U2D5_T Quadratic Equations MCR 3UI

Monday, February 25, 2019 6:22 AM

U2D5_T
Quadratic...

U2D5 MCR 3UI **<u>Solving Quadratic Equations</u>**

<u>Warm Up</u>

Simplify the following.

1. $4\sqrt{99}-7\sqrt{12}+3\sqrt{44}+2\sqrt{75}$

$=4\sqrt{9\times 11}-7\sqrt{4\times 3}+3\sqrt{4\times 11}+2\sqrt{25\times 3}$

$=4(3\sqrt{11})-7(2\sqrt{3})+3(2\sqrt{11})+2(5\sqrt{3})$

$=12\sqrt{11}-14\sqrt{3}+6\sqrt{11}+10\sqrt{3}$

$=18\sqrt{11}-4\sqrt{3}$

2. $(2\sqrt{3}+5)(2\sqrt{3}-8)$

$=4(3)-16\sqrt{3}+10\sqrt{3}-40$

$=12-6\sqrt{3}-40$

$=-28-6\sqrt{3}$

A quadratic equation is a quadratic function where y = 0. Solving a quadratic equation results in the roots or zeroes of the quadratic function. (finds all values of x that makes the equation true)
To solve, <u>FACTOR</u> or use the <u>QUADRATIC FORMULA</u> (if it doesn't factor).

1. Solve, using the most efficient method.

a. $x^2+5x=0$

$x(x+5)=0$

$x=0$ OR $x+5=0$

$x=-5$

b. $x^2+8x-9=0$ M −9
A 8

$(x+9)(x-1)=0$

$x+9=0$ OR $x-1=0$

$x=-9$ $x=1$

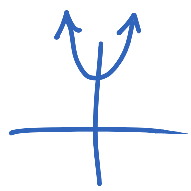
c. $x^2+16=0$

DNF

$x^2=-16$

$x=\pm\sqrt{-16}$

no real Solution



d. $(x+5)^2-81=0$

$(x+5-9)(x+5+9)=0$

$(x-4)(x+14)=0$

$x=4$ OR $x=-14$

$(x+5)^2=81$

$x+5=\pm\sqrt{81}$

$x=5\pm 9$

$x=5+9$ OR $x=5-9$

$x=14$ $x=-4$

2. Locate the roots for the following quadratic functions.

a. $y=4x^2-4x-3$

$4x^2-4x-3=0$

$(2x+1)(2x-3)=0$

$2x+1=0$ OR $2x-3=0$

$2x=-1$ $\quad$ $2x=3$

$x=\frac{-1}{2}$ $\quad$ $x=\frac{3}{2}$

b. $y=2x^2-3x-4$

DNF

* need quadratic formula

3. <u>Solve the following quadratics using the Quadratic Formula</u>

We need this formula for quadratic equations that do not factor.

If $ax^2+bx+c=0$ then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

So … Let's try again

a. $2x^2-3x-4=0$

$$x=\frac{3\pm\sqrt{9-4(2)(-4)}}{4}$$

$$x=\frac{3\pm\sqrt{9+32}}{4}$$

$$x=\frac{3\pm\sqrt{41}}{4}$$

$x=\frac{3+\sqrt{41}}{4}$ OR $x=\frac{3-\sqrt{41}}{4}$

exact answers.

b. $y=3x^2-5-6x$

$3x^2-6x-5=0$

$$x=\frac{6\pm\sqrt{36-4(3)(-5)}}{6}$$

$$x=\frac{6\pm\sqrt{36+60}}{6}$$

$$x=\frac{6\pm\sqrt{96}}{6}$$

$96 = 4 \cdot 24 = 4 \cdot 4 \cdot 6$

$$x=\frac{6\pm\sqrt{16\times 6}}{6}$$

$$x=\frac{6\pm4\sqrt{6}}{6}$$

$$x=\frac{\cancel{2}(3\pm2\sqrt{6})}{\cancel{6}\,3}$$

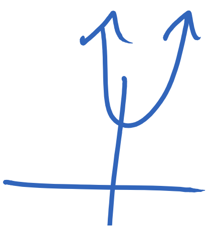
$\therefore$ $\boxed{x=\frac{3+2\sqrt{6}}{3}}$ OR $\boxed{x=\frac{3-2\sqrt{6}}{3}}$

c. $y=x^2-4x+6$

$x^2-4x+6=0$

$$x=\frac{4\pm\sqrt{16-4(1)(6)}}{2}$$

$$x=\frac{4\pm\sqrt{16-24}}{}$$



$$x = \frac{4 \pm \sqrt{16-24}}{2}$$

$$x = \frac{4 \pm \sqrt{-8}}{2}$$

$\therefore$ there is no solution.

