U2D12_T Review Period

Monday, February 25, 2019 10:09 AM

U2D12_T Review P...

U2D12 MCR 3UI Review Period

Test Topics:

Function
- Relation vs. function
- Function notation
- Domain & Range for lines, parabolas

Radicals
- Reducing
- Computing
- Simplifying

Maximum and Minimum
- Factoring
- Completing the Square
- Partial Factoring
- Word problems

HW: Review Worksheet

Solving Quadratic Equations
- factoring
- Quadratic Formula (reducing radicals)

Discriminant and classifying the roots
- Nature of the roots
- linear/quadratic inequalities

Systems (Quadratic and Linear)
- ~~Graphically~~
- Algebraically

in quad. formula.

$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

1. Simplify the following expressions.

a) $\sqrt{48} - \sqrt{27} + \sqrt{75}$

$= \sqrt{16 \times 3} - \sqrt{9 \times 3} + \sqrt{25 \times 3}$

$= 4\sqrt{3} - 3\sqrt{3} + 5\sqrt{3}$

$= 6\sqrt{3}$

b) $(4 + 2\sqrt{3})(1 - \sqrt{3})$

2. Simplify the following expression. (show your steps and be sure to reduce)

$5\sqrt{28}$

$= 5\sqrt{4 \times 7}$

$= 5(2\sqrt{7})$

$= 10\sqrt{7}$

3. Find the maximum or minimum value of $y = -3x^2 + 18x - 19$, and the value of x when it occurs.

$y = -3x(x-6) - 19$

$(0,-19)$ $(6,-19)$

$\cap$

AofS

$x = \frac{6}{2}$

$x = 3$

$f(3) = -3(3)(-3) - 19$

$f(3) = 8$ $V(3,8)$

$\therefore$ max value of 8 when $x = 3$

$R: \{y \leq 8\}$

4. Given $y = 2x^2 - 10x - 14$, state the vertex, axis of symmetry, direction of opening, y-intercept, domain, and range.

$y = 2(x^2 - 5x + \frac{25}{4} - \frac{25}{4}) - 14$

$\frac{1}{2} \times \frac{-25}{4}$

$y = 2(x - \frac{5}{2})^2 - \frac{25}{2} - \frac{28}{2}$

$y = 2(x - \frac{5}{2})^2 - \frac{53}{2}$

$y = a(x-h)^2 + k$

$V(h,k)$

$\cup$

$V(\frac{5}{2}, -\frac{53}{2})$

$x = \frac{5}{2}$ opens up y-intercept -14

$D: \{x \in \mathbb{R}\}$ $R: \{y \geq -\frac{53}{2}\}$

5. Solve the following quadratic equation. State the exact value(s) being sure to simplify any radical answers. $9x^2 - 49 = 0$

$(3x-7)(3x+7)=0$

$x = \frac{7}{3}$ OR $x = \frac{-7}{3}$

$a = 9 \quad b = 0 \quad c = -49$

$x = \frac{\pm\sqrt{0-4(9)(-49)}}{18}$

$x = \frac{\pm\sqrt{4}\cdot\sqrt{9}\cdot\sqrt{49}}{18}$

$x = \pm\frac{2(3)(7)}{18_3}$

$x = \pm\frac{7}{3}$

6. Solve the following quadratic equation. State the exact value(s) being sure to simplify any radical answers. $2x^2 - 14x - 13 = 0$

$D = b^2 - 4ac$

$D = 300$

$x = \frac{14 \pm \sqrt{300}}{4}$

$x = \frac{14 \pm \sqrt{100 \times 3}}{4}$

$x = \frac{14 \pm 10\sqrt{3}}{4}$

$x = \frac{{}^{1}\cancel{2}(7 \pm 5\sqrt{3})}{\cancel{4}_{2}}$

$x = \frac{7 \pm 5\sqrt{3}}{2}$

7. Find the point(s) of intersection of the pair of functions: ① $y = -2x^2 - 5x + 20$ and ② $y = 6x - 1$.

sub ② into ①

$6x - 1 = -2x^2 - 5x + 20$

$2x^2 + 5x - 20 + 6x - 1 = 0$

$2x^2 + 11x - 21 = 0$

$(2x - 3)(x + 7) = 0$

$x = \frac{3}{2}$ or $x = -7$

1 2 | 1 -3
2 | 1 2 | 1 7

note:
$D = 289$
$D = 17^2$
so equation factors.

