U2D5 MCR3UI Worksheet **<u>Solving Quadratic Equations</u>**

1. Determine the roots of each equation by factoring.
   a) $x^2+5x+4=0$ b) $4x^2-9=0$ c) $x^2-11x+18=0$ d) $2x^2-7x-4=0$

2. Use the quadratic formula to determine each of the roots to two decimal places.
   a) $x^2-4x-9=0$ b) $3x^2+2x-8=0$ c) $-2x^2+3x-6=0$ d) $0.5x^2-2.2x-4.7=0$

3. i) For each equation, decide on a strategy to solve it and explain why you chose that strategy.
   ii) Use your strategy to solve the equation. When appropriate, leave your answer in simplest radical form.
   a) $2x^2-3x=x^2+7x$ b) $4x^2+6x+1=0$ c) $x^2+4x-3=0$
   d) $(x+3)^2=-2x$ e) $3x^2-5x=2x^2+4x+10$ f) $2(x+3)(x-4)=6x+6$

4. Locate the x −intercepts of the graph of each function.
   a) $f(x)=3x^2-7x-2$ b) $f(x)=-4x^2+25x-21$

5. The flight of a ball hit from a tree that is 0.6 m tall can be modelled by the function $h(t)=-4.9t^2+6t+0.6$ Where $h(t)$ is the height in metres at time t seconds. How long will it take for the ball to hit the ground?

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.
   a) $P(x)=-x^2+12x+28$ b) $P(x)=-2x^2+18x-40$
   c) $P(x)=-2x^2+22x-17$ d) $P(x)=-0.5x^2+6x-5$

7. A rectangular swimming pool measuring **10 m by 4 m** is surrounded by a deck of uniform width. The **combined area** of the deck and the pool is **135 m^2**. What is the **width** of the deck?

8. The sum of the squares of two consecutive integers is 685. What could the integers be? (list all possibilities)

9. Sally is standing on the top of a river slope and throws a ball. The height of the ball at a given time is modeled by the function $\boldsymbol{h(t)=-5t^2+30t+10}$, where $h(t)$ is the height in metres and t is the time in seconds.
   a) How long is the ball in the air, to the nearest tenth of a second? b) How high is the ball after 4 seconds?
   c) When will the ball be 10m above the ground? d) What is the maximum height of the ball?

10. The height, $h(t)$, in metres, of an object fired upwards from the ground at 50 m/s is given approximately by the equation $h(t)=-5t^2+50t$ where t seconds is the time since the object was launched.
    a) Does an object fired upwards at 50 m/s reach a height of 150 m? If so, after how many seconds is the object at this height?
    b) When will the object hit the ground? c) When does it reach its maximum height?

11. The population of an Ontario city is modeled by the function $P(t)=0.5t^2+10t+300$ where P(t) is the population in thousands and t is the time in years. (Note: $t=0$ corresponds to the year 2000)
    a) What was the population in 2000? b) What will be the population in 2012?
    c) When is the population expected to be 1,050,000?

12. The profit of a skateboard company can be modeled by the function $P(x)=-63+133x-14x^2$, where $P(x)$ is the profit in thousands of dollars and x is the number of skateboards sold, also in thousands.
    a) What is the maximum profit the company can earn?
    b) Determine when the company is profitable by calculating the break-even points.

13. In Vancouver, the height, h, in kilometres, that you would need to climb to see to the east coast of Canada can be modelled by the equation $h^2+12\,740h=20\,000\,000$. If the positive root of this equation is the solution, find the height, to the nearest kilometre.

<u>ANSWERS:</u>

1. a) $\{-4,-1\}$ b) $\{2,9\}$ c) $\left\{\frac{-3}{2},\frac{3}{2}\right\}$ d) $\left\{\frac{-1}{2},4\right\}$ 2. a) $\{-1.61, 5.61\}$ b) $\left\{-2,\frac{4}{3}\right\}$ c) $\{\ \}$ – means there is no solution d) $\{-1.57, 5.97\}$

3.

