U2D12 MCR3UI Worksheet **Quadratics Review**

1. Consider the quadratic function $f(x) = -3(x-2)^2 + 5$.
   a) State the direction of opening, the vertex, and the axis of symmetry.
   b) State the domain and range.
   c) Graph the function.

2. Consider the quadratic function $f(x) = 4(x-2)(x+6)$.
   a) State the direction of opening, and the zeros of the function.
   b) Determine the coordinates of the vertex.
   b) State the domain and range.
   c) Graph the function.

3. Determine the equation of the axis of symmetry of the parabola with points $(-5,3)$ and $(3,3)$ equally distant from the vertex on either side of it.

4. For each quadratic function, state the maximum or minimum value and where it will occur.
   a) $f(x) = -3(x-4)^2 + 7$ b) $f(x) = 4x(x+6)$

5. The height, $h(t)$, in metres, of the trajectory of a football is given by $h(t) = 2 + 28t - \frac{49}{10}t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached. (Use fractions)

6. Express each number as a mixed radical in simplest form.
   a) $\sqrt{98}$ b) $-5\sqrt{32}$ c) $4\sqrt{12} - 3\sqrt{48}$ d) $(3 - 2\sqrt{7})^2$

7. Determine the x-intercepts of the quadratic function $f(x) = 2x^2 + x - 15$.

8. The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40\,000$, where t is the time in years. When $t = 0$, the year is 2007.
   a) According to the model, what was the population expected to be in 2010?
   b) In what year is the population predicted to be 300 000?

9. The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m? Explain.

10. Determine the values of k for which the function $f(x) = 4x^2 - 3x + 2kx + 1$ has two zeros. Check these values in the original equation.

11. Determine the break-even points of the profit function $P(x) = -2x^2 + 7x + 8$, where x is the number of dirt bikes produced, in thousands.

12. Determine the equation of the parabola with roots $2 + \sqrt{3}$ *and* $2 - \sqrt{3}$, and passing through the point $(2,5)$.

13. Describe the characteristics that the members of the family of parabolas $f(x) = a(x+3)^2 - 4$ have in common. Which member passes through the point $(-2, 6)$?

14. An engineer is designing a parabolic arch. The arch must be 15 m high, and 6 m wide at a height of 8 m.
    a) Determine a quadratic function that satisfies these conditions.
    b) What is the width of the arch at its base?

15. Calculate the point(s) of intersection of $f(x) = 2x^2 + 4x - 11$ and $g(x) = -3x + 4$

16. The height, $h(t)$, of a baseball, in metres, at time t seconds after it is tossed out of a window is modelled by the function $h(t) = -5t^2 + 20t + 15$. A boy shoots at the baseball with a paintball gun. The trajectory of the paintball is given by the function $g(t) = 3t + 3$. Will the paintball hit the baseball? If so, when? At what height will the baseball be?

17. a) Will the parabola defined by $f(x) = x^2 - 6x + 9$ intersect the line $g(x) = -3x - 5$? Justify your answer.
    b) Change the slope of the line so that it will intersect the parabola in two locations.

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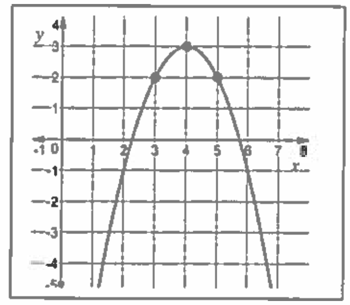
18. You are given $f(x) = -5x^2 + 10x - 5$.
    a) Express the function in factored form and determine the vertex.
    b) Identify the zeros, the axis of symmetry, and the direction of opening.
    c) State the domain and range.
    d) Graph the function.

19. For each function, state whether it will have a maximum or a minimum value.
    Describe the method you would choose to calculate the maximum or minimum value.
    a) $f(x) = -2x^2 - 8x + 3$ b) $f(x) = 3(x-1)(x+5)$

20. Calculate the value of k such that $kx^2 - 4x + k = 0$ has one root.

21. Does the linear function $g(x) = 6x - 5$ intersect the quadratic function
    $f(x) = 2x^2 - 3x + 2$? How can you tell?
    If it does intersect, determine the point(s) of intersection.