4. Solve for the values of x that satisfy the following equation.

$$(2x+1)^2 + (2x+3)^2 = 26$$

$$4x^2+4x+1+4x^2+12x+9-26=0$$

$$8x^2+16x-16=0$$

$$8(x^2+2x-2)=0$$

$$x^2+2x-2=0$$

$$x = \frac{-2 \pm \sqrt{4-4(1)(-2)}}{2}$$

$$x = \frac{-2 \pm \sqrt{12}}{2}$$

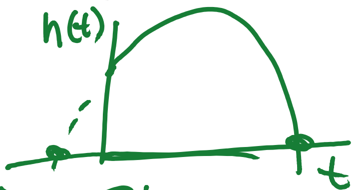
$$x = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$x = \frac{\cancel{2}(-1 \pm \sqrt{3})}{\cancel{2}}$$

$$\therefore \boxed{x = -1+\sqrt{3} \quad \boxed{\text{OR}} \quad x = -1-\sqrt{3}}$$

5. Narein throws a ball that will move through the air in a parabolic path due to gravity. The height, h, in metres, of the ball above the ground after t seconds can be modelled by the function
$h(t) = -4.9t^2 + 40t + 1.5$.
Find the zeros (rounded to the nearest thousandth) of the function and interpret their meaning.

For projectile problems, keep in mind:
i) Object hits ground when the height = 0 m.
ii) If solving for "when" (the time) then need a height (h), if solving for a "how high" (height) then need a time (t).
iii) Object reaches max height at the vertex! (not necessarily at the halfway point if object has an initial height not equal to zero.
iv) Initial height of object can be found at t=0 s

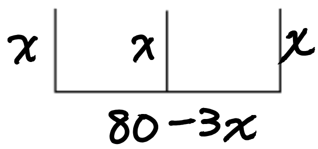


$$-4.9t^2+40t+1.5=0$$
$$4.9t^2-40t-1.5=0$$
$$t=\frac{40\pm\sqrt{1600-4(4.9)(-1.5)}}{9.8}$$
$$t=\frac{40\pm\sqrt{1629.4}}{9.8}$$

$t = 8.200594\ldots$
$t \doteq 8.201$
OR $t = -0.0373\ldots$ ← inadmissible $\{t \geq 0\}$

∴ ball hits the ground after 8.201 sec.

6. A rectangular lot is bounded on one side by a river and on the other three sides by fencing. Then another section of fencing is used to divide the lot into two parts as shown. A total of 80m of fencing is used. Determine all possible dimensions of the lot with a total area of 400 m^2.



$$A(x)=x(80-3x)$$
$$80x-3x^2=400$$
$$-3x^2+80x-400=0$$
⋮

U2D5 MCR3UI **Worksheet** **<u>Solving Quadratic Equations</u>**

1. Determine the roots of each equation by factoring.
   a) $x^2 + 5x + 4 = 0$ b) $4x^2 - 9 = 0$ c) $x^2 - 11x + 18 = 0$ d) $2x^2 - 7x - 4 = 0$

2. Use the quadratic formula to determine each of the roots to two decimal places.
   a) $x^2 - 4x - 9 = 0$ b) $3x^2 + 2x - 8 = 0$ c) $-2x^2 + 3x - 6 = 0$ d) $0.5x^2 - 2.2x - 4.7 = 0$

3. i) For each equation, decide on a strategy to solve it and explain why you chose that strategy.
   ii) Use your strategy to solve the equation. When appropriate, leave your answer in simplest radical form.
   a) $2x^2 - 3x = x^2 + 7x$ b) $4x^2 + 6x + 1 = 0$ c) $x^2 + 4x - 3 = 0$
   d) $(x + 3)^2 = -2x$ e) $3x^2 - 5x = 2x^2 + 4x + 10$ f) $2(x + 3)(x - 4) = 6x + 6$

4. Locate the x –intercepts of the graph of each function.
   a) $f(x) = 3x^2 - 7x - 2$ b) $f(x) = -4x^2 + 25x - 21$

5. The flight of a ball hit from a tree that is 0.6 m tall can be modelled by the function $h(t) = -4.9t^2 + 6t + 0.6$ Where $h(t)$ is the height in metres at time t seconds. How long will it take for the ball to hit the ground?

