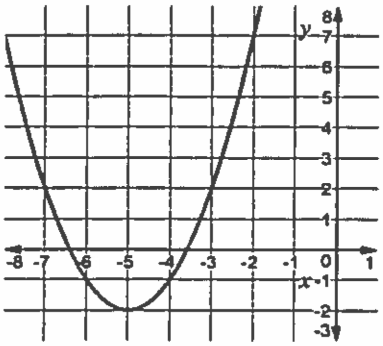
U2D4 MCR 3UI

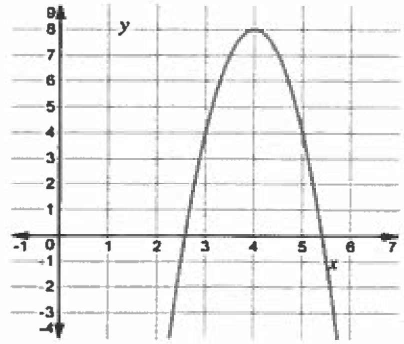
Maximums & Minimums Worksheet

1. Which of the following quadratic functions will have a maximum value? Explain how you know.
   a) $y = -x^2 + 7x$ b) $f(x) = 3(x-1)^2 - 4$ c) $f(x) = -4(x+2)(x-3)$ d) $g(x) = 4x^2 + 3x - 5$

2. State the vertex of each parabola and indicate the maximum or minimum value of the function.
   a)



   b)



3. Determine the maximum or minimum value for each.
   a) $y = -4(x+1)^2 + 6$ b) $f(x) = (x-5)^2$ c) $f(x) = -2x(x-4)$ d) $g(x) = 2x^2 - 7$

4. Determine the maximum or minimum value. Use at least two different methods.
   a) $y = x^2 - 4x - 1$ b) $f(x) = x^2 - 8x + 12$ c) $y = 2x^2 + 12x$
   d) $y = -3x^2 - 12x + 15$ e) $y = 3x(x-2) + 5$ f) $g(x) = -2(x+1)^2 - 5$

5. The height of a ball thrown vertically upward from a rooftop is modelled by $h(t) = -5t^2 + 20t + 50$, where $h(t)$ is the ball's height above the ground, in metres, at time t seconds after the throw.
   a) Determine the maximum height of the ball.
   b) How long does it take for the ball to reach its maximum height? c) How high is the rooftop?

6. Determine by **factoring** the maximum or minimum value of each of the following and state the value of x for which it occurs.
   a) $y = x^2 + 3x - 108$ b) $f(x) = -4x^2 + 12x - 9$ c) $y = -x^2 + 11x$
   d) $g(x) = 4x^2 + 4x - 15$ e) $f(x) = 6t^2 + 33t + 15$ f) $h(x) = -2x^2 - x + 15$

7. Determine by **partial factoring** the maximum or minimum value of each of the following and state the value of x for which it occurs.
   a) $g(x) = x^2 - 4x - 1$ b) $y = -2x^2 - 4x - 3$ c) $y = -3x^2 + 9x + 7$
   d) $g(x) = 4x^2 + 20x - 1$ e) $y = 5x^2 + 35t + 11$ f) $h(x) = -2x^2 + 22x - 15$

8. Determine by **completing the square (CTS)** the maximum or minimum value of each of the following and state the value of x for which it occurs.
   a) $v(t) = 2t^2 + 4t + 3$ b) $y = 8x - 2x^2$ c) $a(t) = -4t^2 - 24t + 29$
   d) $y = 5x^2 - 20x + 18$ e) $h(t) = -3t^2 + 18t + 28$ f) $y = 10x^2 + 20x + 12$

9. The path of the ball for many golf shots can be modeled by a quadratic function. The path of a golf ball hit at an angle of 10° to the horizontal can be modeled by the function $h(d) = -0.002d^2 + 0.4d$, where $h(d)$ is the ball's height above the ground, in metres, at horizontal distance, d metres from the golfer.
   a) Determine the maximum height reached by the ball.
   b) What is the horizontal distance of the ball from the golfer when the ball reaches its maximum height?
   c) What distance does the ball travel horizontally until it first hits the ground? Hint: Use symmetry with answer from part (b)

10. A hockey arena manager in Flin Flon determined that the formula for the dollar revenue $R(n)$, where n is the number of dollars increase over \$5 per ticket is $R(n) = -100n^2 + 500n + 5000$. What is the greatest revenue and at what price per ticket does the maximum occur?