22. Determine the equation in standard form of the parabola shown to the right.



23. a) Simplify $(2 - \sqrt{8})(3 + \sqrt{2})$.
    b) Simplify $(3 + \sqrt{5})(5 - \sqrt{10})$.

ANSWERS:

1. a) down; $V(2,5)$; $x = 2$ b) $D: \{x | x \in \mathbb{R}\}$ c) $R: \{y | y \in \mathbb{R}, y \leq 5\}$
2. a) up; $x = 2, x = -6$ b) $V(-2,-64)$ c) $D: \{x|x \in \mathbb{R}\}$ $R: \{y|y \in \mathbb{R}, y \geq -64\}$
3. $x = -1$ 4. a) Maximum of 7 when $x = 4$ b) Minimum of -36 when $x = -3$
5. 42 m after $\frac{20}{7}$ *second* 6. a) $7\sqrt{2}$ b) $-20\sqrt{2}$ c) $-4\sqrt{3}$ d) $37 - 12\sqrt{7}$
7. $x = \frac{5}{2}, x = -3$ 8. a) 52 428 b) 2124 9. Yes.
10. $\left\{k < -\frac{1}{2} \ or \ k > \frac{7}{2}\right\}$ 11. 4408 bikes 12. $y = \frac{-5}{3}x^2 + \frac{20}{3}x - \frac{5}{3}$
13. $V(-3,-4)$; $y = 10(x+3)^2 - 4$ 14. a) $y = \frac{-7}{9}(x-3)^2 + 15$ b) 8.783 m
15. $\left\{(-5,19), \left(\frac{3}{2}, \frac{-1}{2}\right)\right\}$ 16. Yes, at 15 m after 4 s.
17. a) No. b) $\{m < (-6 - 2\sqrt{14}) or \ m > (-6 + 2\sqrt{14})\}$
18. a) $f(x) = -5(x-1)^2$; $V(1,0)$ b) $x = 1$; Down c) $D: \{x|x \in \mathbb{R}\}$ $R: \{y|y \in \mathbb{R}, y \leq 1\}$
19. a) Maximum (complete the square or partial factor) b) Minimum (use factored form)
20. a) $\{k = \pm 2\}$ 21. Yes; $D > 0$; $\left\{\left(\frac{7}{2}, 16\right), (1,1)\right\}$ 22. $y = -x^2 + 8x - 13$
23. a) $2 - 4\sqrt{2}$ b) $15 - 3\sqrt{10} + 5\sqrt{5} - 5\sqrt{2}$

2-8

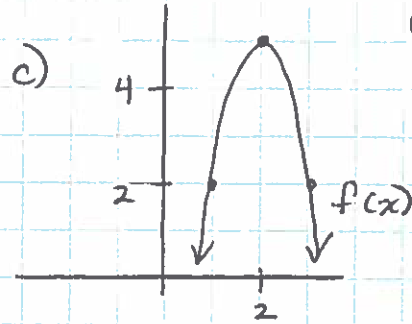
pg. 1

1. $f(x) = -3(x-2)^2 + 5$

a) $\frown$ down $\quad V(2,5)$ $\quad$ AoS: $x = 2$

b) $D = \{x \in \mathbb{R}\}$ $\quad R = \{y \leq 5\}$

c)



2. $f(x) = 4(x-2)(x+6)$

a) $\smile$ up $\quad$ zeros $x = 2, -6$

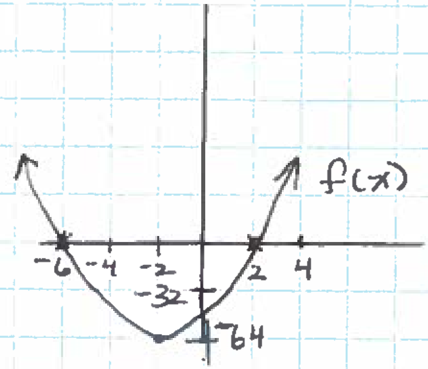
b) AoS: $x = \frac{2-6}{2}$ $\quad f(-2) = -64$

