U2D11-T Family-of-Func

Monday, February 25, 2019 6:26 AM

U2D11-T Family-of-...

U2D11 MCR 3UI **Family of Functions**

WARM UP

1. For what values of k does $kx^2 - 2x + 3 = 0$ have no roots?

$D = b^2 - 4ac \quad a = k, b = -2, c = 3$

$D < 0$

$b^2 - 4ac < 0$

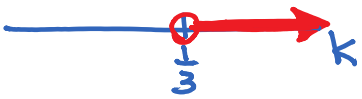
$4 - 4k(3) < 0$

$4 - 12k < 0$

$-12k < -4$

$k > \frac{-4}{-12}$

$k > \frac{1}{3}$



2. For what values of k does $3x^2 + kx + 2 = 0$ have two roots?

$a = 3, b = k, c = 2$

$b^2 - 4ac > 0$

$k^2 - 4(3)(2) > 0$

$k^2 - 24 > 0$

zeros:

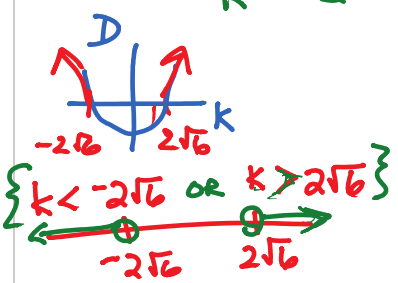
$k^2 - 24 = 0$

$(k - \sqrt{24})(k + \sqrt{24}) = 0$

$k = \pm\sqrt{24}$

$k = \pm\sqrt{4 \times 6}$

$k = \pm 2\sqrt{6}$



$\{k < -2\sqrt{6} \text{ or } k > 2\sqrt{6}\}$

Family of Functions

1. Find the family of quadratic functions that have roots of -3 and 5. Leave your answer in **standard** form. $y = ax^2 + bx + c$

$y = a(x-s)(x-t)$

$y = a(x+3)(x-5)$

$y = a(x^2 - 2x - 15)$

for standard form expand & simplify.

$y = ax^2 - 2ax - 15a, \quad a \in \mathbb{R}.$

2. Determine the standard form equation of a parabola with roots 5 and -1, and goes through the point (-2,14).

$y = a(x-s)(x-t)$

$14 = a(-2-5)(-2+1)$

$14 = a(-7)(-1)$

$14 = 7a$

$a = 2$

Factored Form

$\therefore y = 2(x-5)(x+1)$

$y = 2x^2 - 8x - 10$

standard form

3. Determine the equation of the quadratic function in standard form that goes through (2,5) and has zeroes at 0 and -3.

(x, y) = (2, 5); s = 0; t = -3

$y = a(x-s)(x-t)$

$5 = a(2-0)(2+3)$

$5 = a(2)(5)$

$5 = 10a$

$a = \frac{1}{2}$

$\therefore y = \frac{1}{2}(x-0)(x+3)$

$\boxed{y = \frac{1}{2}x^2 + \frac{3}{2}x}$

4. Find the standard form equation of a parabola with roots of $x = -1 \pm \sqrt{3}$.

$y = a(x-[-1+\sqrt{3}])(x-[-1-\sqrt{3}])$

$y = a(x+1-\sqrt{3})(x+1+\sqrt{3})$

Factored form ↗

* LOTS of work to expand if standard form is required!

However..... There is another way.

Use the sum/product method.

a) Consider $y=(x-3)(x+2)$ or $y=x^2-x-6$

-3, 2 Sum to -1

(-3)(2) have product -6

ROOTS: 3, -2

sum of roots = 3+(-2)

= 1 * wrong sign for b in ax^2+bx+c

product of roots = (3)(-2)

= -6 * exactly the 'c'-value

b) For roots of $x=-1\pm\sqrt{3}$

sum of roots = $(-1+\sqrt{3})+(-1-\sqrt{3})$

= -2

product of roots = $(-1+\sqrt{3})(-1-\sqrt{3})$

$=1-3=-2$

Therefore the quadratic equation is

* $y = a(x^2 - Sx + P)$ *

$\therefore\ y = a(x^2+2x-2)$

$y = ax^2+2ax-2a$

5. Find the family of quadratic functions with roots $\frac{2 \pm \sqrt{7}}{3}$.

RECALL: $y = a(x^2 - Sx + P)$

Sum = $\frac{2+\sqrt{7}}{3} + \frac{2-\sqrt{7}}{3} = \frac{4}{3}$

Product = $\left(\frac{2+\sqrt{7}}{3}\right)\left(\frac{2-\sqrt{7}}{3}\right) = \frac{4}{9} - \frac{7}{9} = \frac{-3}{9} = \frac{-1}{3}$

$y = a\left(x^2 - \frac{4}{3}x - \frac{1}{3}\right)$

One particular quadratic with those roots is

$y = 3x^2 - 4x - 1$

(note: used $a=3$ so function has no fractions).

