U2D10 MCR3UI **Worksheet** **<u>Systems of Equations Involving Quadratics</u>**

1. Determine the point(s) of intersection algebraically.
   a) $f(x) = -x^2 + 6x - 5,\ g(x) = -4x + 19$
   b) $f(x) = 2x^2 - 1,\ g(x) = 3x + 1$
   c) $f(x) = 3x^2 - 2x - 1,\ g(x) = -x - 6$
2. Determine the number of points of intersection of $f(x) = 4x^2 + x - 3$ and $g(x) = 5x - 4$ without solving.
3. Determine the point(s) of intersection of each pair of functions.
   a) $f(x) = -2x^2 - 5x + 20,\ g(x) = 6x - 1$
   b) $f(x) = 3x^2 - 2,\ g(x) = x + 7$
   c) $f(x) = 5x^2 + x - 2,\ g(x) = -3x - 6$
4. The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.
5. Determine the value of k such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.
6. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.
7. Determine through investigation, the equations of lines that have a slope of 2 and intersect the quadratic function $f(x) = x(x - 6)$
   a) Once b) Twice c) Never
8. Solve algebraically. You may confirm graphically.
   a) $y = 3 - x;\ y = x^2 - 8x + 13$
   b) $g(x) = 4x - 1;\ f(x) = -2x^2 + 4x + 1$
   c) $12x - 4y = 19;\ y = 3x^2 - 12x + 14$
   d) $2x - 3y = -6;\ y = -3x^2 + 24x - 50$
   e) $h(x) = 2x^2 + 3;\ g(x) = x^2 - 2x + 7$
   f) $h(x) = -2x^2 + 24x - 69;\ g(x) = x^2 - 10x + 27$
9. An asteroid is moving in a parabolic arc that is modelled by the function $y = -6x^2 - 370x + 100\,900$. For the period of time that it is in the same area, a space probe is moving along a straight path on the same plane as the asteroid according to the linear equation $y = 500x - 83\,024$. A space agency needs to determine if the asteroid will be an issue for the space probe. Will the two paths intersect?
10. The UV index on a sunny day can be modelled by the function $f(x) = -0.15(x - 13)^2 + 7.6$ where x represents the time of day on a 24-hour clock and $f(x)$ represents the UV index. Between what hours was the UV index greater than 7?
11. A parachutist jumps from an airplane and immediately opens his parachute. His altitude, y, in metres, after t seconds is modelled by the equation $y = -4t + 300$. A second parachutist jumps 5 s later and freefalls for a few seconds. Her altitude, in metres, during this time, is modelled by the equation $y = -4.9(t - 5)^2 + 300$. When does she catch up to the first parachutist?

<u>Answers</u>:

1. a) $\{(4,3),(6,-5)\}$ b) $\left\{(2,7),\left(-\frac{1}{2},-\frac{1}{2}\right)\right\}$ c) no intersection
2. one
3. a) $\left\{\left(\frac{3}{2},8\right),(-7,-43)\right\}$ b) $\left\{\left(\frac{1+\sqrt{109}}{6},\frac{43+\sqrt{109}}{6}\right),\left(\frac{1-\sqrt{109}}{6},\frac{43-\sqrt{109}}{6}\right)\right\}$
   c) no intersection
4. \$3 or \$4
5. $k = -5$
6. $k > \frac{73}{12}$
7. a) $y = 2x - 16$ b) $y = 2x + b, b > -16$ c) $y = 2x + b, b < -16$
8. a) $\{(2,1),(5,-2)\}$ b) $\{(1,3),(-1,-5)\}$ c) $\left\{\left(\frac{5}{2},\frac{11}{4}\right)\right\}$ d) no real solution
   e) $\{(-1+\sqrt{5},15-4\sqrt{5}),(-1-\sqrt{5},15+4\sqrt{5})\}$ f) $\left\{(6,3),\left(\frac{16}{3},\frac{19}{9}\right)\right\}$
9. $D > 0$ *so they will intersect.*
10. From 11:00 a.m. until 3:00 p.m.
11. 7.5 seconds after the first parachutist jumps (2.5 seconds after she jumps)