$x = -2$

$V(-2, -64)$

c) $D = \{x \in \mathbb{R}\}$ $\quad R = \{y \geq -64\}$

d)



3. AoS: $x = \frac{-5+3}{2}$

$\boxed{x = -1}$

4 a) $V(4,7)$ $\frown$

Max value of 7 occurs when $x = 4$.

b) $x = \frac{0-6}{2}$ $\smile$ Min value of -36 occurs at $x = -3$.

$x = -3$

$f(-3) = 4(-3)(3)$

$= -36$

5. $h(t) = 2 + 28t - 4.9t^2$

$= -4.9\left(t^2 - \frac{280}{49}t\right) + 2$

$= -4.9\left(t^2 - \frac{40}{7}t + \frac{400}{49} - \frac{400}{49}\right) + 2$

$= -4.9\left(t - \frac{20}{7}\right)^2 + 40 + 2$

$= -4.9\left(t - \frac{20}{7}\right)^2 + 42$

$\therefore$ max height of the ball is 42 m and was reached after $2\frac{6}{7}$ seconds.

6. a) $\sqrt{98}$

$= \sqrt{49 \times 2}$

$= 7\sqrt{2}$

b) $-5\sqrt{32}$

$= -5\sqrt{16 \times 2}$

$= -20\sqrt{2}$

c) $4\sqrt{12} - 3\sqrt{48}$

$= 4(2\sqrt{3}) - 3(4\sqrt{3})$

$= 8\sqrt{3} - 12\sqrt{3}$

$= -4\sqrt{3}$

d) $(3 - 2\sqrt{7})^2$

$= 9 - 12\sqrt{7} + 4(7)$

$= 37 - 12\sqrt{7}$

7. $f(x) = 2x^2 + x - 15$ $\quad$ M -30, A 1, $6, -5$, $\div 2$

$= (2x-5)(x+3)$

x-intercepts are $\frac{5}{2}, -3$.

8. a) $t = 13$

$P(13) = 12(13)^2 + 800(13) + 40\,000$

$= 52\,428$

$\therefore$ Population will be 52428 in 2020.

8b). $12t^2 + 800t + 40000 = 300\,000$

$3t^2 + 200t - 65000 = 0$

$\longrightarrow$

8b) $3t^2+200t-65000=0$

$$t=\frac{-200\pm\sqrt{820000}}{6}$$

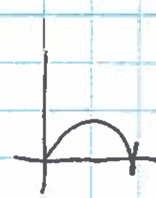
$t=117.5897...$ or $t<0$

$$\begin{array}{r} 2007 \\ +\ 117.589 \\ \hline 2124.589... \end{array}$$

In 2124, the population is predicted to reach 300000.

9. $h(t)=14t-5t^2$, set $h(t)=9\,m$.

$14t-5t^2=9$
$5t^2-14t+9=0$
b^2-4ac
$=16$
>0 $\therefore$ 2 solutions.



$\therefore$ the projectile will reach a height of 9m.

10. $f(x)=4x^2-3x+2kx+1$
$b^2-4ac>0$

$(-3+2k)^2-4(4)(1)>0$
$9-12k+4k^2-16>0$
$4k^2-12k-7>0$ $m=-28$
$(2k-7)(2k+1)>0$ $-14, 2$
$\div 2$ $\div 2$

$-\frac{1}{2}$ $\frac{7}{2}$

$\therefore \{k<-\frac{1}{2} \text{ or } k>\frac{7}{2}\}$

11. set $P(x)=0$
$-2x^2+7x+8=0$ m-16 A 7

$$x=\frac{-7\pm\sqrt{49+64}}{-4}$$

$$x=\frac{7\pm\sqrt{113}}{4}$$

$x=4.40753$ since $x>0$

$\therefore$ 4408 dirt bikes should be produced.

