U2D6 MCR 3UI

Worksheet Quadratic Applications

1. A cliff diver dives from 17m above the water. The diver's height above the water, h(*t*) in metres after *t* seconds is modelled by $h(t) = -4.9t^2 + 1.5t + 17$. Determine when the diver was 5 m above the water.
2. The function $P(x) = -30x^2 + 360x + 785$ models the profit, P(*x*), earned by a theatre owner on the basis of a ticket price, *x*. Both the profit and ticket price are in dollars. What is the maximum profit and how much should the tickets cost?
3. The population of a town is modelled by the function $P(t) = 6t^2 + 110t + 4000$, where P(*t*) is the population and *t* is the time in years since 2000.
   a. What will the population be in 2020?
   b. When will the population be 6000?
   c. Will the population ever be 0? Explain your answer.
4. The profit of a shoe company is modelled by the function $P(x) = -5(x - 4)^2 + 45$, where x is the number of pairs of shoes produced in thousands, and P(x) is the profit, in thousands of dollars. How many thousands of pairs of shoes will the company need to sell to earn a profit?
5. Beth wants to plant a garden at the back of her house. She has 32m of fencing. The area that can be enclosed is modelled by the function $A(x) = -2x^2 + 32x$, where x is the width of the garden in metres and A(x) is the area in square metres. What is the maximum area that can be enclosed?
6. A rectangle is 7 cm longer than it is wide. The diagonal is 13cm. What are the rectangle's dimensions?
7. A photo framer wants to place a matte of uniform width all around a photo. The area of the matte should be equal to the area of the photo. The photo measures 40 cm by 60cm. How wide should the matte be?
8. The stopping distance for a boat in calm water is modelled by the function $d(v) = 0.004v^2 + 0.2v + 6$, where d(v) is in metres and v is in kilometres per hour.
   a. What is the stopping distance if the speed is 10km/h?
   b. What is the initial speed of the boat if it takes 11.6m to stop?
9. Mario wants to install a wooden deck around his rectangular swimming pool. The cost is modelled by the function $C(w) = 120w^2 + 1800w$, where C(w) is the cost in dollars and w is the width in metres. How wide will the deck be if he has $4080 to spend?
10. The population of a rural town can be modelled by the function $P(x) = 3x^2 - 102x + 25000$, where x is the number of years since 2000. According to the model in what year will the population be lowest?
11. A bowling alley has a $5 cover charge on Friday nights. The manager is considering increasing the cover charge in 50¢ increments. The revenue modelled by the function $R(x) = -12.5x^2 + 75x + 2000$, where revenue R(x) is in dollars and x is the number of 50¢ increments.
    a. What cover charge will maximize revenue?
    b. What will the cover charge be if revenue is $2000?
12. The height of a soccer ball kicked in the air is given by the equation $h(t) = -4.9(t - 2.1)^2 + 23$, where t, is the time in seconds and h(t) is the height in metres
    a. What is the height of the ball when it was kicked?
    b. What is the maximum height of the ball?
    c. Is the ball still in the air after 6 seconds? Explain
    d. For how long was the ball <u>at least</u> 10m high?

Answers

1.	1.73 s	**2.**	$1865 & $6	**3.**	a.)8600	b)2011	c)no	**4.**	between 1000 & 7000
5.	$128m^2$	**6.**	5 cm X 12 cm	**7.**	10 cm		**8.**	a)8.4km	b) 20km/h
9.	2 m	**10.**	2017	**11.**	a)$6.50	b) $5 or $8			
12.	a)1.39m		b)23m		c)no hits ground at 4.3s				d) 3.26s

1. $-4.9t^2+1.5t+17=5$ } $\times -10$

$49t^2-15t-120=0$

$t=\frac{15\pm\sqrt{225-4(49)(-120)}}{98}$

$t=\frac{15\pm\sqrt{23745}}{98}$

$t\doteq 1.7254...$ OR $-1.4193...$

∴ the diver will be 5m above the water after 1.73 seconds.

2. $P(x)=-30(x^2-12x+36-36)+785$ — By completing the square

$P(x)=-30(x-6)^2+1080+785$

$P(x)=-30(x-6)^2+1865$

There is a maximum profit of \$1865 with ticket price \$6.

By partial factoring

$P(x)=-30x(x-12)+785$

$P(6)=-30(6)(-6)+785$

$P(6)=1865$

3. $P(t)=6t^2+110t+4000$

a) In 2020, $t=20$

$P(20)=6(20)^2+110(20)+4000$

$P(20)=8600$

The population will be 8600 in 2020.

b) $P(x)=6000$

$6t^2+110t+4000=6000$

$6t^2+110t-2000=0$

$3t^2+55t-1000=0$

$D=15025$ ∴ DNF

$t=\frac{-55\pm\sqrt{15025}}{6}$

$t=11.26...$ OR -29.596

∴ the population reaches 6000 in 2011.

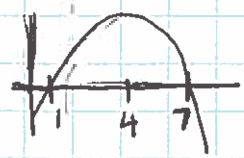
c) $6t^2+110t+4000=0$

$D=-83900$ ← no solution

∴ the population will never be zero.

4. $P(x) = -5(x-4)^2 + 45$ x, P both in thousands.