U2D10

pg 1

1 a) $f(x) = -x^2+6x-5$

$g(x) = -4x+19$

set. $g(x) = f(x)$

$-4x+19 = -x^2+6x-5$

$x^2-10x+24=0$

$(x-4)(x-6)=0$

$x=4$ or $x=6$

$g(4) = -16+19$
$= 3$

$g(6) = -24+19$
$= -5$

$\therefore (x,y) = \{(4,3), (6,-5)\}$

b) $f(x) = 2x^2-1$, $g(x) = 3x+1$

set $f(x) = g(x)$

$2x^2-1 = 3x+1$

$2x^2-3x-2=0$ m −4

$(x-2)(2x+1)=0$ A −3

−4, 1

$x=2$ or $x=\frac{-1}{2}$ ÷2

$g(2) = 7$

$g(-\frac{1}{2}) = \frac{-3}{2}+\frac{2}{2}$
$= -\frac{1}{2}$

$\therefore (x,y) = \{(2,7), (-\frac{1}{2}, -\frac{1}{2})\}$

c) $f(x) = 3x^2-2x-1$, $g(x) = -x-6$

set $f(x) = g(x)$

$3x^2-2x-1 = -x-6$

$3x^2-x+5=0$ m −15

A −1

$x = \frac{1 \pm \sqrt{1-12(5)}}{6}$

no solution

$\therefore$ there are no points of intersection.

set $f(x) = g(x)$

2. $4x^2 + x - 3 = 5x - 4$

$4x^2 - 4x + 1 = 0$

$b^2 - 4ac$

$16 - 4(4)(1)$

$= 0$

$\therefore$ there is one point of intersection.

3\. a) $f(x) = -2x^2 - 5x + 20$, $g(x) = 6x - 1$

set $g(x) = f(x)$

$6x - 1 = -2x^2 - 5x + 20$

$2x^2 + 11x - 21 = 0$

M -42
A 11 14×-3

$x = \dfrac{-11 \pm \sqrt{121 - 4(2)(-21)}}{4}$

$= \dfrac{-11 \pm \sqrt{289}}{4}$

$= \dfrac{-11 \pm 17}{4}$

$x = \frac{3}{2}$ OR $x = -7$

$g(\frac{3}{2}) = 6(\frac{3}{2}) - 1$

$= 8$

$g(-7) = 6(-7) - 1$

$= -43$

$\therefore (x, y) = \{(\frac{3}{2}, 8), (-7, -43)\}$

3b) $f(x) = 3x^2 - 2$, $g(x) = x + 7$

set $f(x) = g(x)$

$$3x^2 - 2 = x + 7$$
$$3x^2 - x - 9 = 0$$
$$x = \frac{1 \pm \sqrt{1 - 12(-9)}}{6}$$
$$x = \frac{1 \pm \sqrt{109}}{6}$$

$$g\left(\frac{1+\sqrt{109}}{6}\right) = \frac{1+\sqrt{109}}{6} + \frac{42}{6} = \frac{43+\sqrt{109}}{6}$$

$$g\left(\frac{1-\sqrt{109}}{6}\right) = \frac{1-\sqrt{109}}{6} + \frac{42}{6} = \frac{43-\sqrt{109}}{6}$$

$$\therefore (x, y) = \left\{\left(\frac{1+\sqrt{109}}{6}, \frac{43+\sqrt{109}}{6}\right), \left(\frac{1-\sqrt{109}}{6}, \frac{43-\sqrt{109}}{6}\right)\right\}$$

c) $f(x) = 5x^2 + x - 2$, $g(x) = -3x - 6$

set $f(x) = g(x)$

$$5x^2 + x - 2 = -3x - 6$$
$$5x^2 + 4x + 4 = 0$$
$$b^2 - 4ac$$
$$= 16 - 4(5)(4)$$
$$< 0$$

$\therefore$ there is no point(s) of intersection.

3d) $f(x) = -4x^2 - 2x + 3$, $g(x) = 5x + 4$

set $g(x) = f(x)$

$$5x + 4 = -4x^2 - 2x + 3$$
$$4x^2 + 7x + 1 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 4(4)(1)}}{8}$$

$$x = \frac{-7 \pm \sqrt{33}}{8}$$

$$g\left(\frac{-7+\sqrt{33}}{8}\right) = 5\left(\frac{-7+\sqrt{33}}{8}\right) + 4$$
$$= \frac{-35 + 5\sqrt{33}}{8} + \frac{32}{8}$$
$$= \frac{-3 + 5\sqrt{33}}{8}$$

$$g\left(\frac{-7-\sqrt{33}}{8}\right) = 5\left(\frac{-7-\sqrt{33}}{8}\right) + 4$$
$$= \frac{-35 - 5\sqrt{33} + 32}{8}$$
$$= \frac{-3 - 5\sqrt{33}}{8}$$

$$\therefore (x, y) = \left\{\left(\frac{-7+\sqrt{33}}{8}, \frac{-3+5\sqrt{33}}{8}\right), \left(\frac{-7-\sqrt{33}}{8}, \frac{-3-5\sqrt{33}}{8}\right)\right\}$$