12. Sum $=2+\sqrt{3}+2-\sqrt{3}$
$=4$
Product $=4-3$
$=1$

$y=a(x^2-4x+1)$
$5=a(2^2-8+1)$
$5=a(-3)$
$-\frac{5}{3}=a$

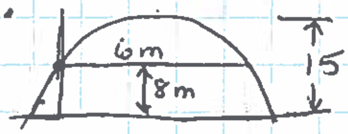
$\therefore y=-\frac{5}{3}(x^2-4x+1)$

$y=-\frac{5}{3}x^2+\frac{20}{3}x-\frac{5}{3}$

13. $f(x) = a(x+3)^2 - 4$ Vertex at $(-3, -4)$

$6 = a(-2+3)^2 - 4$
$6 = a - 4$
$a = 10$ $\therefore y = 10(x+3)^2 - 4$

14.



a) $(0,8)$ $(6,8)$ $(3,15)$

$y = a(x-3)^2 + 15$

$8 = a(-3)^2 + 15$
$8 = 9a + 15$
$-7 = 9a$
$a = \frac{-7}{9}$

$\therefore y = \frac{-7}{9}(x-3)^2 + 15$

b) $\frac{-7}{9}(x-3)^2 + 15 = 0$
$-7(x-3)^2 = -135$
$(x-3)^2 = \frac{135}{7}$
$x - 3 = \pm\sqrt{\frac{135}{7}}$

zeros $x = 3 \pm \sqrt{\frac{135}{7}}$

width at the base is
$\left(3 + \sqrt{\frac{135}{7}} - 3\right) \times 2$

$= 2\sqrt{\frac{135}{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{2\sqrt{945}}{7}$
$\doteq 8.783$ m.

15. $f(x) = 2x^2 + 4x - 11$,
$g(x) = -3x + 4$
set $f(x) = g(x)$
$2x^2 + 4x - 11 = -3x + 4$
$2x^2 + 7x - 15 = 0$ m -30
$(x+5)(2x-3) = 0$ A 7
$10, -3$
$\div 2$
$x = -5$ or $x = \frac{3}{2}$

$g(-5) = 15 + 4$
$= 19$

$g\left(\frac{3}{2}\right) = \frac{-9+8}{2}$
$= \frac{-1}{2}$

$\therefore (x,y) = \left\{(-5, 19), \left(\frac{3}{2}, \frac{-1}{2}\right)\right\}$

16. set $h(t) = g(t)$
$3t + 3 = -5t^2 + 20t + 15$
$5t^2 - 17t - 12 = 0$ m -60
$(t-4)(5t+3) = 0$ A -17
$-20, 3$
$\div 5$
$t = 4$ or $t = \frac{-3}{5}$

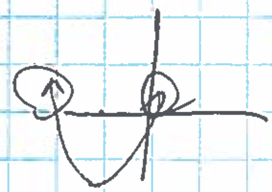
$g(4) = 3(4) + 3$
$= 12 + 3$
$= 15$

$\therefore$ the paintball will hit the baseball at 15 m in the air, after 4 seconds.

17. $f(x) = g(x)$

a) $x^2 - 6x + 9 = -3x - 5$
$x^2 - 3x + 14 = 0$
$b^2 - 4ac$
$= 9 - 4(1)(14)$
< 0 $\therefore$ no intersection.

b) $x^2 - 6x + 9 = mx - 5$
$x^2 + (-6-m)x + 14 = 0$
$b^2 - 4ac > 0$
$(-6-m)^2 - 4(1)(14) > 0$
$(m+6)^2 - 56 > 0$
$(m+6)^2 > 56$



$m + 6 > \sqrt{56}$ OR $m + 6 < -\sqrt{56}$
$m > -6 + \sqrt{56}$ OR $m < -6 - \sqrt{56}$

$\therefore$ for two intersections.