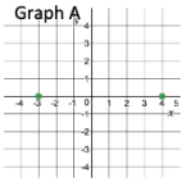
U2D11 Workshee...

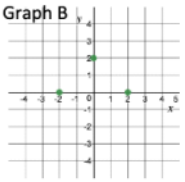
U2D11 MCR3UI Worksheet **Families of Quadratics**

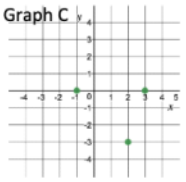
Determine the maximum number of parabolas that could be drawn through the points given in each of the graphs to the right.

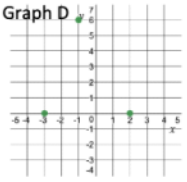
Number of Points:

Number of Possible Parabola:

Graph A 

Graph B 

Graph C 

Graph D 

What is the minimum number of points required to define a unique parabola?

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1. What characteristics will two parabolas in the family $f(x) = a(x-2)(x+5)$ share?
2. How are the parabolas $f(x) = -2(x-3)^2 - 5$ and $g(x) = 6(x-3)^2 - 5$ the same? How are they different?
3. What point do the parabolas $f(x) = 3x^2 + 5x - 9$ and $g(x) = -5x^2 + 5x - 9$ have in common?
4. Determine the equation of the parabola with x-intercepts
   a) -4 and 3, and that passes through (2, 7)
   b) 0 and 8, and that passes through (-3, -6)
   c) $\sqrt{7}$ *and* $-\sqrt{7}$, and that passes through (-5, 3)
   d) $1-\sqrt{2}$ *and* $1+\sqrt{2}$, and that passes through (2, 4)
5. Determine the equation of the parabola with vertex
   a) (-2, 5) and that passes through (4, -8)
   b) (1, 6) and that passes through (0, -7)
   c) (4, -5) and that passes through (-1, -3)
   d) (4, 0) and that passes through (11, 8)
6. Determine the equation of the quadratic function $f(x) = ax^2 - 6x - 7$ if $f(2) = 3$
7. Determine the equation of the parabola with x-intercepts ± 4 and passing through (3, 6)
8. Determine the equation of the quadratic function that passes through (-4, 5) if its zeros are $2+\sqrt{3}$ and $2-\sqrt{3}$.
9. What is the equation of the parabola with zeros -1, -3 if the point (-4, -9) is on the graph?
10. a) Write the equation of the family of quadratic functions whose roots are 5 and -6.
    b) Determine the equation of the *specific member* of the above family that passes through the point (1, -3)
11. Write one possible quadratic equation, given each pair of roots:

a) 7 and -2 b) $-\frac{3}{5}$ *and* $-\frac{2}{3}$ c) $2-\sqrt{5}$ *and* $2+\sqrt{5}$ d) $\frac{3+2\sqrt{6}}{2}$ *and* $\frac{3-2\sqrt{6}}{2}$

12. Determine the standard form equation of the quadratic function that has an optimal value of -12, if the roots of the corresponding quadratic equation are $3+2\sqrt{3}$ *and* $3-2\sqrt{3}$.
13. Determine the standard form equation of the quadratic function that goes through $(-4,-1)$, if the only root of the corresponding quadratic equation is $-\frac{7}{2}$.
14. Determine the standard form equation of the quadratic function that represents the family of parabolas, if the roots of the corresponding quadratic equation are $-\frac{\sqrt{5}}{2}$ *and* $\frac{\sqrt{5}}{2}$.

Answers:

1. Same zeros, Same Axis of Symmetry 2. Same vertex, same A of S, different direction of opening, different stretch
3. $f(x), g(x)$ have the same y-intercept at -9

4. a) $y = \frac{-7}{6}(x+4)(x-3)$ 4. b) $y = \frac{-2}{11}(x)(x-8)$ 4. c) $y = \frac{1}{6}(x^2-7)$ 4. d) $y = -4x^2+8x+4$

5. a) $y = \frac{-13}{36}(x+2)^2+5$ 5.b) $y = -13(x-1)^2+6$ 5.c) $y = \frac{2}{25}(x-4)^2-5$ 5. d) $y = \frac{8}{49}(x-4)^2$

6. $y = \frac{11}{2}x^2-6x-7$ 7. $y = \frac{-6}{7}(x^2-16)$ 8. $y = \frac{5}{33}(x^2-4x+1)$ 9. $y = -3x^2-12x-9$

10. a) $y = k(x-5)(x+6)$ 10. b) $y = \frac{3}{28}(x-5)(x+6)$

11. a) $x^2-5x-14=0$ 11. b) $15x^2+19x+6=0$ 11. c) $x^2-4x-1=0$ 11. d) $4x^2-12x-15=0$

12. $f(x) = x^2-6x-3$ 13. $f(x) = -4x^2-28x-49$ 14. $f(x) = 4kx^2-5k,\ k \in \mathbb{R}$