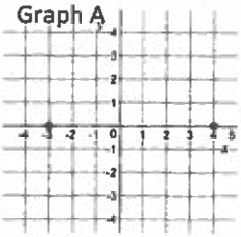
U2D11 MCR3UI Worksheet

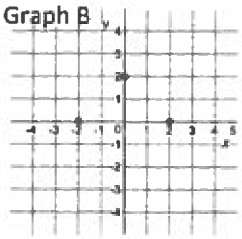
Families of Quadratics

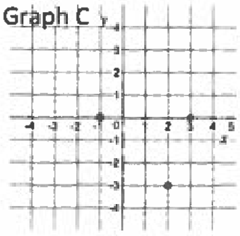
Determine the maximum number of parabolas that could be drawn through the points given in each of the graphs to the right.

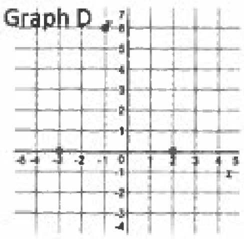
Number of Points:

Number of Possible Parabola:


Graph A


Graph B


Graph C


Graph D

What is the minimum number of points required to define a unique parabola?

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1. What characteristics will two parabolas in the family $f(x) = a(x-2)(x+5)$ share?

2. How are the parabolas $f(x) = -2(x-3)^2 - 5$ and $g(x) = 6(x-3)^2 - 5$ the same? How are they different?

3. What point do the parabolas $f(x) = 3x^2 + 5x - 9$ and $g(x) = -5x^2 + 5x - 9$ have in common?

4. Determine the equation of the parabola with x-intercepts
   - a) -4 and 3, and that passes through (2, 7)
   - b) 0 and 8, and that passes through (-3, -6)
   - c) $\sqrt{7}$ *and* $-\sqrt{7}$, and that passes through (-5, 3)
   - d) $1-\sqrt{2}$ *and* $1+\sqrt{2}$, and that passes through (2, 4)

5. Determine the equation of the parabola with vertex
   - a) (-2, 5) and that passes through (4, -8)
   - b) (1, 6) and that passes through (0, -7)
   - c) (4, -5) and that passes through (-1, -3)
   - d) (4, 0) and that passes through (11, 8)

6. Determine the equation of the quadratic function $f(x) = ax^2 - 6x - 7$ if $f(2) = 3$

7. Determine the equation of the parabola with x-intercepts ± 4 and passing through (3, 6)

8. Determine the equation of the quadratic function that passes through (-4, 5) if its zeros are $2+\sqrt{3}$ and $2-\sqrt{3}$.

9. What is the equation of the parabola with zeros -1, -3 if the point (-4, -9) is on the graph?

10. a) Write the equation of the family of quadratic functions whose roots are 5 and -6.
    b) Determine the equation of the *specific member* of the above family that passes through the point (1, -3)

11. Write one possible quadratic equation, given each pair of roots:

a) 7 and -2 b) $-\frac{3}{5}$ *and* $-\frac{2}{3}$ c) $2-\sqrt{5}$ *and* $2+\sqrt{5}$ d) $\frac{3+2\sqrt{6}}{2}$ *and* $\frac{3-2\sqrt{6}}{2}$

12. Determine the standard form equation of the quadratic function that has an optimal value of -12, if the roots of the corresponding quadratic equation are $3+2\sqrt{3}$ *and* $3-2\sqrt{3}$.

13. Determine the standard form equation of the quadratic function that goes through $(-4,-1)$, if the only root of the corresponding quadratic equation is $-\frac{7}{2}$.

14. Determine the standard form equation of the quadratic function that represents the family of parabolas, if the roots of the corresponding quadratic equation are $-\frac{\sqrt{5}}{2}$ *and* $\frac{\sqrt{5}}{2}$.