$g(x) = mx - 5$ where
$\therefore \{m \in \mathbb{R} \mid m > (-6 + 2\sqrt{14})$ OR $m < (-6 - 2\sqrt{14})\}$

18. $f(x) = -5x^2 + 10x - 5$

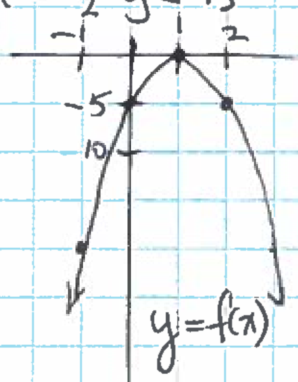
a) $f(x) = -5(x^2 - 2x + 1)$
$= -5(x-1)^2$
zeros: $x = 1$

$V(1, 0)$

b) zeros: just $x = 1$
axis of symmetry $x = 1$
$\therefore$ opens down

c) $D = \{x \in \mathbb{R}\}$
$R = \{y \leq 1\}$

d)



19. a) $f(x) = -2x^2 - 8x + 3$
$= -2(x^2 + 4x \quad) + 3$
$= -2(x^2 + 4x + 4 - 4) + 3$
$= -2(x+2)^2 + 8 + 3$
$= -2(x+2)^2 + 11$

Completing the Square
(Didn't actually have to do it – just describe it)
$\therefore$ maximum of 11 occurs at $x = -2$.

*could also use Partial Factoring.

b) $f(x) = 3(x-1)(x+5)$
use factored form
① find zeros $x = 1, -5$
② find AoS: $x = \frac{1 - 5}{2}$
$\boxed{x = -2}$
③ find $f(-2) = 3(-3)(3)$
$= -27$
$\therefore$ Min value of -27 occurs at $x = -2$.

☺

20. $kx^2 - 4x + k = 0$

$b^2 - 4ac = 0$

$16 - 4(k)(k) = 0$

$-4k^2 + 16 = 0$

$k^2 - 4 = 0$

$k = \pm 2$

$\therefore$ the equation has one root when $k = +2$ OR $k = -2$.

21. set $f(x) = g(x)$

$2x^2 - 3x + 2 = 6x - 5$

$2x^2 - 9x + 7 = 0$

$b^2 - 4ac$

$= 81 - 56$

> 0 $\therefore$ 2 points of intersection

$2x^2 - 9x + 7 = 0$ M 14

$(2x - 7)(x - 1) = 0$ A -9

$x = \frac{7}{2}$ OR $x = 1$ $-7, -2$ $\div 2$

$g\left(\frac{7}{2}\right) = 6\left(\frac{7}{2}\right) - 5$

$= 21 - 5$

$= 16$

$g(1) = 6 - 5$

$= 1$

$\therefore (x, y) = \left\{\left(\frac{7}{2}, 16\right), (1, 1)\right\}$

22. $y = a(x-4)^2 + 3$

use $(3, 2)$

$2 = a(3-4)^2 + 3$

$2 = a + 3$

$-1 = a$

$\therefore y = -(x-4)^2 + 3$ ← vertex form

$y = -(x^2 - 8x + 16) + 3$

$\boxed{y = -x^2 + 8x - 13}$

↑ standard form

23a) $(2 - \sqrt{8})(3 + \sqrt{2})$

$= 6 + 2\sqrt{2} - 3\sqrt{8} - \sqrt{16}$

$= 6 + 2\sqrt{2} - 3\sqrt{4 \times 2} - 4$

$= 2 + 2\sqrt{2} - 3(2\sqrt{2})$

$= 2 + 2\sqrt{2} - 6\sqrt{2}$

$= 2 - 4\sqrt{2}$

b) $(3 + \sqrt{5})(5 - \sqrt{10})$

$= 15 - 3\sqrt{10} + 5\sqrt{5} - \sqrt{50}$

$= 15 - 3\sqrt{10} + 5\sqrt{5} - \sqrt{25 \times 2}$

$= 15 - 3\sqrt{10} + 5\sqrt{5} - 5\sqrt{2}$