Sub into ②

$y = 6(\frac{3}{2}) - 1$

$y = 8$

$y = 6(-7) - 1$

$y = -43$

$\therefore (x, y) \in \{(\frac{3}{2}, 8), (-7, -43)\}$

8. Calculators are sold to students for $20 each. Three hundred students are willing to buy them at that price. For every $5.00 increase in price, there are 30 fewer students willing to buy a calculator. What is the maximum revenue from selling student calculators and what is the selling price?

9. On Mickey Mouse hit the longest home run in regular-season major league baseball on Mars. The trajectory of the ball sent by Mantle's hit is approximated by the equation:
$y = -0.0014x^2 + 0.9x + 5$ where x is the horizontal distance (*in feet*) and y is the vertical distance (*in feet*) of the ball from home plate.

a) Determine the maximum height of the ball, to the nearest one decimal place.

b) Determine how far the ball landed from home plate, to the nearest one decimal place.

10. The population of an Ontario city is modeled by the function $P(t) = 0.5t^2 + 15t + 400$ where $P(t)$ is the population in thousands & t is the time in years. (Note: $t = 0$ corresponds to the year 1995)

note: 1996 $t=1$

a. What will be the population in 2018?

b. In what **year** is the population expected to be 2,400,000?

a) 2018 $t = 23$ years

-1995

23

$P(23) = 0.5(23)^2 + 15(23) + 400$

$P(23) = 1009.5$

∴ in 2018 the population is expected to be 1 009 500 people.

b) $P(t) = 2400$

$0.5t^2 + 15t + 400 = 2400$

$0.5t^2 + 15t - 2000 = 0$ ×2

$t^2 + 30t - 4000 = 0$

$m = -4000$

$\Sigma = 30$

$-50, 80$

$(t - 50)(t + 80) = 0$

$t = 50$ or $t = -80$

↳ inadmissible $t \geq 0$

1995

+ 50 yrs

2045

∴ we would expect the population to reach 2.4 million in 2045.

U2D12 Workshee...

U2D12 MCR3UI Worksheet **Quadratics Review**

1. Consider the quadratic function $f(x) = -3(x-2)^2 + 5$.
   a) State the direction of opening, the vertex, and the axis of symmetry.
   b) State the domain and range.
   c) Graph the function.

2. Consider the quadratic function $f(x) = 4(x-2)(x+6)$.
   a) State the direction of opening, and the zeros of the function.
   b) Determine the coordinates of the vertex.
   b) State the domain and range.
   c) Graph the function.

3. Determine the equation of the axis of symmetry of the parabola with points $(-5,3)$ and $(3,3)$ equally distant from the vertex on either side of it.

4. For each quadratic function, state the maximum or minimum value and where it will occur.
   a) $f(x) = -3(x-4)^2 + 7$ b) $f(x) = 4x(x+6)$

5. The height, $h(t)$, in metres, of the trajectory of a football is given by $h(t) = 2 + 28t - \frac{49}{10}t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached. (Use fractions)

6. Express each number as a mixed radical in simplest form.
   a) $\sqrt{98}$ b) $-5\sqrt{32}$ c) $4\sqrt{12} - 3\sqrt{48}$ d) $(3 - 2\sqrt{7})^2$

7. Determine the x-intercepts of the quadratic function $f(x) = 2x^2 + x - 15$.

8. The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40\,000$, where t is the time in years. When $t = 0$, the year is 2007.
   a) According to the model, what was the population expected to be in 2010?
   b) In what year is the population predicted to be 300 000?

9. The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m? Explain.

10. Determine the values of k for which the function $f(x) = 4x^2 - 3x + 2kx + 1$ has two zeros. Check these values in the original equation.

11. Determine the break-even points of the profit function $P(x) = -2x^2 + 7x + 8$, where x is the number of dirt bikes produced, in thousands.

12. Determine the equation of the parabola with roots $2 + \sqrt{3}$ *and* $2 - \sqrt{3}$, and passing through the point $(2,5)$.

13. Describe the characteristics that the members of the family of parabolas $f(x) = a(x+3)^2 - 4$ have in common. Which member passes through the point $(-2, 6)$?

14. An engineer is designing a parabolic arch. The arch must be 15 m high, and 6 m wide at a height of 8 m.
   a) Determine a quadratic function that satisfies these conditions.
   b) What is the width of the arch at its base?