11. A grappling iron is thrown vertically to catch a ledge above the thrower. If its height, $h(t)$, in metres, at t seconds after being thrown is represented by the function $h(t) = -4.9t^2 + 11t + 1.5$. a) Determine the maximum height of the grappling hook. b) Will the grappling hook reach a ledge 7.5 m above the thrower?

U2D4 Worksheet Answers:

1. Negative 'a' values mean maximum -- so only a, & c have maximums.
2. a) $V(-5,-2)$; Min value of -2 b) $V(4,8)$; Max value of 8
3. a) max value of 6 b) min value of 0 c) max value of 8 d) min value of -7
4. a) min -5 b) min -4 c) min -18 d) max 27 e) min 2 f) max -5
5. a) $70\ m$ b) 2 seconds c) $50\ m$
6. a) $min\ of\ \frac{-441}{4}\ at\ x = \frac{-3}{2}$ b) $max\ of\ 0\ at\ x = \frac{3}{2}$ c) $max\ of\ \frac{121}{4}\ at\ x = \frac{11}{2}$
   d) $min\ of\ -16\ at\ x = \frac{-1}{2}$ e) $min\ of\ \frac{-243}{8}\ at\ x = \frac{-11}{4}$ f) $max\ of\ \frac{121}{16}\ at\ x = \frac{1}{4}$
7. a) $min\ of\ -5\ at\ x = 2$ b) $max\ of\ -1\ at\ x = -1$ c) $max\ of\ \frac{55}{4}\ at\ x = \frac{3}{2}$
   d) $min\ of\ -26\ at\ x = \frac{-5}{2}$ e) $min\ of\ \frac{-201}{4}\ at\ x = \frac{-7}{2}$ f) $max\ of\ \frac{91}{2}\ at\ x = \frac{11}{2}$
8. a) $min\ of\ 1\ at\ t = -1$ b) $max\ of\ 8\ at\ x = 2$ c) $max\ of\ 65\ at\ t = -3$
   d) $min\ of\ -2\ at\ x = 2$ e) $max\ of\ 55\ at\ t = 3$ f) $min\ of\ 2\ at\ x = -1$
9. a) $20\ m$ b) $100\ m$ c) $200\ m$
10. *The maximum Revenue of* \$5625 *occurs with a ticket price is* \$7.50.

11. a) $\frac{376}{49}\ m$ b) yes.

1a) $\cap$ maximum $a=-1$ b) $\cup$ minimum $a=3$

c) $\cap$ maximum $a=-4$ d) $\cup$ minimum $a=4$

2\. a) $V(-5,-2)$
min value -2
b) $V(4,8)$
max value 8

3\. a) $\cap$ max value 6 b) $\cup$ min value 0

c) $\cap$ max value 8 d) $\cup$ min value -7

$$f\left(\frac{0+4}{2}\right)=f(2)$$
$$=-2(2)(-2)$$
$$=8$$

4\. a) $y=x^2-4x-1$
$y=(x^2-4x+4-4)-1$
$y=(x-2)^2-5$

$y=x(x-4)-1$
A of S: $x=\frac{0+4}{2}$
$x=2$
$y=2(-2)-1$
$y=-5$

$\cup$

Min Value -5 occurs at $x=2$.

b) $f(x)=x^2-8x+12$
$f(x)=(x^2-8x+16-16)+12$
$f(x)=(x-4)^2-4$

$f(x)=x(x-8)+12$
A of S: $x=\frac{8}{2}$
$x=4$
$f(4)=4(4-8)+12$
$=4(-4)+12$
$=-4$

$f(x)=(x-2)(x-6)$
zeros: 2, 6
A of S: $x=\frac{8}{2}$
$x=4$
$f(4)=2(-2)$
$=-4$

$\cup$ Min value of -4 occurs at $x=4$.

c) $y=2x^2+12x$
$y=2x(x+6)$
A of S $x=\frac{0-6}{2}$
$\boxed{x=-3}$

$y=2(-3)(-3+6)$
$=-18$

$y=2(x^2+6x+9-9)$
$=2(x+3)^2-18$

$\cup$

$\therefore$ Min value of -18 occurs at $x=-3$.

