U2D10 MCR3UI Worksheet Systems of Equations Involving Quadratics

1. Determine the point(s) of intersection algebraically.
   a) $f(x) = -x^2 + 6x - 5,\ g(x) = -4x + 19$
   b) $f(x) = 2x^2 - 1,\ g(x) = 3x + 1$
   c) $f(x) = 3x^2 - 2x - 1,\ g(x) = -x - 6$
2. Determine the number of points of intersection of $f(x) = 4x^2 + x - 3$ and $g(x) = 5x - 4$ without solving.
3. Determine the point(s) of intersection of each pair of functions.
   a) $f(x) = -2x^2 - 5x + 20,\ g(x) = 6x - 1$
   b) $f(x) = 3x^2 - 2,\ g(x) = x + 7$
   c) $f(x) = 5x^2 + x - 2,\ g(x) = -3x - 6$
4. The revenue function for a production by a theatre group is $R(t) = -50t^2 + 300t$, where t is the ticket price in dollars. The cost function for the production is $C(t) = 600 - 50t$. Determine the ticket price that will allow the production to break even.
5. Determine the value of k such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.
6. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.
7. Determine through investigation, the equations of lines that have a slope of 2 and intersect the quadratic function $f(x) = x(x - 6)$
   a) Once b) Twice c) Never
8. Solve algebraically. You may confirm graphically.
   a) $y = 3 - x;\ y = x^2 - 8x + 13$
   b) $g(x) = 4x - 1;\ f(x) = -2x^2 + 4x + 1$
   c) $12x - 4y = 19;\ y = 3x^2 - 12x + 14$
   d) $2x - 3y = -6;\ y = -3x^2 + 24x - 50$
   e) $h(x) = 2x^2 + 3;\ g(x) = x^2 - 2x + 7$
   f) $h(x) = -2x^2 + 24x - 69;\ g(x) = x^2 - 10x + 27$
9. An asteroid is moving in a parabolic arc that is modelled by the function $y = -6x^2 - 370x + 100\,900$. For the period of time that it is in the same area, a space probe is moving along a straight path on the same plane as the asteroid according to the linear equation $y = y = 500x - 83\,024$. A space agency needs to determine if the asteroid will be an issue for the space probe. Wil the two paths intersect?
10. The UV index on a sunny day can be modelled by the function $f(x) = -0.15(x - 13)^2 + 7.6$ where x represents the time of day on a 24-hour clock and $f(x)$ represents the UV index. Between what hours was the UV index greater than 7?
11. A parachutist jumps from an airplane and immediately opens his parachute. His altitude, y, in metres, after t seconds is modelled by the equation $y = -4t + 300$. A second parachutist jumps 5 s later and freefalls for a few seconds. Her altitude, in metres, during this time, is modelled by the equation $y = -4.9(t - 5)^2 + 300$. When does she catch up to the first parachutist?

Answers:

1. a) $\{(4,3), (6,-5)\}$ b) $\left\{(2,7), \left(-\frac{1}{2}, -\frac{1}{2}\right)\right\}$ c) no intersection
2. one
3. a) $\left\{\left(\frac{3}{2}, 8\right), (-7, -43)\right\}$ b) $\left\{\left(\frac{1+\sqrt{109}}{6}, \frac{43+\sqrt{109}}{6}\right), \left(\frac{1-\sqrt{109}}{6}, \frac{43-\sqrt{109}}{6}\right)\right\}$
   c) no intersection d) $\left\{\left(\frac{-7+\sqrt{33}}{8}, \frac{-3+5\sqrt{33}}{8}\right), \left(\frac{-7-\sqrt{33}}{8}, \frac{-3-5\sqrt{33}}{8}\right)\right\}$
4. \$3 or \$4 5. $k = -5$ 6. $k > \frac{73}{12}$
7. a) $y = 2x - 16$ b) $y = 2x + b, b > -16$ c) $y = 2x + b, b < -16$
8. a) $\{(2,1), (5,-2)\}$ b) $\{(1,3), (-1,-5)\}$ c) $\left\{\left(\frac{5}{2}, \frac{11}{4}\right)\right\}$ d) no real solution
   e) $\{(-1+\sqrt{5}, 15-4\sqrt{5}), (-1-\sqrt{5}, 15+4\sqrt{5})\}$ f) $\left\{(6,3), \left(\frac{16}{3}, \frac{19}{9}\right)\right\}$
9. $D > 0$ *so they will intersect.*
10. From 11:00 a.m. until 3:00 p.m.
11. 7.5 seconds after the first parachutist jumps (2.5 seconds after she jumps)