U2D10_T Systems of Equations Involving Quadratics MCR 3UI

Monday, February 25, 2019 6:25 AM

Word Problem Review for Test

The profit, P(x), of a video company, in **thousands of dollars**, is given by $P(x) = -5x^2 + 550x - 5000$, where x is the amount spent on advertising, in **thousands of dollars**. **For full marks, use proper function notation** in answering these questions.

a) Determine the maximum profit that the company could make. [3

b) Determine the amounts spent on advertising that will result in the company breaking even. (Remember x and P(x) are both in thousands of dollars.) [4]

use $P(x) \geq 4000$

c) Determine the amounts spent on advertising that will result in a profit of at least $4 000 000. [4]

a) By Partial Factoring

$P(x) = -5x(x-110) - 5000$

$(0, -5000)$ $(110, -5000)$

Axis of Symmetry

$x = \frac{110}{2}$

$x = 55$

$P(55) = -5(55)(-55) - 5000$

$P(55) = 10\,125$

By Completing the Square

$P(x) = -5(x^2 - 110x + 3025 - 3025) - 5000$

$P(x) = -5(x-55)^2 + 15125 - 5000$

$P(x) = -5(x-55) + 10125$

$\therefore$ $V(55, 10125)$

max value

$\therefore$ maximum profit of \$10 125 000 (\$10125 thousand) can be expected with \$55 000 (\$55 thousand) spent on advertising.

b) $P(x) = 0$ for break-even.

$-5x^2 + 550x - 5000 = 0$

$-5(x^2 - 110x + 1000) = 0$

$(x-10)(x-100) = 0$

$x = 10$ or $x = 100$

$\therefore$ to break even, the company should spend \$10 000 or \$100 000 on advertising

(Spending between \$10 thousand and \$100 thousand on advertising will result in a profit.)

U2D10 MCR 3UI **Systems of Equations involving Quadratics**

Warm Up

Determine the solution to the linear system: ① $2x - y = 14$ → $y = 2x - 14$

② $5y = x + 11$

point of intersection

isolate a variable in one equation

"substitution" method from gr. 10

Sub ① into ②

$5(2x-14) = x + 11$

$10x - 70 = x + 11$

$10x - x = 11 + 70$

$9x = 81$

$x = 9$

may substitute this x-value into either equation.

sub $x = 9$ into ①

$y = 2(9) - 14$

$y = 18 - 14$

$y = 4$

$\therefore (x, y) = (9, 4)$

[The ... ... is (9, 4)]

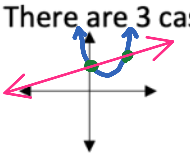
$\therefore (x, y) = (9, 4)$

[The point of intersection is $(9,4)$]

Systems of Equations

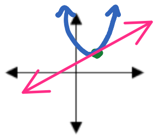
To solve a system of equations, we can use substitution (or elimination) to determine all points where the parabola intersects the line. (i.e. points of intersection)

There are 3 cases:



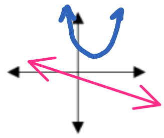
Two solutions

- two distinct roots
- discriminant is > 0



One solution

- two identical roots
- discriminant is = 0



No solution

- no roots
- discriminant is < 0

Solve the following systems of Equations:

a) $f(x) = x^2 - 6x + 9 \rightarrow (x-3)^2$
$g(x) = 1x - 1 \rightarrow b = -1 \quad m = \frac{1}{1}$

set $f(x) = g(x)$

$x^2 - 6x + 9 = x - 1$

$x^2 - 6x + 9 - x + 1 = 0$

$x^2 - 7x + 10 = 0$

to solve we need form $ax^2 + bx + c = 0$

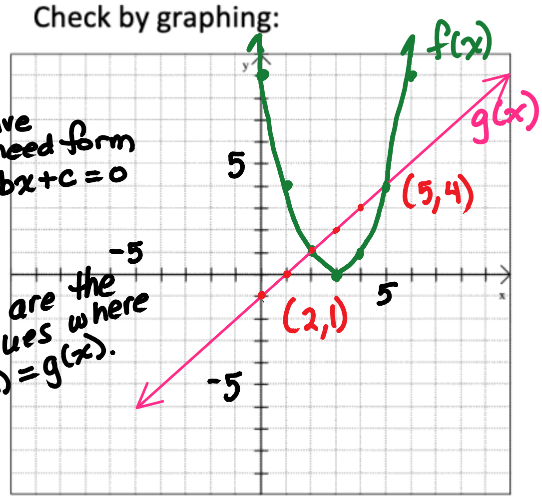
$(x-5)(x-2) = 0$

$x = 5$ OR $x = 2$ ← these are the x-values where $f(x) = g(x)$.