$-5(x-4)^2 + 45 > 0$



zeros:

$-5(x^2 - 8x + 16) + 45 = 0$

$-5x^2 + 40x - 80 + 45 = 0$

$-5x^2 + 40x - 35 = 0$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 1$ OR 7

∴ to make a profit, the company must sell between 1000, 7000 pairs of shoes.

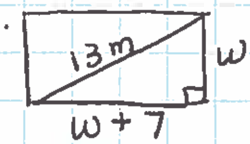
5. $A(x) = -2x^2 + 32x$

$= -2(x^2 - 16x + 64 - 64)$

$= -2(x-8)^2 + 128$

∴ the maximum area that can be enclosed is $128 m^2$.

6.



$(w+7)^2 + w^2 = 13^2$

$w^2 + 14w + 49 + w^2 - 169 = 0$

$2w^2 + 14w - 120 = 0$

$w^2 + 7w - 60 = 0$

$(w+12)(w-5) = 0$

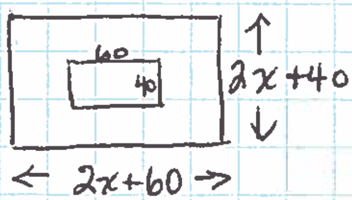
$w = -12$ OR $w = 5$

↑ inadmissible

length: $5 + 7 = 12$

∴ the rectangle is 12m × 5m.

7.



$(2x+60)(2x+40) = 2(40)(60)$

$4x^2 + 80x + 120x + 2400 - 4800 = 0$

$4x^2 + 200x - 2400 = 0$

$x^2 + 50x - 600 = 0$

$(x+60)(x-10) = 0$

$x = -60$ OR $x = 10$

∴ the matte is 10cm wide.

8. $d(v) = 0.004v^2 + 0.2v + 6$

a) $v = 10\text{ km/h}$

$d(10) = 0.004(10)^2 + 0.2(10) + 6$

$= 0.4 + 2 + 6$

$= \boxed{8.4\text{ km}}$ ∴ the stopping distance is 8.4 km.

b) $d = 11.6\text{ m}, v = ?$

$0.004v^2 + 0.2v + 6 = 11.6$

$0.004v^2 + 0.2v - 5.6 = 0$

$4v^2 + 200v - 5600 = 0$

$v^2 + 50v - 1400 = 0$

$(v + 70)(v - 20) = 0$

$v = -70$ OR $v = 20$

driving backwards at 70 km/h is unrealistic

∴ a velocity of 20 km/h results in a stopping distance of 11.6 km.

9. $C = \$4080$ $C(w) = 120w^2 + 1800w$

$120w^2 + 1800w = 4080$

$120w^2 + 1800w - 4080 = 0$

$2w^2 + 30w - 68 = 0$

$D = 1444$ ← (perfect square so equation is factorable)

$w = \frac{-30 \pm \sqrt{1444}}{4}$

$w = \frac{-30 \pm 38}{4}$

$w = \frac{-68}{4}$ OR $w = \frac{8}{4}$

↑ inadmissible $w = 2$

∴ a 2 m wide deck would cost \$4080.

10. $P(x) = 3x^2 - 102x + 25000$

$P(x) = 3(x^2 - 34x + 289 - 289) + 25000$

$P(x) = 3(x - 17)^2 - 867 + 25000$

$P(x) = 3(x - 17)^2 + 24133$

$P(x) = 3x(x - 34) + 25000$

P(17) is lowest ∴ population.

∴ In 2017, population will be at its lowest, 24133 people.

11. a) $R(x) = -12.5x^2 + 75x + 2000$
$R(x) = -12.5x(x-6) + 2000$
A of S $x = 3$
∴ 3 50¢ increments will maximize revenue.

$\$5 + 3 \times 50¢$
$= 5 + 1.50$
$= 6.50$ ∴ the cover charge of \$6.50 will maximize revenue.

b) $R(x) = 2000$
$-12.5x^2 + 75x + 2000 = 2000$
$-12.5x(x-6) = 0$
$x = 0$ OR $x = 6$
Cover charge is
\$5 + 0 OR \$5 + \$0.5 × 6
= \$5 = \$8.
∴ a cover charge of \$5 OR \$8 results in a Revenue of \$2000.

12. $h(t) = -4.9(t-2.1)^2 + 23$

a) $h(0) = -4.9(-2.1)^2 + 23$
$h(0) = 1.391$ ∴ the ball is kicked at a height of 1.391 m.

b) $V(2.1, 23)$ ∴ maximum height of ball is 23 m.

c) $h(6) = -4.9(6-2.1)^2 + 23$
$h(6) = -51.529$
∴ no, after 6 seconds the height is negative so the ball hits the ground before 6 seconds. (Can set $h(t)=0$ to determine when the ball hits the ground).

d) $-4.9(t-2.1)^2 + 23 > 10$
$-4.9(t-2.1)^2 > -13$
$(t-2.1)^2 < \frac{13}{4.9}$

zeros:
$t = 2.1 + \sqrt{\frac{13}{4.9}}$ OR $t = 2.1 - \sqrt{\frac{13}{4.9}}$
$t = 3.7288$ $t = 0.4711$

$3.7288 - 0.4711$
$= 3.2576$ ∴ the ball is in the air for about 3.26 seconds.