4. set $C(t) = R(t)$

$$600 - 50t = -50t^2 + 300t$$
$$50t^2 - 350t + 600 = 0$$
$$t^2 - 7t + 12 = 0$$
$$(t-3)(t-4) = 0$$
$$t = 3 \text{ OR } t = 4$$

$\therefore$ a ticket price of \$3 OR \$4 will result in a break even production.

5. set $f(x) = g(x)$

$$2x^2 - 5x + 3 = 3x + k$$
$$2x^2 - 8x + 3 - k = 0$$

$$b^2 - 4ac = 0$$
$$64 - 4(2)(3-k) = 0$$
$$64 - 24 + 8k = 0$$
$$40 + 8k = 0$$
$$k = \frac{-40}{8}$$
$$k = -5$$

$\therefore$ when $k = -5$, $f(x)$, $g(x)$ intersect in exactly one point.

6. set $g(x) = f(x)$, find k such that $b^2 - 4ac < 0$

$$4x + k = -3x^2 - x + 4$$

$$3x^2 + 5x + (k-4) = 0$$

$$b^2 - 4ac < 0$$
$$25 - 12(k-4) < 0$$
$$25 - 12k + 48 < 0$$
$$-12k + 73 < 0$$
$$-12k < -73$$
$$k > \frac{73}{12}$$

$\therefore$ for $k > \frac{73}{12}$, there is no intersection.

* answer to number 7 is after number 8

8. a) set ② = ①

$x^2 - 8x + 13 = 3 - x$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x = 2$ OR $x = 5$

① $y = 3 - x$
② $y = x^2 - 8x + 13$

sub $x = 2$ into ①

$y = 3 - 2$
$y = 1$

sub $x = 5$ into ①

$y = 3 - 5$
$y = -2$

$\therefore (x, y) = \{(2, 1), (5, -2)\}$

use Desmos to check.

b) $g(x) = 4x - 1$, $f(x) = -2x^2 + 4x + 1$

set $g(x) = f(x)$

$4x - 1 = -2x^2 + 4x + 1$

$2x^2 - 2 = 0$

$2(x^2 - 1) = 0$

$x = \pm 1$

$g(1) = 4 - 1$
$= 3$

$g(-1) = -4 - 1$
$= -5$

$\therefore (x, y) = \{(1, 3), (-1, -5)\}$

c) $12x - 4y = 19$

① $4y = 12x - 19$

$y = 3x^2 - 12x + 14$

$4y = 12x^2 - 48x + 56$ ②

set ② = ①

$12x^2 - 48x + 56 = 12x - 19$

$12x^2 - 60x + 75 = 0$

$4x^2 - 20x + 25 = 0$

$(2x - 5)^2 = 0$

$x = \frac{5}{2}$

sub $x = \frac{5}{2}$ into ①

$4y = 12\left(\frac{5}{2}\right) - 19$

$4y = 30 - 19$

$4y = 11$

$y = \frac{11}{4}$

$\therefore (x, y) = \left(\frac{5}{2}, \frac{11}{4}\right)$

8d) $y = -3x^2 + 24x - 50$ $\quad 2x - 3y = -6$

① $3y = -9x^2 + 72x - 150$ $\quad$ ② $3y = 2x + 6$

set ② = ①

$2x + 6 = -9x^2 + 72x - 150$

$9x^2 - 70x + 156 = 0$

$b^2 - 4ac$

$= 4900 - 5616$

< 0

$\therefore$ no solution.

e) $h(x) = 2x^2 + 3$, $g(x) = x^2 - 2x + 7$

set $h(x) = g(x)$

$2x^2 + 3 = x^2 - 2x + 7$

$x^2 + 2x - 4 = 0$

$x = \dfrac{-2 \pm \sqrt{4 - 4(-4)}}{2}$

$x = \dfrac{-2 \pm \sqrt{20}}{2}$

$x = -1 \pm \sqrt{5}$

$h(-1+\sqrt{5}) = 2(-1+\sqrt{5})^2 + 3$
$= 2(1 - 2\sqrt{5} + 5) + 3$
$= 15 - 4\sqrt{5}$