$g(5) = 5 - 1$ $\qquad$ $g(2) = 2 - 1$

$g(5) = 4$ $\qquad$ $g(2) = 1$

Check by graphing:



$\therefore (x,y) \in \{(2,1), (5,4)\}$

b) ① $y = -x^2 + 4x + 2$
② $2x + y - 7 = 0$

sub ① into ②

$2x + (-x^2 + 4x + 2) - 7 = 0$

$2x - x^2 + 4x + 2 - 7 = 0$

$-x^2 + 6x - 5 = 0$

$x^2 - 6x + 5 = 0$ $\qquad$ M 5, A −6

$(x-1)(x-5) = 0$

$x = 1$ OR $x = 5$

sub $x = 1$ into ②

$2(1) + y - 7 = 0$

$2 + y - 7 = 0$

$y - 5 = 0$

$y = 5$

sub $x = 5$ into ②

$2(5) + y - 7 = 0$

$10 + y - 7 = 0$

$y + 3 = 0$

$y = -3$

$\therefore (x,y) \in \{(1,5), (5,-3)\}$

c) ① $y = 2x^2 + 12x + 13$
② $2x - 3y - 6 = 0$

sub ① into ②

$2x - 3(2x^2 + 12x + 13) - 6 = 0$

$2x - 6x^2 - 36x - 39 - 6 = 0$

$-6x^2 - 34x - 45 = 0$

$6x^2 + 34x + 45 = 0$

large numbers so I can use $D = b^2 - 4ac$ to determine if it factors & how many solutions there are.

$D = b^2 - 4ac$

$D = 76$

∴ 76 is not a perfect square so $6x^2 + 34x - 45$ does not factor.

$76 > 0$ so there are 2 roots.

∴ must use quadratic formula to solve.

$x = \frac{-34 \pm \sqrt{76}}{12}$

$x = \frac{-34 \pm 2\sqrt{19}}{12}$

$x = \frac{2(-17 \pm \sqrt{19})}{12}$

$\boxed{x = \frac{-17 \pm \sqrt{19}}{6}}$

sub $x = \frac{-17+\sqrt{19}}{2}$ into ②

$2x - 3y - 6 = 0$

$3y = 2x - 6$

$y = \frac{2}{3}x - 2$

$y = \frac{\cancel{2}^{1}}{3}\left(\frac{-17+\sqrt{19}}{\cancel{6}_{3}}\right) - \frac{2}{1}$

$y = \frac{-17+\sqrt{19}}{9} - \frac{18}{9}$

$\boxed{y = \frac{-35+\sqrt{19}}{9}}$

sub $x = \frac{-17-\sqrt{19}}{6}$ into ②

$y = \frac{-17-\sqrt{19}}{9} - \frac{18}{9}$

$\boxed{y = \frac{-35-\sqrt{19}}{9}}$

$\therefore (x, y) \in \left\{\left(\frac{-17+\sqrt{19}}{6}, \frac{-35+\sqrt{19}}{9}\right), \left(\frac{-17-\sqrt{19}}{6}, \frac{-35-\sqrt{19}}{9}\right)\right\}$

d) ① $x + 3f(x) = 15$
$g(x) = -x^2 + 6x - 7$

sub $g(x)$ into ①

$x + 3(-x^2 + 6x - 7) = 15$

$x - 3x^2 + 18x - 21 - 15 = 0$

$-3x^2 + 19x - 36 = 0$

$3x^2 - 19x + 36 = 0$

$b^2 - 4ac = -71$

∴ there is no intersection.

$x - 3x^2 + 18x - 21 - 15 = 0$
$-3x^2 + 19x - 36 = 0$

$\therefore$ there is no intersection.

Relevance

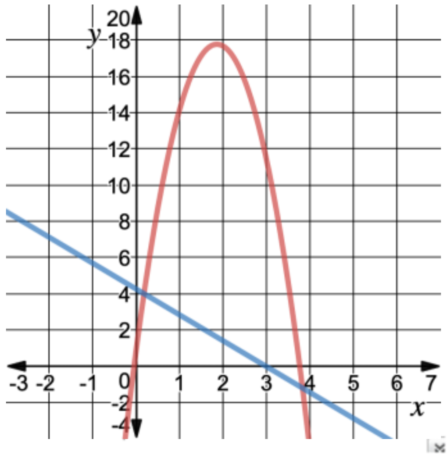
A punter kicks a football. The ball's height is given by the equation:

$$y_1 = -4.9x^2 + 18.24x + 0.8$$

The height of an approaching blocker's hands is modelled by the equation:

where x represents the same time. $y_2 = -1.43x + 4.26$

Can the blocker knock down the punt?
If so, at what point will it happen?



