U2D4_T Max and Mins MCR 3UI

Monday, February 25, 2019 6:21 AM

U2D4_T
Max and ...

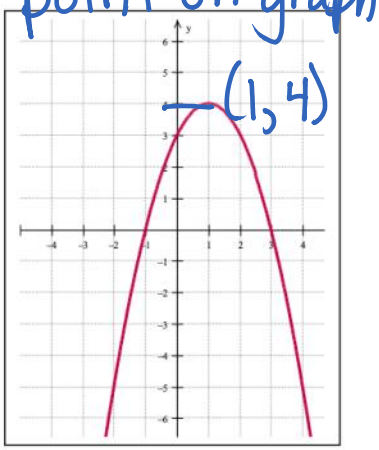
U2D4 MCR 3UI **QUADRATIC FUNCTIONS**

Maximum or Minimum Values

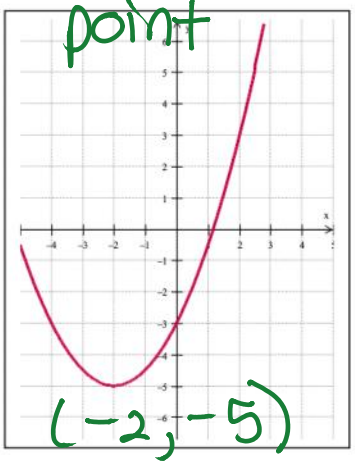
Consider the following graphs.

Is the optimal Value a Maximum or a minimum? Why?

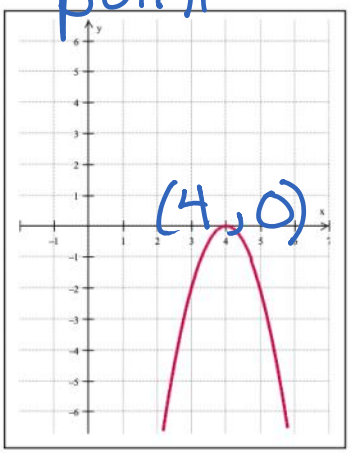
Max - Vertex is highest point on graph



Min - Vertex lowest point



Max - Vertex highest point



Indicate the maximum or minimum.

Max 4 Min -5 max 0

When does the optimal value occur?

when $x = 1$ when $x = -2$ when $x = 4$

Determine the maximum or minimum value for each of the following quadratics **and** state the value of x for which this occurs.
The Domain of each is $D: \{x \in \mathbb{R}\}$. State the Range of each.

1. By Factoring.

a) $y = -x^2 + 2x + 48$

$y = -(x^2 - 2x - 48)$

$y = -(x-8)(x+6)$

M -48
A -2
-8, 6

AofS
$x = \frac{-6+8}{2}$

$x = 1$

$f(1) = -(1-8)(1+6)$
$f(1) = -(-7)(7)$
$f(1) = 49$

∴ Max value of 49 occurs when $x = 1$. $R: \{y \leq 49\}$

2 | 48
2 | 24
2 | 12
2 | 6
3

b) $y = 3x^2 + 11x - 4$

$y = (3x-1)(x+4)$

1 3 | 1 2
3 1 | 4 2

AofS
$x = \frac{\frac{1}{3} - \frac{4}{1}}{2}$

$x = \left[\frac{1}{3} - \frac{12}{3}\right] \div \frac{2}{1}$

$x = \frac{-11}{3} \times \frac{1}{2}$

$x = -\frac{11}{6}$

$f\left(-\frac{11}{6}\right) = \left[\frac{3}{1}\left(-\frac{11}{6}\right) - 1\right]\left[-\frac{11}{6} + \frac{24}{6}\right]$

$= \left(-\frac{11}{2} - \frac{2}{2}\right)\left(\frac{13}{6}\right)$

$= \left(-\frac{13}{2}\right)\left(\frac{13}{6}\right)$

$= -\frac{169}{12}$

∴ Min of $-\frac{169}{12}$ when $x = -\frac{11}{6}$

$R: \{y \geq -\frac{169}{12}\}$

2. By Partial Factoring.

a) $f(x) = 2x^2 - 8x + 5$

$\therefore \cup$

$f(x) = 2x(x-4) + 5$

(0,5) (4,5)

(2, ↑

AofS $x = \frac{4}{2}$

$x = 2$

$f(2) = 2(2)(2-4)+5$

$f(2) = 2(2)(-2)+5$

$f(2) = -3$

$\therefore$ Min value of -3 when $x=2$

$R: \{y \geq -3\}$

b) $y = 1 - 4x^2 - 8x$

$y = -4x^2 - 8x + 1$

$\therefore \cap$

$y = -4x(x+2) + 1 \Rightarrow$

y-int. Vertex

(0,1) (-1,5) (-2,1)

