U2D3 MCR 3UI Quadratic Functions Worksheet

1. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.
   a) How long will the rocket be in the air? How do you know?
   b) How high will the rocket be after 3 seconds?
   c) What is the maximum height that the rocket will reach?

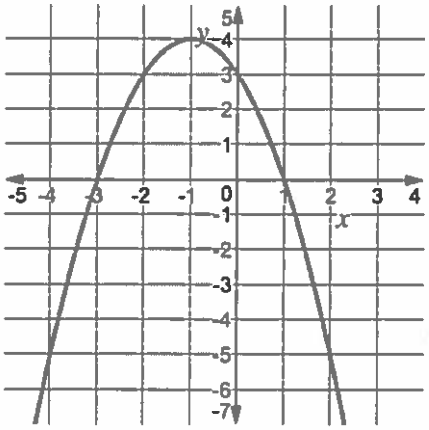
2. State whether each parabola opens up or down.
   a) $f(x) = 3x^2$ b) $f(x) = -2(x-3)(x+6)$ c) $f(x) = -(x+5)^2 - 1$ d) $f(x) = \frac{2}{3}x^2 - 2x - 1$

3. Given $f(x) = -3(x-2)(x+6)$, state:
   a) the zeros b) the direction of opening c) the equation of the axis of symmetry

4. Examine the parabola shown at the right.
   a) Copy and complete this table.

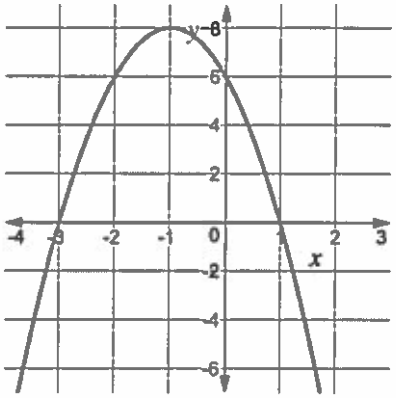
x	-2	-1	0	1	2
$f(x)$					

   b) Calculate the second differences of the function. How could you have predicted their signs?



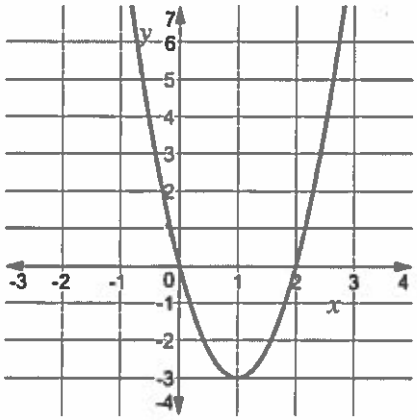
5. For each function, state the direction of opening, the vertex, and the equation of the axis of symmetry.
   a) $f(x) = x^2 - 3$ b) $f(x) = -(x+3)^2 - 4$
   c) $f(x) = 2(x-4)(x+2)$
   d) $f(x) = -\frac{1}{2}x^2 + 4$

6. Express each quadratic function in standard form. State the y-intercept of each.
   a) $f(x) = -3(x-1)^2 + 6$ b) $f(x) = 4(x-3)(x+7)$



7. Examine the parabola at the left.
   a) State the direction of opening.
   b) Name the coordinates of the vertex.
   c) List the values of the *x*-intercepts.
   d) State the Domain & Range
   e) If you calculated the second differences, what would their sign be? How do you know?
   f) Determine the algebraic model for this quadratic function.

8. Examine the parabola at the right.
   a) State the direction of opening.
   b) Name the coordinates of the vertex.
   c) What is the equation of the axis of symmetry?
   d) State the Domain & Range of the function.
   e) If you calculated the second differences, what would their sign be? Explain.



9. Each pair of points (x, y) are the same distance from the vertex of their parabola. Determine the equation of the axis of symmetry of each parabola.
   a) (-2, 2), (2, 2) b) (-9, 1), (-5, 1)
   c) (6, 3), (18, 3)

Answers:

1. a) 8 seconds since $h(8) = 0$ b) 60 m ..... c) 64 m
2. a) $a > 0$ so opens up b) $a < 0$ so opens down c) $a < 0$ so opens down d) $a > 0$ so opens up
3. a) zeros: $x = -6, x = 2$ b) $a < 0$ so opens down c) $x = -2$
4. a)

x	-2	-1	0	1	2
$f(x)$	3	4	3	0	-5
Δy	1	-1	-3	-5	
Second Differences	-2	-2	-2		

b) $2a = -2$ so $a = -1$
Parabola opens down so second differences are negative.

5.