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.
   a) $P(x) = -x^2 + 12x + 28$ b) $P(x) = -2x^2 + 18x - 40$
   c) $P(x) = -2x^2 + 22x - 17$ d) $P(x) = -0.5x^2 + 6x - 5$

7. A rectangular swimming pool measuring **10 m by 4 m** is surrounded by a deck of uniform width. The **combined area** of the deck and the pool is **135 m²**. What is the **width** of the deck?

8. The sum of the squares of two consecutive integers is 685. What could the integers be? (list all possibilities)

9. Sally is standing on the top of a river slope and throws a ball. The height of the ball at a given time is modeled by the function $\boldsymbol{h(t) = -5t^2 + 30t + 10}$, where $h(t)$ is the height in metres and t is the time in seconds.
   a) How long is the ball in the air, to the nearest tenth of a second? b) How high is the ball after 4 seconds?
   c) When will the ball be 10m above the ground? d) What is the maximum height of the ball?

10. The height, $h(t)$, in metres, of an object fired upwards from the ground at 50 m/s is given approximately by the equation $h(t) = -5t^2 + 50t$ where t seconds is the time since the object was launched.
    a) Does an object fired upwards at 50 m/s reach a height of 150 m? If so, after how many seconds is the object at this height?
    b) When will the object hit the ground? c) When does it reach its maximum height?

11. The population of an Ontario city is modeled by the function $P(t) = 0.5t^2 + 10t + 300$ where P(*t*) is the population in thousands and *t* is the time in years. (Note: *t* = 0 corresponds to the year 2000)
    a) What was the population in 2000? b) What will be the population in 2012?
    c) When is the population expected to be 1,050,000?

12. The profit of a skateboard company can be modeled by the function $P(x) = -63 + 133x - 14x^2$, where $P(x)$ is the profit in thousands of dollars and x is the number of skateboards sold, also in thousands.
    a) What is the maximum profit the company can earn?
    b) Determine when the company is profitable by calculating the break-even points.

13. In Vancouver, the height, h, in kilometres, that you would need to climb to see to the east coast of Canada can be modelled by the equation $h^2 + 12\,740h = 20\,000\,000$. If the positive root of this equation is the solution, find the height, to the nearest kilometre.

<u>ANSWERS:</u>

1. a) $\{-4, -1\}$ b) $\{2, 9\}$ c) $\left\{\frac{-3}{2}, \frac{3}{2}\right\}$ d) $\left\{\frac{-1}{2}, 4\right\}$ 2. a) $\{-1.61, 5.61\}$ b) $\left\{-2, \frac{4}{3}\right\}$ c) $\{\ \}$ – means there is no solution d) $\{-1.57, 5.97\}$

3.

Easiest method	Roots		Easiest Method	Roots
a) Common Factoring	$\{0, 10\}$		b) DNF -- Quadratic Formula	$\left\{\frac{-3 \pm \sqrt{5}}{4}\right\}$
c) DNF – use Quadratic Formula	$\{-2 \pm \sqrt{7}\}$		d) DNF – use Quad. Formula	$\{-4 \pm \sqrt{7}\}$
e) Simple Trinomial Factoring	$\{-1, 10\}$		f) DNF – use Quad. Formula	$\{2 \pm \sqrt{19}\}$

4. a) $\left\{\frac{7 \pm \sqrt{73}}{6}\right\}$ b) $\left\{1, \frac{21}{4}\right\}$ 5. $\frac{30 \pm \sqrt{1194}}{49}$ or about 1.32 seconds. 6. a) 14 000 units
6. b) 4000 *units or* 5000 *units* 6. c) 836 *units or* 10 164 *units*
6.d) 901 *units or* 11 099 *units* 7. 2.5 *m* 8. −19, −18 *OR* 18, 19
9. a) 6.3 *seconds* b) 50 *m* c) 6.2 *sec.* d) 55 *m*
10. a) *no* b) 10 *sec.* c) 5 *sec.* 11. a) 300 000 b) 492 000 c) 2030
12. a) \$252 875 b) 500 *units or* 9000 *units* 13. 1413 *km*