$\frac{0+(-2)}{2}$

AofS $x = -1$

$f(-1) = -4(-1)(1)+1$

$f(-1) = 5$

Max of 5 at $x = -1$

$R: \{y \leq 5\}$

$(a+b)^2 = a^2 + 2ab + b^2$

3. By Completing the Square

a) $g(x) = -x - 2x^2$

$g(x) = -2x^2 - x$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$g(x) = -2\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right)$

$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\cap$ $g(x) = -2\left(x + \frac{1}{4}\right)^2 + \frac{1}{8}$

$\therefore$ Max value of $\frac{1}{8}$ occurs when $x = -\frac{1}{4}$

$R: \{y \leq \frac{1}{8}\}$

b) $y = 3x^2 - 5x + 4$

$\cup$ $y = 3\left(x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}\right) + \frac{4}{1}$

$\frac{1}{\cancel{3}} \times \frac{-25}{\cancel{36}}{}^{12}$

$\frac{4}{1} \times 12 = \frac{48}{12}$

$y = 3\left(x - \frac{5}{6}\right)^2 - \frac{25}{12} + \frac{48}{12}$

$y = 3\left(x - \frac{5}{6}\right)^2 + \frac{23}{12}$

$\therefore$ Min value of $\frac{23}{12}$ when $x = \frac{5}{6}$.

$R: \{y \geq \frac{23}{12}\}$

c) $h(x) = x^2 - 0.06x + 0.4$

$\cup$ $h(x) = (x^2 - 0.06x + 0.0009 - 0.0009) + 0.4$

$h(x) = (x - 0.03)^2 + 0.3991$

$\therefore$ Min value of 0.3991 when $x = 0.03$

$R: \{y \geq 0.3991\}$

Maximum and Minimum Word Problems

1. A football is kicked straight up in the air. Its height above the ground is approximated by the relation $h = 24t - 5t^2$, where h is the height in metres and t is the time in seconds.
What maximum height will the ball reach? How long does it take to reach the maximum height?

For projectile problems, keep in mind:
i) Object hits ground when the height = 0 m.
ii) If solving for "when" (the time) then need a height (h), if solving for a "how high" (height) then need a time (t).
iii) Object reaches max height at the vertex! (not necessarily at the halfway point if object has an initial height not equal to zero.
iv) Initial height of object can be found at t=0 s

$h(t) = -5t^2 + 24t$

$h(t) = -5t(t - \frac{24}{5})$

AofS $x = \frac{12}{5}$ $h(\frac{12}{5}) = -5(\frac{12}{5})(-\frac{12}{5})$

$h(\frac{12}{5}) = \frac{144}{5}$

∴ Max height of the ball is $\frac{144}{5}$ m (or 28.8 m) after $\frac{12}{5}$ seconds (2.4 seconds).

making $4/scarf

2. Raven and Ben are knitting scarves to sell at the craft show. The wool for each scarf costs $6. They were planning to sell the scarves for $10 each, the same as last year when they sold 40 scarves. However, they know that if they raise the price, they will be able to make more profit, even if they end up selling fewer scarves. They have been told that for every 50¢ increase in price, they can expect to sell four fewer scarves. What selling price will maximize their profit and what will the profit be?

Profit = Number Sold × Price/item.

$P(n) = (40 - 4n)(4 + 0.5n)$

where $P(n)$ is the profit given n 50¢ increases.

$P(n) = 160 + 20n - 16n - 2n^2$

$P(n) = -2n^2 + 4n + 160$

$P(n) = -2n(n-2) + 160$

A of S $n = 1$

$P(1) = -2(1)(-1) + 160$

$P(1) = 162$

Selling price: $\$10 + \$0.50(1)$

$= \$10.50$

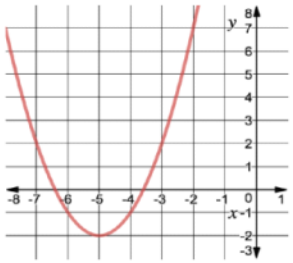
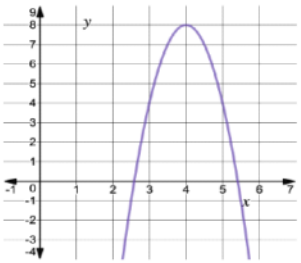
one $.50 increase

∴ Max profit is $162 when the selling price is $10.50

U2D4 Workshee...

U2D4 MCR 3UI Worksheet **Maximums & Minimums**

1. Which of the following quadratic functions will have a maximum value? Explain how you know.
   a) $y = -x^2 + 7x$ b) $f(x) = 3(x-1)^2 - 4$ c) $f(x) = -4(x+2)(x-3)$ d) $g(x) = 4x^2 + 3x - 5$

2. State the vertex of each parabola and indicate the maximum or minimum value of the function.
   a)

   b)


3. Determine the maximum or minimum value for each.
   a) $y = -4(x+1)^2 + 6$ b) $f(x) = (x-5)^2$ c) $f(x) = -2x(x-4)$ d) $g(x) = 2x^2 - 7$

4. Determine the maximum or minimum value. Use at least two different methods.
   a) $y = x^2 - 4x - 1$ b) $f(x) = x^2 - 8x + 12$ c) $y = 2x^2 + 12x$
   d) $y = -3x^2 - 12x + 15$ e) $y = 3x(x-2) + 5$ f) $g(x) = -2(x+1)^2 - 5$

5. The height of a ball thrown vertically upward from a rooftop is modelled by $h(t) = -5t^2 + 20t + 50$, where $h(t)$ is the ball's height above the ground, in metres, at time t seconds after the throw.
   a) Determine the maximum height of the ball.
   b) How long does it take for the ball to reach its maximum height? c) How high is the rooftop?