Answers:

1. Same zeros, Same Axis of Symmetry 2. Same vertex, same A of S, different direction of opening, different stretch

3. $f(x), g(x)$ have the same y-intercept at -9

4. a) $y = \frac{-7}{6}(x+4)(x-3)$ 4. b) $y = \frac{-2}{11}(x)(x-8)$ 4. c) $y = \frac{-1}{6}(x^2-7)$ 4. d) $y = -4x^2+8x+4$

5. a) $y = \frac{-13}{36}(x+2)^2+5$ 5.b) $y = -13(x-1)^2+6$ 5.c) $y = \frac{2}{25}(x-4)^2-5$ 5. d) $y = \frac{8}{49}(x-4)^2$

6. $y = \frac{11}{2}x^2-6x-7$ 7. $y = \frac{-6}{7}(x^2-16)$ 8. $y = \frac{5}{33}(x^2-4x+1)$ 9. $y = -3x^2-12x-9$

10. a) $y = k(x-5)(x+6)$ 10. b) $y = \frac{3}{28}(x-5)(x+6)$

11. a) $x^2-5x-14=0$ 11. b) $15x^2+19x+6=0$ 11. c) $x^2-4x-1=0$ 11. d) $4x^2-12x-15=0$

12. $f(x) = x^2-6x-3$ 13. $f(x) = -4x^2-28x-49$ 14. $f(x) = 4kx^2-5k,\ k \in \mathbb{R}$

Graph	Number of Points	Number of Possible Parabolas
A	2	many
B	3	one
C	3	one
D	3	one

∴ We need 3 points to define a unique parabola.

1. Both have the same two zeros 2, −5.

2. $V(3,-5)$ is shared
$f(x)$ opens down $g(x)$ opens up
$g(x)$ is stretched 3 x's as much as $f(x)$.
$f(x)$ has a max of −5 at $x=3$
$g(x)$ has a min of −5 at $x=3$.

3. $f(x)$, $g(x)$ have the same y-intercept $(0,-9)$

4. a) $7=a(2+4)(2-3)$
$7=a(6)(-1)$
$\frac{-7}{6}=a$
$\therefore\ y=\frac{-7}{6}(x+4)(x-3)$

ⓄⓇ $S\ -1\ ,\ P\ -12$
$y=a(x^2+x-12)$
$7=a(4+2-12)$
$7=a(-6))$
$\frac{-7}{6}=a$
$\therefore\ y=\frac{-7}{6}(x^2+x-12)$

$y=a(x-s)(x-t)$
b) $-6=a(-3-0)(-3-8)$
$-6=a(-3)(-11)$
$\frac{-6}{33}=a$
$\frac{-2}{11}=a$
$\therefore\ y=\frac{-2}{11}(x)(x-8)$
$y=\frac{-2x^2+16x}{11}$

c) $S=0\quad P=-7$
$y=a(x^2-7)$
$3=a(25-7)$
$3=a(18)$
$\frac{1}{6}=a$
$\therefore\ y=\frac{1}{6}(x^2-7)$

d) $S=2\quad P=1-2$
$P=-1$
$y=a(x^2-2x-1)$
$4=a(4-4-1)$
$4=-a$
$a=-4\qquad \therefore\ y=-4(x^2-2x-1)$
$y=-4x^2+8x+4$

5.a) $y=a(x-h)^2+k$

$-8=a(4+2)^2+5$

$-8=a(36)+5$

$\frac{-13}{36}=a$

$\therefore y=\frac{-13}{36}(x+2)^2+5$

b) $-7=a(0-1)^2+6$

$-13=a$

$\therefore y=-13(x-1)^2+6$

c) $-3=a(-1-4)^2-5$

$2=25a$

$a=\frac{2}{25}$

$\therefore y=\frac{2}{25}(x-4)^2-5$

d) $8=a(11-4)^2+0$

$8=a(49)$

$a=\frac{8}{49}$

$\therefore y=\frac{8}{49}(x-4)^2$

6. $f(x)=ax^2-6x-7$ $f(2)=3$

$3=a(4)-6(2)-7$

$10=4a-12$

$22=4a$

$a=\frac{11}{2}$

$\therefore f(x)=\frac{11}{2}x^2-6x-7$

7. Sum $=0$

Product $=-16$

$y=a(x^2-16)$

$6=a(9-16)$

$6=a(-7)$

$\frac{-6}{7}=a$

$\therefore y=\frac{-6}{7}(x^2-16)$

8. Sum $=4$

Product $=4-3$

$=1$

$y=a(x^2-4x+1)$

$5=a(16-4(-4)+1)$

$5=a(16+16+1)$

$a=\frac{5}{33}$

$\therefore y=\frac{5}{33}(x^2-4x+1)$

9. Sum -4

Product 3

$y=a(x^2+4x+3)$

$-9=a(16-16+3)$

$-3=a$

$\therefore y=-3(x^2+4x+3)$

$y=-3x^2-12x-9$

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10. a) $S = 5 + (-6)$ $P = 5(-6)$