4d) $y = -3x^2 - 12x + 15$ $\therefore$

$y = -3(x^2 + 4x - 5)$
$y = -3(x+5)(x-1)$

A of S $x = \frac{-5+1}{2}$

$x = -2$

$y = -3(-2+5)(-2-1)$
$= -3(3)(-3)$
$= 27$

$\therefore$ Max value of 27

$y = -3(x^2+4x+4-4)+15$
$= -3(x+2)^2 + 12 + 15$
$= -3(x+2)^2 + 27$

$\therefore$ Max value 27

$y = -3x(x+4)+15$
A of S: $x = \frac{0-4}{2}$
$x = -2$

$y = -3(-2)(-2+4)+15$
$= 6(2)+15$
$\therefore$ $= 12+15$
$= 27$
$\therefore$ Max value 27.

4e) $y = 3x(x-2)+5$ $\therefore$

A of S: $x = \frac{0+2}{2}$

$\boxed{x=1}$

$y = 3(1)(1-2)+5$
$y = 3(-1)+5$
$= 2$

$\therefore$ min value of 2.

$y = 3x^2 - 6x + 5$
$y = 3(x^2-2x+1-1)+5$
$y = 3(x-1)^2 - 3 + 5$
$y = 3(x-1)^2 + 2$

$\therefore$ Min value 2.

* does not factor

4f) $g(x) = -2(x+1)^2 - 5$

$\therefore$ Max Value of -5

$g(x) = -2(x^2+2x+1) - 5$
$= -2x^2 - 4x - 2 - 5$
$= -2x^2 - 4x - 7$ ← does not factor
$= -2x(x+2) - 7$

A of S: $x = \frac{-2}{2}$

$\boxed{x = -1}$

$g(-1) = -2(-1)(-1+2) - 7$
$= 2(1) - 7$
$= -5$

$\therefore$ max value of -5.

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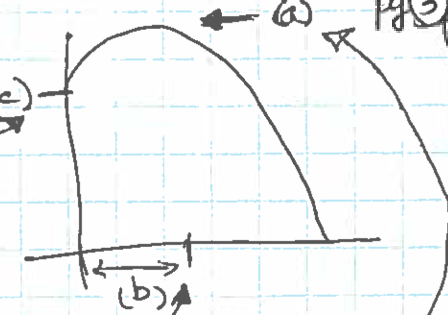
5. $h(t) = -5t^2 + 20t + 50$ initial height



a) $h(t) = -5t(t-4) + 50$

A of S: $t = \frac{4}{2}$

$t = 2$

$h(2) = -5(2)(2-4) + 50$
$= -10(-2) + 50$
$= 70$

a) $\therefore$ the Maximum height of the ball is 70 m.

b) it takes 2 seconds for the ball to reach the maximum height

c) The ball was thrown from the rooftop so the initial height is the height of the rooftop.

$\therefore$ the height of the rooftop is 50 m.

6. a) $y = x^2 + 3x - 108$ $\quad -9, 12$

☺ $y = (x-9)(x+12)$

A of S $x = \frac{9-12}{2}$

$x = \frac{-3}{2}$

$y = (\frac{-3}{2} - 9)(\frac{-3}{2} + 12)$

$y = (\frac{-3-18}{2})(\frac{-3+24}{2})$

$y = (\frac{-21}{2})(\frac{21}{2})$

$y = \frac{-441}{4}$

Min of $-\frac{441}{4}$ when $x = \frac{-3}{2}$

d) $g(x) = 4x^2 + 4x - 15$ $\quad$ m -60, A 4, ~~5 x 12~~, -6×10, $\div 2 \quad \div 2$

$g(x) = (2x-3)(2x+5)$

☺ A of S $x = \left(\frac{3}{2} + (\frac{-5}{2})\right) \div 2$

$x = (-1) \times \frac{1}{2}$

$x = -\frac{1}{2}$

$g(-\frac{1}{2}) = [2(-\frac{1}{2}) - 3][2(-\frac{1}{2}) + 5]$

$= (-4)(4)$

☺ $= -16$

Min of -16 when $x = \frac{-1}{2}$

b) $f(x) = -4x^2 + 12x - 9$

$f(x) = -(4x^2 - 12x + 9)$ $\quad$ m 36, A -12

☺ $= -(2x-3)^2$

Max of 0 when $x = \frac{3}{2}$

c) $y = -x^2 + 11x$

$y = -x(x-11)$

A of S: $x = \frac{11}{2}$

$y = -\frac{11}{2}(\frac{11}{2} - \frac{22}{2})$

☺ $= \frac{-11}{2}(\frac{-11}{2})$

$= \frac{121}{4}$

$\therefore$ Max of $\frac{121}{4}$ when $x = \frac{11}{2}$.