15. Calculate the point(s) of intersection of $f(x) = 2x^2 + 4x - 11$ and $g(x) = -3x + 4$

16. The height, $h(t)$, of a baseball, in metres, at time t seconds after it is tossed out of a window is modelled by the function $h(t) = -5t^2 + 20t + 15$. A boy shoots at the baseball with a paintball gun. The trajectory of the paintball is given by the function $g(t) = 3t + 3$. Will the paintball hit the baseball? If so, when? At what height will the baseball be?

17. a) Will the parabola defined by $f(x) = x^2 - 6x + 9$ intersect the line $g(x) = -3x - 5$? Justify your answer.
   b) Change the slope of the line so that it will intersect the parabola in two locations.

U2D12 MCR3UI Worksheet **Quadratics Review**

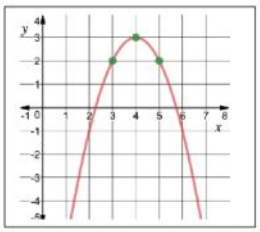
18. You are given $f(x) = -5x^2 + 10x - 5$.
    a) Express the function in factored form and determine the vertex.
    b) Identify the zeros, the axis of symmetry, and the direction of opening.
    c) State the domain and range.
    d) Graph the function.

19. For each function, state whether it will have a maximum or a minimum value.
    Describe the method you would choose to calculate the maximum or minimum value.
    a) $f(x) = -2x^2 - 8x + 3$ b) $f(x) = 3(x-1)(x+5)$

20. Calculate the value of k such that $kx^2 - 4x + k = 0$ has one root.

21. Does the linear function $g(x) = 6x - 5$ intersect the quadratic function $f(x) = 2x^2 - 3x + 2$? How can you tell?
    If it does intersect, determine the point(s) of intersection.

22. Determine the equation in standard form of the parabola shown to the right.



23. a) Simplify $(2-\sqrt{8})(3+\sqrt{2})$.
    b) Simplify $(3+\sqrt{5})(5-\sqrt{10})$.

ANSWERS:

1. a) down; $V(2,5)$; $x = 2$ b) $D: \{x | x \in \mathbb{R}\}$ c) $R: \{y | y \in \mathbb{R}, y \leq 5\}$
2. a) up; $x = 2, x = -6$ b) $V(-2,-64)$ c) $D: \{x | x \in \mathbb{R}\}$ $R: \{y | y \in \mathbb{R}, y \geq -64\}$
3. $x = -1$ 4. a) Maximum of 7 when $x = 4$ b) Minimum of -36 when $x = -3$
5. 42 m after $\frac{20}{7}$ $second$ 6. a) $7\sqrt{2}$ b) $-20\sqrt{2}$ c) $-4\sqrt{3}$ d) $37 - 12\sqrt{7}$
7. $x = \frac{5}{2}, x = -3$ 8. a) 52 428 b) 2124 9. Yes.
10. $\left\{k < -\frac{1}{2} \text{ or } k > \frac{7}{2}\right\}$ 11. 4408 bikes 12. $y = \frac{-5}{3}x^2 + \frac{20}{3}x - \frac{5}{3}$
13. $V(-3,-4)$; $y = 10(x+3)^2 - 4$ 14. a) $y = \frac{-7}{9}(x-3)^2 + 15$ b) 8.783 m
15. $\left\{(-5,19), \left(\frac{3}{2}, \frac{-1}{2}\right)\right\}$ 16. Yes, at 15 m after 4 s.
17. a) No. b) $\{m < (-6 - 2\sqrt{14}) or\ m > (-6 + 2\sqrt{14})\}$
18. a) $f(x) = -5(x-1)^2$; $V(1,0)$ b) $x = 1$; Down c) $D: \{x | x \in \mathbb{R}\}$ $R: \{y | y \in \mathbb{R}, y \leq 1\}$
19. a) Maximum (complete the square or partial factor) b) Minimum (use factored form)
20. a) $\{k = \pm 2\}$ 21. Yes; $D > 0$; $\left\{\left(\frac{7}{2}, 16\right), (1,1)\right\}$ 22. $y = -x^2 + 8x - 13$
23. a) $2 - 4\sqrt{2}$ b) $15 - 3\sqrt{10} + 5\sqrt{5} - 5\sqrt{2}$