Easiest method	Roots		Easiest Method	Roots
a) Common Factoring	$\{0,10\}$		b) DNF -- Quadratic Formula	$\left\{\frac{-3\pm\sqrt{5}}{4}\right\}$
c) DNF – use Quadratic Formula	$\{-2\pm\sqrt{7}\}$		d) DNF – use Quad. Formula	$\{-4\pm\sqrt{7}\}$
e) Simple Trinomial Factoring	$\{-1,10\}$		f) DNF – use Quad. Formula	$\{2\pm\sqrt{19}\}$

4. a) $\left\{\frac{7\pm\sqrt{73}}{6}\right\}$ b) $\left\{1,\frac{21}{4}\right\}$ 5. $\frac{30\pm\sqrt{1194}}{49}$ or about 1.32 seconds. 6. a) 14 000 units
6. b) 4000 *units or* 5000 *units* 6. c) 836 *units or* 10 164 *units*
6.d) 901 *units or* 11 099 *units* 7. 2.5 *m* 8. −19, −18 *OR* 18, 19
9. a) 6.3 *seconds* b) 50 *m* c) 6.2 *sec.* d) 55 *m*
10. a) *no* b) 10 *sec.* c) 5 *sec.* 11. a) 300 000 b) 492 000 c) 2030
12. a) \$252 875 b) 500 *units or* 9000 *units* 13. 1413 *km*

1. a) $x^2+5x+4=0$
$(x+1)(x+4)=0$
$x=-1$ OR $x=-4$

b) $x^2-11x+18=0$
$(x-2)(x-9)=0$
$x=2$ OR $x=9$

c) $4x^2-9=0$
$(2x+3)(2x-3)=0$
$x=\frac{-3}{2}$ OR $x=\frac{3}{2}$

d) $2x^2-7x-4=0$
$(2x+1)(x-4)=0$
$x=\frac{-1}{2}$ OR $x=4$

M -8
A -7
$-8, 1$
$\div 2$

2a) $x^2-4x-9=0$
$x=\frac{4\pm\sqrt{16-4(1)(-9)}}{2}$
$x=\frac{4\pm\sqrt{52}}{2}$
$x=\frac{4\pm2\sqrt{13}}{2}$
$x=2\pm\sqrt{13}$

$x=5.61$ OR $x=-1.61$

b) $3x^2+2x-8=0$
$x=\frac{-2\pm\sqrt{4-4(3)(-8)}}{6}$
$x=\frac{-2\pm\sqrt{100}}{6}$
$x=\frac{-2\pm10}{6}$

$x=-2$ OR $x=\frac{4}{3}$

c) $-2x^2+3x-6=0$
$2x^2-3x+6=0$

$x=\frac{3\pm\sqrt{9-4(2)(6)}}{4}$

$x=$ no real solutions.

note
$(-22)^2$
$=22^2$
$=(2\times11)^2$
$=4\times121$
$=484$

d) $0.5x^2-2.2x-4.7=0$
$5x^2-22x-47=0$

$x=\frac{22\pm\sqrt{22^2-4(5)(-47)}}{10}$

$x=\frac{22\pm\sqrt{484+940}}{10}$

$x=\frac{22\pm\sqrt{1424}}{10}$

$x=\frac{22\pm\sqrt{16\times89}}{10}$

$x=\frac{22\pm4\sqrt{89}}{10}$

$x=\frac{11\pm2\sqrt{89}}{5}$ ← exact answers

for approximate answers
$x=(22+\sqrt{1424})\div10$
$x\doteq5.97$
OR $x=(22-\sqrt{1424})\div10$
$x\doteq-1.57$

3\. a) $2x^2 - 3x = x^2 + 7x$

$x^2 - 10x = 0$

* easiest to common factor.

$x(x-10) = 0$

$x = 0$ OR $x = 10$

b) $4x^2 + 6x + 1 = 0$ m4, A6 DNF

easiest to use formula

$x = \frac{-6 \pm \sqrt{36-16}}{8}$

$x = \frac{-6 \pm \sqrt{20}}{8}$

$x = \frac{-6 \pm 2\sqrt{5}}{8}$

$x = \frac{-3 \pm \sqrt{5}}{4}$

c) $x^2 + 4x - 3 = 0$ DNF

use formula.

$x = \frac{-4 \pm \sqrt{16+12}}{2}$

$x = \frac{-4 \pm \sqrt{28}}{2}$

$x = \frac{-4 \pm 2\sqrt{7}}{2}$

$x = -2 \pm \sqrt{7}$

d) $(x+3)^2 = -2x$

$x^2 + 6x + 9 + 2x = 0$

$x^2 + 8x + 9 = 0$ DNF

so use formula.