$S = -1$ $P = -30$

$y = a(x^2 - Sx + P)$

$y = a(x^2 + x - 30)$

OR $y = a(x-5)(x+6)$

b) $(1, -3)$

$-3 = a(1^2 + 1 - 30)$

$-3 = -28a$

$a = \frac{3}{28}$

$\therefore y = \frac{3}{28}(x^2 + x - 30)$

OR $y = \frac{3}{28}(x-5)(x+6)$

11. a) $S = 7 + (-2)$ $P = 7(-2)$

$S = 5$ $P = -14$

one quadratic in the family:

$y = x^2 - 5x - 14$ ← quadratic function

$\boxed{x^2 - 5x - 14 = 0}$ ← quadratic equation

b) $S = \frac{-3}{5} + \left(\frac{-2}{3}\right)$ $P = -\frac{\cancel{3}}{5}\left(\frac{-2}{\cancel{3}}\right)$

$S = \frac{-9 - 10}{15}$ $P = \frac{2}{5}$

$S = \frac{-19}{15}$

$x^2 + \frac{19}{15}x + \frac{2}{5} = 0$ ← this is a quadratic equation satisfying the conditions

×15

$\boxed{15x^2 + 19x + 6 = 0}$ ← this is a 'nicer' quadratic equation satisfying the conditions.

11c) $2-\sqrt{5}$, $2+\sqrt{5}$

$S = 4$ $\quad$ $P = 4-5$

$P = -1$

$$\boxed{x^2-4x-1=0}$$

d) $S = \frac{3+2\sqrt{6}}{2} + \frac{3-2\sqrt{6}}{2}$

$S = \frac{6}{2}$ $\quad$ $P = \frac{9-4(6)}{4}$

$S = 3$ $\quad$ $P = \frac{-15}{4}$

$x^2-3x-\frac{15}{4}=0$ $\quad$ ×4

$$\boxed{4x^2-12x-15=0}$$

12. $(h,-12)$ $\quad$ roots: $3+2\sqrt{3}$, $3-2\sqrt{3}$

Sum $= 6$

Product $= 9-4(3)$

$P = -3$

A of S: $x = \frac{(3+2\sqrt{3})+(3-2\sqrt{3})}{2}$

$x = \frac{6}{2}$

$x = 3$ $\quad$ $h = 3$

sub in $V(\underset{x}{3},\underset{y}{-12})$ $\quad$ $y = a(x^2-6x-3)$

$-12 = a(3^2-6(3)-3)$

$-12 = a(-12)$

$1 = a$

$\therefore\ y = x^2-6x-3$

13. $(\underset{x}{-4}, \underset{y}{-1})$ $\frac{-7}{2}$ ← only root

NICER!

$y = a\left(x+\frac{7}{2}\right)^2$ → OR $y = a(2x+7)^2$

$-1 = a\left(-\frac{4}{1}+\frac{7}{2}\right)^2$

$-1 = a\left(\frac{-8+7}{2}\right)^2$

$-1 = a\left(\frac{-1}{2}\right)^2$

$-1 = \frac{1}{4}a$

$\frac{-1}{1} \times \frac{4}{1} = a$

$a = -4$

$-1 = a(2(-4)+7)^2$

$-1 = a(-1)^2$

$-1 = 1a$

$a = -1$

$\therefore \boxed{y = -(2x+7)^2}$ Vertex Form

OR $\boxed{y = -4x^2 - 28x - 49}$

← standard form.

14. $S = -\frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2}$

$S = 0$

$P = \left(\frac{\sqrt{5}}{2}\right)\left(\frac{\sqrt{5}}{2}\right)$

$P = \frac{-5}{4}$

$y = a\left(x^2 - 0x - \frac{5}{4}\right)$

Let $a = 4k$, $k \in \mathbb{R}$

↑ to clear fractions.

$y = 4k\left(x^2 - \frac{5}{4}\right)$

$\boxed{y = 4kx^2 - 5k}$