U7D3_T Angle Relationships in Polygons

Thursday, May 3, 2018 2:15 PM

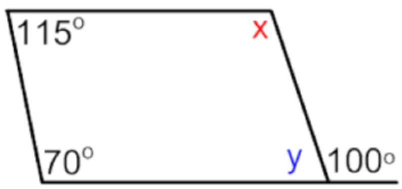
U7D3_T
Angle Rel...

U7D3 MPM1DI

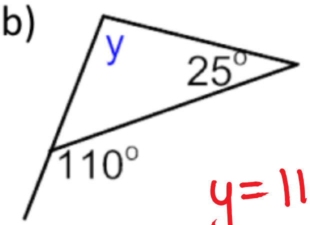
Warm Up:

Find the unknown measure of each angle (make sure you SHOW the justification for your solution):

a)



b)



$y + 25° = 110°$ (EAT)

$y = 110° - 25°$

$y = 85°$

$y = 80°$ (SA)

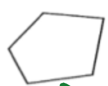
$x = 360° - (115° + 70° + 80°)$ (ASQT)

$x = 95°$

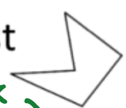
7.3 Angle Relationships in Polygons

Terms:

Convex Polygon: A polygon where all interior angles are less than 180° (obtuse or acute).



Concave Polygon: A polygon where there is at least one interior angle greater than 180°. (reflex angle).



U7D3 MPM1DI

Angles in a Pentagon

1. Make a hypothesis about the sum of interior angles in a pentagon. 540°
2. Draw a pentagon on geogebra.
3. Measure each interior angle (select the angle tool, then click once on the inside of the pentagon).
4. Calculate the sum of interior angles of the pentagon.

poly1 = 4.05

a = 1.86

b = 1.42

c = 1.24

d = 1.85

e = 1.54

α = 80.61°

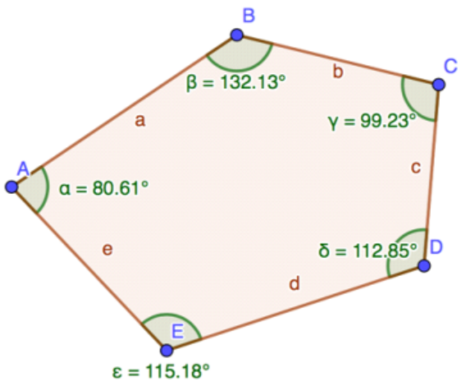
→ β = 132.13°

→ γ = 99.23°

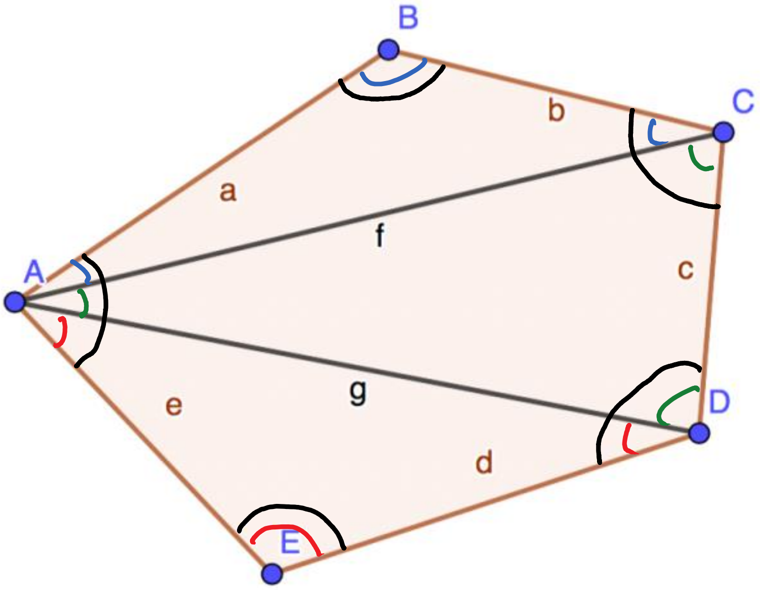
→ δ = 112.85°

→ ε = 115.18°

Sum = 540°



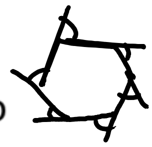
5. Pick one vertex, and draw two diagonals (from your chosen vertex to the two non-adjacent vertices). How many triangles do these diagonals create? 3



6. How do the interior angles of the triangles relate to the interior angles of the pentagon?

$3 \times 180°$
$= 540°$

The sum of all the interior angles in the triangles equals the sum of the interior angles of the pentagon.



