8=0$$
$$(x-8)(x-1)=0$$
$$\therefore x \in \{1, 8\}$$

12e) $\sqrt{2x^2-2} - x = 1$

$$\left(\sqrt{2x^2-2}\right)^2 = (x+1)^2$$
$$2x^2-2 = x^2+2x+1$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x=3 \text{ OR } x=-1$$
$$\therefore x \in \{-1, 3\}$$

$$2x^2-2 \geq 0$$
$$x^2-1 \geq 0$$
$$(x-1)(x+1) \geq 0$$



$D: \{x \mid x \in \mathbb{R},\ x \leq -1 \text{ or } x \geq 1\}$

13. two equal roots (1 solution) $\Rightarrow D=0$

a) $3x^2-kx+8=0$

$b^2-4ac=0$

$k^2-4(3)(8)=0$

$k^2-96=0$

$k=\pm\sqrt{96}$

$k=\pm\sqrt{16\times 6}$

$k=\pm 4\sqrt{6}$

$\therefore k\in\{\pm 4\sqrt{6}\}$

b) $5x^2+8x-2k=0$

$b^2-4ac=0$

$64-4(5)(-2k)=0$

$64+40k=0$

$40k=-64$

$k=\frac{-64}{40}$ $\leftarrow 8\times 8$, $\leftarrow 8\times 5$

$k=\frac{-8}{5}$

13c) $kx^2+9=18x$

$kx^2-18x+9=0$

$b^2-4ac=0$

$324-4(k)(9)=0$

$324-36k=0$

$36k=324$

$k=\frac{324}{36}$

$k=9$

$\therefore k\in\{9\}$

13d) $(3k+1)x^2+kx+1=0$

$b^2-4ac=0$

$k^2-4(3k+1)(1)=0$

$k^2-12k-4=0$

$k=\frac{12\pm\sqrt{144-4(1)(-4)}}{2}$

$k=\frac{12\pm\sqrt{160}}{2}$

$k=\frac{12\pm\sqrt{16\times 10}}{2}$

$k=\frac{12\pm 4\sqrt{10}}{2}$

$k=\frac{2(6\pm 2\sqrt{10})}{2}$

$k=6\pm 2\sqrt{10}$.

$\therefore k\in\{6\pm 2\sqrt{10}\}$

14. two distinct real roots. $\underline{D>0}$

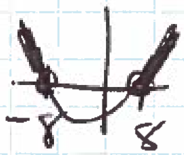
a) $2x^2+mx+8=0$

$b^2-4ac>0$

$m^2-4(2)(8)>0$

$m^2-64>0$

$(m+8)(m-8)>0$



$\therefore \{m<-8 \text{ or } m>8\}$

$m<-8 \quad 8<m$

b) $5mx^2+6x+2=0$

$b^2-4ac>0$

$36-4(5m)(2)>0$

$36-40m>0$

$-40m>-36$

$m<\frac{36}{40}$

$m<\frac{9}{10}$

$\therefore \{m<\frac{9}{10}\}$

c) $3(x^2-2m)=9x$

$3x^2-9x-6m=0$

$b^2-4ac>0$

$81-4(3)(-6m)>0$

$81+72m>0$

$72m>-81$

$m>\frac{-81}{72}$

$m>-\frac{9}{8}$

$\therefore \{m>-\frac{9}{8}\}$

d) $4x^2-2mx+3=0$

$b^2-4ac>0$

$(-2m)^2-4(4)(3)>0$

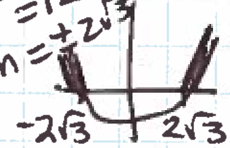
$4m^2-48>0$

$4(m^2-12)>0$

$m^2-12>0$

$m^2=12$

$m=\pm 2\sqrt{3}$



$m<-2\sqrt{3},\ 2\sqrt{3}<m$

$\therefore \{m<-2\sqrt{3} \text{ OR } m>2\sqrt{3}\}$

15. a) $5kx^2 + 6x + 2 = 0$ 2 real roots $\Rightarrow D > 0$

$b^2 - 4ac > 0$

$36 - 4(5k)(2) > 0$

$36 - 40k > 0$

$-40k > -36$

$k < \frac{-36}{-40}$

$k < \frac{9}{10}$

$\therefore \{k < \frac{9}{10}\}$

15b) $2x^2 + kx + 9 = 0$ no real roots $\Rightarrow D < 0$

$b^2 - 4ac < 0$

$k^2 - 4(2)(9) < 0$

$k^2 - 72 < 0$

$\sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$

$-6\sqrt{2} < k < 6\sqrt{2}$

$\therefore \{k \mid k \in \mathbb{R}, -6\sqrt{2} < k < 6\sqrt{2}\}$ OR $\{-6\sqrt{2} < k < 6\sqrt{2}\}$

15c) $4x^2 - 2kx + 3 = 0$ 2 real roots $\Rightarrow D > 0$

$b^2 - 4ac > 0$

$(2k)^2 - 4(4)(3) > 0$

$4k^2 - 48 > 0$

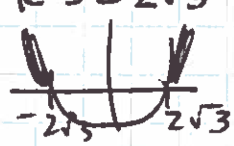
$4(k^2 - 12) > 0$

$k^2 - 12 > 0$

$k^2 = 12$

$k = \pm\sqrt{12}$

$k = \pm 2\sqrt{3}$



$k < -2\sqrt{3}$, $2\sqrt{3} < k$

$\therefore \{k < -2\sqrt{3}$ or $k > 2\sqrt{3}\}$