$h(-1-\sqrt{5}) = 2(-1-\sqrt{5})^2 + 3$
$= 2(1 + 2\sqrt{5} + 5) + 3$
$= 15 + 4\sqrt{5}$

$\therefore (x, y) = \{(-1+\sqrt{5}, 15-4\sqrt{5}), (-1-\sqrt{5}, 15+4\sqrt{5})\}$

8f) $h(x) = -2x^2+24x-69,\ g(x) = x^2-10x+27$
set $g(x) = h(x)$

$$x^2-10x+27 = -2x^2+24x-69$$
$$3x^2-34x+96 = 0$$
$$(x-6)(3x-16) = 0$$
$$x = 6 \text{ OR } x = \frac{16}{3}$$

M 288
A −34
18, 16
÷3

$$g(6) = 36-60+27$$
$$= 3$$

$$g\left(\tfrac{16}{3}\right) = \left(\tfrac{16}{3}\right)^2 - 10\left(\tfrac{16}{3}\right)+27$$
$$= \frac{256-480+243}{9}$$
$$= \frac{19}{9}$$

$$\therefore (x,y) = \left\{(6,3), \left(\tfrac{16}{3}, \tfrac{19}{9}\right)\right\}$$

7. ② $y = 2x+b$ $f(x) = x(x-6)$
① $f(x) = x^2-6x$

set ①=② $x^2-6x = 2x+b$
$$x^2-8x-b = 0$$

a) $b^2-4ac = 0$
$64-4(-b) = 0$
$64+4b = 0$
$4b = -64$
$b = -16$

b) $b^2-4ac > 0$
$64+4b > 0$
$4b > -64$
$b > -16$

c) $b^2-4ac < 0$
$64+4b < 0$
$b < -16$

$\therefore$ $y = 2x-16$ intersects $f(x)$ once
$y = 2x+b$, $\{b \in \mathbb{R} \mid b > -16\}$, intersects $f(x)$ twice.
$y = 2x+b$, $\{b \in \mathbb{R} \mid b < -16\}$, does not intersect $f(x)$.

9. $y=-6x^2-370x+100900$. $y=500x-83024$.

$-6x^2-370x+100900=500x-83024$

$6x^2+370x+500x-100900-83024=0$

$6x^2+870x-183924=0$

$x^2+145x-30654=0$

$$x=\frac{-145\pm\sqrt{21025+122616}}{2}$$

$$x=\frac{-145\pm\sqrt{143641}}{2}$$

$$x=\frac{-145+379}{2} \quad \text{OR} \quad x=\frac{-145-379}{2}$$

$x=117$ $\qquad$ $x=^{-}262$

not required to actually find the intersections... just will they intersect

$D=b^2-4ac$

$D=21025-4(1)(-30654)$

$D=143641$

$D>0$ so there are 2 points of intersection.

U2D10

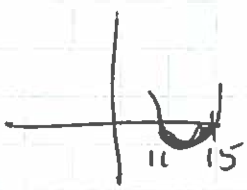
10. $f(x) = -0.15(x-13)^2+7.6$

$$-0.15(x-13)^2+7.6>7$$

$$\frac{-0.15(x-13)^2+0.6}{-0.15} > \frac{0}{-0.15}$$

flip sign when dividing by a negative.

$$(x-13)^2-4<0$$
$$[(x-13)-2][(x-13)+2]<0$$
$$(x-15)(x-11)<0$$



$$\{11<x<15\}$$

∴ the UV index is greater than 7 between 11am and 3pm.

11. $y=-4t+300$, $y=-4.9(t-5)^2+300$

$$-4t+300=-4.9(t-5)^2+300$$

$$4.9(t-5)^2-300-4t+300=0$$

$$4.9(t^2-10t+25)-4t=0$$
$$4.9t^2-49t+122.5-4t=0 \quad \times 10$$
$$49t^2-530t+1225=0$$
$$t=\frac{530\pm\sqrt{40800}}{98}$$
$$t=\frac{530\pm20\sqrt{102}}{98}$$
$$t=\frac{265\pm10\sqrt{102}}{49}$$

$t=3.347\ldots$ inadmissible (second hasn't jumped yet)

OR $t=7.469\ldots$

∴ the second parachutist catches up after about 7.5 seconds (2.5 s. after she jumps).