7. Now you need to extend each side in order to create an exterior angle at each vertex (remember, to do this you will need to create a ray on each side).
8. Measure each exterior angle (select the angle tool, then click on the two lines that you want to calculate the angle between).
9. Calculate the sum of exterior angles.

I = (-1.74, -1.28)

J = (-4.29, -1.37)

$\zeta = 99.39°$

$\eta = 47.87°$

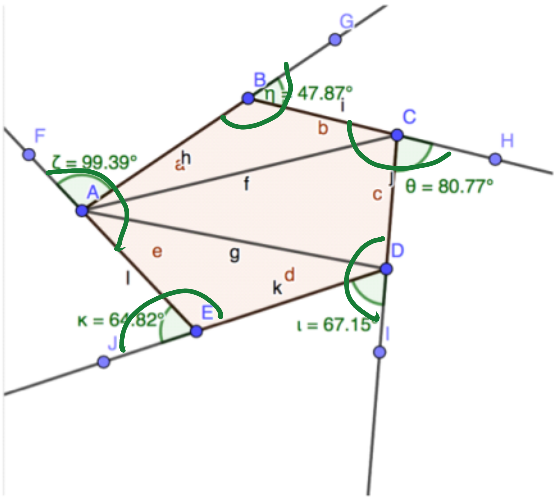
$\theta = 80.77°$

$\iota = 67.15°$

$\kappa = 64.82°$

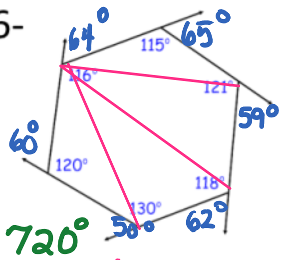
$\lambda = \zeta + \eta + \theta + \iota + \kappa$

Input...



$\lambda = 360°$

Example 1: a) For the following hexagon (6-sided figure), calculate the sum of the interior and exterior angles.



Interior Angles

$116° + 115° + 121° + 118° + 130° + 120° = 720°$

NOTE: 4 Δ's, $4 \times 180° = 720°$

Exterior Angles

$64° + 65° + 59° + 62° + 50° + 60° = 360°$

agrees with 'PEAST'

b) Can you come up with a formula to find the sum of the interior angles for any polygon?

$\text{Sum} = 180°(n-2)$ OR $180°n - 360°$

Example 2: Complete the following chart.

Polygon	Number of Sides	Number of Diagonals from one vertex	Number of Triangles in the Polygon	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	0	1	180°	360°
Quadrilateral	4	1	2	360°	360°
Pentagon	5	2	3	540°	360°
Hexagon	6	3	4	720°	360°
Heptagon	7	4	5	900°	360°
Octagon	8	5	6	1080°	360°
	47	44	45	45×180°	360°
	n	$n-3$	$n-2$	$180°(n-2)$	360°

Example 3: a) Calculate the sum of the interior angles of a decagon (10-sided figure).

$180°(n-2)$
$= 180°(10-2)$
$= 180° \times 8$
$= 1440°$

OR

$180° \times 10 - 360°$
$= 1800° - 360°$
$= 1440°$

b) Determine the measure of *one* interior angle for a regular decagon.

↳ all sides same length
↳ all interior angles same measure.

$1440° \div 10$
$= 144°$

OR if part (a) was not completed first...
each exterior angle is
$360° \div 10 = 36°$
so each interior angle is
$180° - 36° = 144°$.

Example 4: How many sides does a polygon have, if the sum of its interior angles is 1980°?

* dividing by 180° determines how many Δ's are inside the polygon.

$180(n-2) = 1980$

$\frac{180(n-2)}{180} = \frac{1980}{180}$

$n-2 = 11$

$n = 13$

∴ there are 13 sides.

Summary:

Angle Sum Polygon Theorem (ASPT)

To calculate the sum of interior angles for any polygon we use the formula $S_n = 180°(n-2)$, where *n* is the number of sides in the polygon and S_n is the total sum of the interior angles.

Polygon Exterior Angle Sum Theorem (PEAST)

The sum of ***exterior*** angles for ***any*** polygon is always $360°$.

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