e) $y = 6x^2 + 33x + 15$

$y = 3(2x^2 + 11x + 5)$ $\quad$ m 10, A 11, $1, 10$, $\div 2$

$y = 3(2x+1)(x+5)$

A of S: $x = (\frac{-1}{2} - 5) \times \frac{1}{2}$

$= \frac{-1-10}{2} \times \frac{1}{2}$

☺ $= \frac{-11}{4}$

$y = 3[2(\frac{-11}{4}) + 1](\frac{-11}{4} + 5)$

$y = 3(\frac{-11}{2} + \frac{2}{2})(\frac{-11}{4} + \frac{20}{4})$

$y = 3(\frac{-9}{2})(\frac{9}{4})$

$y = \frac{-243}{8}$

$\therefore$ Min of $\frac{-243}{8}$ when $x = \frac{-11}{4}$

6 f) $h(x) = -2x^2 + x + 15$

$h(x) = -(2x^2 - x - 15)$

M −30
A −1
−6, 5
÷2

$= -(x-3)(2x+5)$

A of S: $x = \left(3 - \frac{5}{2}\right) \times \frac{1}{2}$

$x = \frac{6-5}{2} \times \frac{1}{2}$

$x = \frac{1}{4}$

$h\left(\frac{1}{4}\right) = -\left(\frac{1}{4} - \frac{12}{4}\right)\left(\cancel{2}^{1}\left(\frac{1}{\cancel{4}_2}\right) + \frac{10}{2}\right)$

$= -\left(\frac{-11}{4}\right)\left(\frac{11}{4}\right)$

$\cap$ $= \frac{121}{16}$

$\therefore$ Max of $\frac{121}{16}$ when $x = \frac{1}{4}$

7. a) $g(x) = x^2 - 4x - 1$

$g(x) = x(x-4) - 1$

A of S: $x = \frac{4}{2}$

$x = 2$

$g(2) = 2(2-4) - 1$

$= 2(-2) - 1$

$\cup$ $= -4 - 1$

$= -5$

$\therefore$ Min of -5 when $x = 2$

7b) $y = -2x^2 - 4x - 3$

$y = -2x(x+2) - 3$

A of S: $x = \frac{-2}{2}$

$x = -1$

$y = -2(-1)(-1+2) - 3$

$= +2(1) - 3$

$\cap$ $= -1$

$\therefore$ Max of -1 when $x = -1$.

7. c) $y = -3x^2 + 9x + 7$

$y = -3x(x-3) + 7$

A of S: $x = \frac{3}{2}$

$y = -3\left(\frac{3}{2}\right)\left(\frac{3}{2} - \frac{6}{2}\right) + 7$

$= -\frac{9}{2}\left(\frac{-3}{2}\right) + \frac{7}{1}$

$= \frac{27}{4} + \frac{28}{4}$

$= \frac{55}{4}$

$\therefore$ max of $\frac{55}{4}$ when $x = \frac{3}{2}$

d) $g(x) = 4x^2 + 20x - 1$

$g(x) = 4x(x+5) - 1$

A of S: $x = -\frac{5}{2}$

$g\left(-\frac{5}{2}\right) = 4\left(-\frac{5}{2}\right)\left(\frac{5}{2}\right) - 1$

$= -25 - 1$

$= -26$

$\therefore$ Min of -26 when $x = \frac{-5}{2}$

e) $y = 5x^2 + 35x + 11$

$y = 5x(x+7) + 11$

A of S: $x = \frac{-7}{2}$

$y = 5\left(-\frac{7}{2}\right)\left(\frac{7}{2}\right) + 11$

$= \frac{5(-49)}{4} + \frac{44}{4}$

$= \frac{-245 + 44}{4}$

$= -\frac{201}{4}$

$\therefore$ Min of $-\frac{201}{4}$ when $x = \frac{-7}{2}$

f) $h(x) = -2x^2 + 22x - 15$

$h(x) = -2x(x-11) - 15$

A of S: $x = \frac{11}{2}$

$h\left(\frac{11}{2}\right) = -\cancel{2}\left(\frac{11}{\cancel{2}_1}\right)\left(-\frac{11}{2}\right) - 15$