Function	Direction of Opening	Vertex	Axis of Symmetry
$f(x) = x^2 - 3$	Up	$(0, -3)$	$x = 0$
$f(x) = -(x+3)^2 - 4$	Down	$(-3, -4)$	$x = -3$
$f(x) = 2(x-4)(x+2)$	Up	$(1,18)$	$x = 1$
$f(x) = -\frac{1}{2}x^2 + 4$	Down	$(0,4)$	$x = 0$

6. a) $f(x) = -3x^2 + 6x + 3$ $y - intercept\ 3$ b) $f(x) = 4x^2 + 16x - 84$ $y - intercept - 84$
7. a) down b) $V(-1,8)$ c) $(-3,0), (1,0)$ d) $D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R}, y \leq 8\}$
   e) negative (opens down) f) $f(x) = -2(x+1)^2 + 8$
8. a) up b) $V(1, -3)$ c) $x = 1$ d) $D: \{x \in \mathbb{R}\}$ $R: \{y \in \mathbb{R}, y \geq -3\}$
   e) positive since parabola opens up
9. a) $x = 0$ b) $x = -7$ c) $x = 12$

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SOLUTIONS

1. $h(t) = -4t^2 + 32t$

a) $-4t(t-8) = 0$

$t = 0$ or $t = 8$

∴ the rocket is in the air 8 seconds (when it hits the ground $h = 0$).

b) $h(3) = -4(3)^2 + 32(3)$

$= -4(9) + 96$

$= 60$

∴ after 3 seconds, the ball is 60m.

c) $h(t) = -4(t^2 - 8t + 16 - 16)$ (OR) axis of symmetry at $x = \frac{0+8}{2}$

$= -4(t-4)^2 + 64$ $x = 4$

↑ t_{max} $h(4) = -4(4)^2 + 32(4)$

$= 64$

∴ the rocket reaches a maximum height of 64m.

2. a) $f(x) = 3x^2$

↑ >0 so ☺ 'smiley' opens up.

b) $f(x) = -2(x-3)(x+1)$

$a = -2$

$a < 0$ ☹ 'frowny' opens down

c) $f(x) = -(x+5)^2 - 1$

$a = -1$

'frowny' ☹ opens down

d) $f(x) = \frac{2}{3}x^2 - 2x - 1$

$a = \frac{2}{3}$ ☺ 'smiley' opens up.

3. $f(x) = -3(x-2)(x+6)$

a) zeros are $x - 2 = 0$, $x + 6 = 0$

$x = 2$ $x = -6$.

b) $a = -3$ ☹ opens down

'frowny'

c) axis of symmetry $x = \frac{2-6}{2}$ $\boxed{x = -2}$

4. a)

	x	$f(x)$	first diff (Δy)	second diff.
Δx	-2	3		
1			1	
	-1	4		-2
1			-1	
	0	3		-2
1			-3	
	1	0		-2
1			-5	
	2	-5		

$2a = -2$
$a = -1$

opens

b) down. so 2nd differences are negative.

	function	direction of opening	Vertex	Axis of Sym.
5. a)	$f(x) = x^2 - 3$	∪ up	$(0,-3)$	$x=0$
b)	$f(x) = -(x+3)^2 - 4$	down	$(-3,-4)$	$x=-3$
c)	$f(x) = 2(x-4)(x+2)$	up	$(1,-18)$	$x=1$
d)	$f(x) = -\frac{1}{2}x^2 + 4$	down	$(0,4)$	$x=0$.

c) $x = \frac{4-2}{2} = 1$ $\quad f(1) = 2(-3)(3) = -18$

6. a) $f(x) = -3(x-1)^2 + 6$
$= -3(x^2 - 2x + 1) + 6$
$= -3x^2 + 6x - 3 + 6$
$= -3x^2 + 6x + 3$

y-int 3.

b) $f(x) = 4(x-3)(x+7)$
$= 4(x^2 + 4x - 21)$
$= 4x^2 + 16x - 84$

y-int -84

7. a) down b) $(-1,8)$ c) $(-3,0)$ $(1,0)$ (x-int's at $-3, 1$).

d) $D: \{x \in \mathbb{R}\}$ $\quad R: \{y \in \mathbb{R} \mid y \leq 8\}$

e) negative (opens down)

(f) $f(x) = a(x+1)^2 + 8$ ↔ (ⓡ)
sub in $(0,6)$
$6 = a(0+1)^2 + 8$
$-2 = a$
$\therefore f(x) = -2(x+1)^2 + 8$

(or Find 'a' by identifying the 'step' factor).

(f) $f(x) = a(x+3)(x-1)$
y-int 6 sub in $x=0, y=6$
$a(0+3)(0-1) = 6$
$-3a = 6$
$a = -2$
$\therefore f(x) = -2(x+3)(x-1)$

U2D3 Worksheet Solutions

8. a) opens up b) V(1, −3) c) x = 1 d) $R = \{y \in \mathbb{R} \mid y \geq -3\}$, $D = \{x \in \mathbb{R}\}$

e) Second differences would be positive since the parabola opens up.

9. a) (−2, 2), (2, 2) $x = \frac{-2+2}{2}$ $\boxed{x = 0}$

b) (−9, 1) (−5, 1) $x = \frac{-9-5}{2}$ $\boxed{x = -7}$

c) (6, 3), (18, 3) $x = \frac{6+18}{2}$ $\boxed{x = 12}$