U2D10 Workshee...

U2D10 MCR3UI Worksheet Systems of Equations Involving Quadratics

1. Determine the point(s) of intersection algebraically.
   a) $f(x) = -x^2 + 6x - 5,\ g(x) = -4x + 19$
   b) $f(x) = 2x^2 - 1,\ g(x) = 3x + 1$
   c) $f(x) = 3x^2 - 2x - 1,\ g(x) = -x - 6$
2. Determine the number of points of intersection of $f(x) = 4x^2 + x - 3$ and $g(x) = 5x - 4$ without solving.
3. Determine the point(s) of intersection of each pair of functions.
   a) $f(x) = -2x^2 - 5x + 20,\ g(x) = 6x - 1$
   b) $f(x) = 3x^2 - 2,\ g(x) = x + 7$
   c) $f(x) = 5x^2 + x - 2,\ g(x) = -3x - 6$
4. The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.
5. Determine the value of k such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.
6. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.
7. Determine through investigation, the equations of lines that have a slope of 2 and intersect the quadratic function $f(x) = x(x - 6)$
   a) Once b) Twice c) Never
8. Solve algebraically. You may confirm graphically.
   a) $y = 3 - x;\ y = x^2 - 8x + 13$
   b) $g(x) = 4x - 1;\ f(x) = -2x^2 + 4x + 1$
   c) $12x - 4y = 19;\ y = 3x^2 - 12x + 14$
   d) $2x - 3y = -6;\ y = -3x^2 + 24x - 50$
   e) $h(x) = 2x^2 + 3;\ g(x) = x^2 - 2x + 7$
   f) $h(x) = -2x^2 + 24x - 69;\ g(x) = x^2 - 10x + 27$
9. An asteroid is moving in a parabolic arc that is modelled by the function $y = -6x^2 - 370x + 100\,900$. For the period of time that it is in the same area, a space probe is moving along a straight path on the same plane as the asteroid according to the linear equation $y = y = 500x - 83\,024$. A space agency needs to determine if the asteroid will be an issue for the space probe. Wil the two paths intersect?
10. The UV index on a sunny day can be modelled by the function $f(x) = -0.15(x - 13)^2 + 7.6$ where x represents the time of day on a 24-hour clock and $f(x)$ represents the UV index. Between what hours was the UV index greater than 7?
11. A parachutist jumps from an airplane and immediately opens his parachute. His altitude, y, in metres, after t seconds is modelled by the equation $y = -4t + 300$. A second parachutist jumps 5 s later and freefalls for a few seconds. Her altitude, in metres, during this time, is modelled by the equation $y = -4.9(t - 5)^2 + 300$. When does she catch up to the first parachutist?

Answers:

1. a) $\{(4,3), (6,-5)\}$ b) $\left\{(2,7), \left(-\frac{1}{2}, -\frac{1}{2}\right)\right\}$ c) no intersection
2. one
3. a) $\left\{\left(\frac{3}{2}, 8\right), (-7, -43)\right\}$ b) $\left\{\left(\frac{1+\sqrt{109}}{6}, \frac{43+\sqrt{109}}{6}\right), \left(\frac{1-\sqrt{109}}{6}, \frac{43-\sqrt{109}}{6}\right)\right\}$
   c) no intersection d) $\left\{\left(\frac{-7+\sqrt{33}}{8}, \frac{-3+5\sqrt{33}}{8}\right), \left(\frac{-7-\sqrt{33}}{8}, \frac{-3-5\sqrt{33}}{8}\right)\right\}$
4. \$3 or \$4 5. $k = -5$ 6. $k > \frac{73}{12}$
7. a) $y = 2x - 16$ b) $y = 2x + b, b > -16$ c) $y = 2x + b, b < -16$
8. a) $\{(2,1), (5,-2)\}$ b) $\{(1,3), (-1,-5)\}$ c) $\left\{\left(\frac{5}{2}, \frac{11}{4}\right)\right\}$ d) no real solution
   e) $\{(-1 + \sqrt{5}, 15 - 4\sqrt{5}), (-1 - \sqrt{5}, 15 + 4\sqrt{5})\}$ f) $\left\{(6,3), \left(\frac{16}{3}, \frac{19}{9}\right)\right\}$
9. $D > 0$ *so they will intersect.*
10. From 11:00 a.m. until 3:00 p.m.
11. 7.5 seconds after the first parachutist jumps (2.5 seconds after she jumps)