$= \frac{121}{2} - \frac{30}{2}$

$= \frac{91}{2}$

$\therefore$ Max of $\frac{91}{2}$ when $x = \frac{11}{2}$

8. a) $v(t) = 2(t^2 + 2t + 1 - 1) + 3$
$v(t) = 2(t+1)^2 - 2 + 3$
$v(t) = 2(t+1)^2 + 1$
$\therefore$ Min of 1 when $t = -1$

b) $y = -2x^2 + 8x$
$y = -2(x^2 - 4x + 4 - 4)$
$y = -2(x-2)^2 + 8$
$\therefore$ max of 8 when $x = 2$

c) $a(t) = -4t^2 - 24t + 29$
$a(t) = -4(t^2 + 6t + 9 - 9) + 29$
$a(t) = -4(t+3)^2 + 36 + 29$
$a(t) = -4(t+3)^2 + 65$
$\therefore$ Max of 65 when $t = -3$

d) $y = 5x^2 - 20x + 18$
$y = 5(x^2 - 4x + 4 - 4) + 18$
$y = 5(x-2)^2 - 20 + 18$
$\therefore$ $y = 5(x-2)^2 - 2$
$\therefore$ Min of -2 when $x = 2$.

e) $h(t) = -3(t^2 - 6t + 9 - 9) + 28$
$h(t) = -3(t-3)^2 + 27 + 28$
$h(t) = -3(t-3)^2 + 55$
$\therefore$ Max of 55 when $t = 3$

f) $y = 10x^2 + 20x + 12$
$y = 10(x^2 + 2x + 1 - 1) + 12$
$y = 10(x+1)^2 - 10 + 12$
$\therefore$ $y = 10(x+1)^2 + 2$
Min of 2 when $x = -1$.

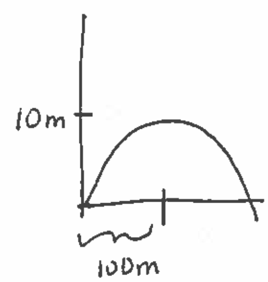
9. $h(d) = -0.002d^2 + 0.4d$

a) $h(d) = -0.002(d^2 - 200d + 10000 - 10000)$
$= -0.002(d-100)^2 + 20$

$\dfrac{0.400}{-0.002} = \dfrac{400}{-2} = -200$

Max height is 20 metres

b) 100 m is the horizontal distance covered when ball reaches it's maximum vertical height



c) The ball hits the ground 200 m horizontally from where it was hit initially.

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10. $R(n) = -100n^2 + 500n + 5000$

$\therefore$ $R(n) = -100\left(n^2 - 5n + \frac{25}{4} - \frac{25}{4}\right) + 5000$

$= -100\left(n - \frac{5}{2}\right)^2 + 25(25) + 5000$

$= -100\left(n - \frac{5}{2}\right)^2 + 5625$

$\therefore$ the maximum revenue is \$5625 when the price is \$5 + \$2.5 = \$7.50.

11. a) $h(t) = -4.9t^2 + 11t + 1.5$

$h(t) = -4.9\left(t^2 - \frac{11}{4.9}t\right) + 1.5$

$= \frac{-49}{10}\left(t^2 - \frac{110}{49}t + \frac{3025}{2401} - \frac{3025}{2401}\right) + \frac{3}{2}$

$= \frac{-49}{10}\left(t - \frac{55}{49}\right)^2 + \frac{49}{10} \times \frac{5 \times 605}{49 \times 49} + \frac{147}{98}$

$= \frac{-49}{10}\left(t - \frac{55}{49}\right)^2 + \frac{752}{98}$

$= \frac{-49}{10}\left(t - \frac{55}{49}\right)^2 + \frac{376}{49}$

a) Max height of grappling hook is $\frac{376}{49}$ m.

b) $\frac{376}{49} = 7.673\ldots$ $\therefore$ the grappling hook will reach 7.5 m.