$x = \frac{-8 \pm \sqrt{64-36}}{2}$

$x = \frac{-8 \pm \sqrt{28}}{2}$

$x = \frac{-8 \pm 2\sqrt{7}}{2}$

$x = -4 \pm \sqrt{7}$

e) $3x^2 - 5x = 2x^2 + 4x + 10$

$x^2 - 9x - 10 = 0$ m -10, A -9, -10, 1

factors easily so do that ☺

$(x-10)(x+1) = 0$

$x = 10$ OR $x = -1$

f) $2(x+3)(x-4) = 6x + 6$

$2(x^2 - x - 12) - 6x - 6 = 0$

$2x^2 - 2x - 24 - 6x - 6 = 0$

$2x^2 - 8x - 30 = 0$

$x^2 - 4x - 15 = 0$ DNF

so use formula

$x = \frac{4 \pm \sqrt{16+60}}{2}$

$x = \frac{4 \pm \sqrt{76}}{2}$

$x = \frac{4 \pm 2\sqrt{19}}{2}$

$x = 2 \pm \sqrt{19}$

4. a) $3x^2-7x-2=0$ m−6 A−7 DNF

$$x=\frac{7\pm\sqrt{49+24}}{6}$$

$$x=\frac{7\pm\sqrt{73}}{6}$$

$(x \doteq 2.59$ or $-0.26)$

b) $-4x^2+25x-21=0$

$4x^2-25x+21=0$

factors!

$(x-1)(4x-21)=0$

$x=1$ or $x=\frac{21}{4}$

m 84
A −25
1×84
~~2×42~~
~~3×28~~
4×21
÷4

5. $h(t)=-4.9t^2+6t+0.6$

set $h(t)=0$

$49t^2-60t-6=0$

$$t=\frac{60\pm\sqrt{3600+196(6)}}{98}$$

$$t=\frac{60\pm\sqrt{4776}}{98}$$

$$t=\frac{60\pm2\sqrt{1194}}{98}$$

$$t=\frac{30\pm\sqrt{1194}}{49}$$

$(t \doteq 1.32$ seconds , $t>0)$.

So $t=\frac{30+\sqrt{1194}}{49}$, $t>0$.

6. Break even means profit $=0$.

a) $-x^2+12x+28=0$

$x^2-12x-28=0$ −14, 2

$(x-14)(x+2)=0$

$x=14$ or $\cancel{x=-2}$

$x \{x\geq 0\}$

∴ break even is 14 000 units.

b) $-2x^2+18x-40=0$

$x^2-9x+20=0$

$$x=\frac{9\pm\sqrt{81-80}}{2}$$

$$x=\frac{9\pm1}{2}$$

$x=5$ or $x=4$

Break even is 5000 units or 4000 units.

c) $-2x^2+22x-17=0$

$2x^2-22x+17=0$

$$x=\frac{22\pm\sqrt{484-136}}{4}$$

$$x=\frac{22\pm\sqrt{348}}{4}$$

$$x=\frac{11\pm\sqrt{87}}{2}$$

Break even is 10 164 units or 836 units.

6d) $-0.5x^2+6x-5=0$ $2x-2$

$x^2-12x+10=0$

DNF

$$x=\frac{12\pm\sqrt{144-40}}{2}$$

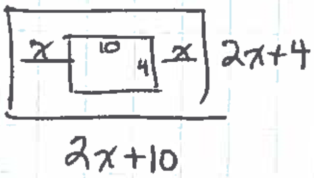
$$x=\frac{12\pm\sqrt{104}}{2}$$

$$x=6\pm\sqrt{26}$$

Break even 11099 units OR 901 units.

U2D5 Worksheet Solutions.

7.



$$(2x+10)(2x+4)=135$$
$$4x^2+28x+40-135=0$$
$$4x^2+28x-95=0$$

$$x=\frac{-28\pm\sqrt{28^2+16(95)}}{8}$$
$$=\frac{-28\pm\sqrt{2304}}{8}$$
$$=\frac{-28\pm 48}{8}$$

$x=-\frac{76}{8}$ OR $\boxed{x=\frac{20}{8}\Rightarrow x=\frac{10}{4}}$

inadmissible

$\therefore$ the deck is $\frac{10}{4}$ OR 2.5 m wide.

8. Let x, $x+1$ represent the integers.

$$x^2+(x+1)^2=685$$

$$x^2+x^2+2x-684=0$$
$$2x^2+2x-684=0$$
$$x^2+x-342=0$$
$$(x+19)(x-18)=0$$
$x=-19$ or $x=18$

$x+1=-18$ $\quad$ $x+1=19$

$\therefore$ the numbers are $-19, -18$ OR $18, 19$.