6. Determine by **factoring** the maximum or minimum value of each of the following and state the value of x for which it occurs.
   a) $y = x^2 + 3x - 108$ b) $f(x) = -4x^2 + 12x - 9$ c) $y = -x^2 + 11x$
   d) $g(x) = 4x^2 + 4x - 15$ e) $f(x) = 6t^2 + 33t + 15$ f) $h(x) = -2x^2 - x + 15$

7. Determine by **partial factoring** the maximum or minimum value of each of the following and state the value of x for which it occurs.
   a) $g(x) = x^2 - 4x - 1$ b) $y = -2x^2 - 4x - 3$ c) $y = -3x^2 + 9x + 7$
   d) $g(x) = 4x^2 + 20x - 1$ e) $y = 5x^2 + 35t + 11$ f) $h(x) = -2x^2 + 22x - 15$

8. Determine by **completing the square (CTS)** the maximum or minimum value of each of the following and state the value of x (or t) for which it occurs.
   a) $v(t) = 2t^2 + 4t + 3$ b) $y = 8x - 2x^2$ c) $a(t) = -4t^2 - 24t + 29$
   d) $y = 5x^2 - 20x + 18$ e) $h(t) = -3t^2 + 18t + 28$ f) $y = 10x^2 + 20x + 12$

9. The path of the ball for many golf shots can be modeled by a quadratic function. The path of a golf ball hit at an angle of 10° to the horizontal can be modeled by the function $h(d) = -0.002d^2 + 0.4d$, where $h(d)$ is the ball's height above the ground, in metres, at horizontal distance, d metres from the golfer.
   a) Determine the maximum height reached by the ball.
   b) What is the horizontal distance of the ball from the golfer when the ball reaches its maximum height?
   c) What distance does the ball travel horizontally until it first hits the ground? Hint: Use symmetry with answer from part (b)

10. A hockey arena manager in Flin Flon determined that the formula for the dollar revenue $R(n)$, where n is the number of dollars increase over \$5 per ticket is $R(n) = -100n^2 + 500n + 5000$. What is the greatest revenue and at what price per ticket does the maximum occur?

11. A grappling iron is thrown vertically to catch a ledge above the thrower. If its height, $h(t)$, in metres, at t seconds after being thrown is represented by the function $h(t) = -4.9t^2 + 11t + 1.5$. a) Determine the maximum height of the grappling hook. b) Will the grappling hook reach a ledge 7.5 m above the thrower?

U2D4 Worksheet Answers:

1. Negative 'a' values mean maximum -- so only a, & c have maximums.
2. a) $V(-5,-2)$; Min value of -2 b) $V(4,8)$; Max value of 8
3. a) max value of 6 b) min value of 0 c) max value of 8 d) min value of -7
4. a) min -5 b) min -4 c) min -18 d) max 27 e) min 2 f) max -5
5. a) 70 m b) 2 seconds c) 50 m
6. a) $min\ of\ \frac{-441}{4}\ at\ x=\frac{-3}{2}$ b) $max\ of\ 0\ at\ x=\frac{3}{2}$ c) $max\ of\ \frac{121}{4}\ at\ x=\frac{11}{2}$
   d) $min\ of\ -16\ at\ x=\frac{-1}{2}$ e) $min\ of\ \frac{-243}{8}\ at\ x=\frac{-11}{4}$ f) $max\ of\ \frac{121}{16}\ at\ x=\frac{1}{4}$
7. a) $min\ of\ -5\ at\ x=2$ b) $max\ of\ -1\ at\ x=-1$ c) $max\ of\ \frac{55}{4}\ at\ x=\frac{3}{2}$
   d) $min\ of\ -26\ at\ x=\frac{-5}{2}$ e) $min\ of\ \frac{-201}{4}\ at\ x=\frac{-7}{2}$ f) $max\ of\ \frac{91}{2}\ at\ x=\frac{11}{2}$
8. a) $min\ of\ 1\ at\ t=-1$ b) $max\ of\ 8\ at\ x=2$ c) $max\ of\ 65\ at\ t=-3$
   d) $min\ of\ -2\ at\ x=2$ e) $max\ of\ 55\ at\ t=3$ f) $min\ of\ 2\ at\ x=-1$
9. a) 20 m b) 100 m c) 200 m
10. *The maximum Revenue of* \$5625 *occurs with a ticket price is* \$7.50.