9. $h(t)=-5t^2+30t+10$

a) set $h=0$

$$-5(t^2-6t-2)=0$$
$$t^2-6t-2=0$$
$$t=\frac{6\pm\sqrt{36-4(-2)}}{2}$$
$$t=\frac{6\pm\sqrt{44}}{2}$$
$$t=\frac{6\pm2\sqrt{11}}{2}$$
$$t=3\pm\sqrt{11}$$

$t=3+\sqrt{11}$ OR $t=3-\sqrt{11}$

$t\doteq 6.31\ldots$ $\quad$ $t\doteq -0.31\ldots$

$t\doteq 6.3$ $\quad$ $t\doteq -0.3$

inad.

$\therefore$ it will be in the air about 6.3 seconds.

9 b) $h(4) = -5(4)^2 + 30(4) + 10$
$= -80 + 120 + 10$
$= 50$

$\therefore$ Ball is 50 m in air at 4 seconds.

c) $-5t^2 + 30t + 10 = 5$ (set $h(t) = 5$ metres)

$t^2 - 6t - 1 = 0$

$t = \dfrac{6 \pm \sqrt{36 - 4(1)(-1)}}{2}$

$t = \dfrac{6 \pm \sqrt{40}}{2}$

$t = \dfrac{2(3 \pm \sqrt{10})}{2}$

$t = 3 \pm \sqrt{10}$

$t = 6.162\ldots$ OR $t = -0.162\ldots$ ← inad.

$\therefore$ the ball will be 10 m above the air after about 6.2 seconds ($3 + \sqrt{10}$ seconds).

d) $h(t) = -5t(t-6) + 10$

max height is at $t = \dfrac{0+6}{2}$

$t = 3$ seconds.

"Partial Factoring"

$h(3) = -5(3)(-3) + 10$
$= 45 + 10$
$= 55$ m.

10. a) Does $h(t)$ ever equal 150?

$-5t^2 + 50t = 150$

$t^2 - 10t + 30 = 0$

$t = \dfrac{10 \pm \sqrt{100 - 4(30)}}{2}$

$b^2 - 4ac$ is -20 so there is no real solution.

$\therefore$ the object never reaches 150 m.

b) $h(t) = 0$ when $t = ?$

$-5t^2 + 50t = 0$

$t^2 - 10t = 0$

$t(t-10) = 0$

$t = 0,\ t = 10$

$\therefore$ object hits the ground after 10 seconds.

10 c) $h(t) = -5(t^2 - 10t + 25 - 25)$
$= -5(t-5)^2 + 125$

$\therefore$ it reaches maximum after 5 seconds.

11. $P(t) = 0.5t^2 + 10t + 300$ P in thousands

a) $P(0) = 300$
initial population was 300000.

b) $P(12) = 0.5(144) + 120 + 300$
$= 72 + 420$
$= 492$ $\therefore$ 2012 → popⁿ 492000.

c) $P(t) = 1050$ when $t = ?$
remember P in thousands so 1050000 means
P = 1050.

$0.5t^2 + 10t + 300 = 1050$
$t^2 + 20t - 1500 = 0$
$(t + 50)(t - 30) = 0$ 50, −30
$t = -50,\ t = 30.$

$\therefore$ in year 2030 popⁿ will be 1050000
if trend continues.

12. a) P in thousands, x in thousands.

$P(x) = -14x^2 + 133x - 63$
$= -14x(x - 9.5) - 63$
at max, $x = \frac{0 + 9.5}{2}$
$x = 4.75$

$P(4.75) = 252.875$ thousand.

$\therefore$ the maximum profit is \$252875.

b) set $P(x) = 0$
$-14x^2 + 133x - 63 = 0$
$14x^2 - 133x + 63 = 0$ ⟹

12b) $x = \frac{133 \pm \sqrt{133^2 - 4(14)(63)}}{28}$

$x = \frac{133 \pm \sqrt{14161}}{28}$

$x = \frac{133 \pm 119}{28}$

$x = \frac{252}{28}$ OR $x = \frac{14}{28}$

$x = 9$ $\quad$ $x = \frac{1}{2}$

$\therefore$ 9000 units OR 500 units will produce a break-even point.

13. $h^2 + 12740h - 20\,000\,000 = 0$

$h = \frac{-12740 \pm \sqrt{12740^2 - 4(1)(-20\,000\,000)}}{2}$

$h = \frac{-12740 \pm \sqrt{242\,307\,600}}{2}$

$h = \frac{-12740 + \sqrt{242307600}}{2}$, $h > 0$.

$h = 1413.116$

$h \doteq 1413\text{km}$.