MAP 4CI Unit 6 – Algebraic Models **Lesson 1: Exponent Laws**

1. **Multiplication Law:** $x^m \times x^n = x^{m+n}$

When multiplying powers with the same base, keep the base the same and add the exponents.

Ex 1. $x^3 \times x^2$ (Note: $x^3 \times x^2 =$) Ex. 2 $2^3 \times 2^4$

2. **Division Law:** $x^m \div x^n = x^{m-n}$

When dividing powers with the same base, keep the base the same and subtract the exponents.

Ex. 1 $x^5 \div x^2$ Ex. 2 $2^4 \div 2^3$ Note: $2^4 \div 2^3$

$$= \frac{2^4}{2^3}$$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$$

3. **Power of a Power Law:** $(x^m)^n = x^{m \times n}$

If a power is raised to an exponent, **multiply the exponents.**

Ex. $(x^3)^2 =$ NOTE: $(x^3)^2 =$

4. **Power of a Product Law:** $(x \cdot y)^m = x^m y^m$

If a Product is raised to an exponent, distribute the exponent to each factor in the base.

NOTE: This rule does NOT apply to the power of a sum or difference!

Ex. 1 $(x \cdot y)^5 =$ Ex. 2 $(3x^5y^3)^2$

5. Power of a Quotient Law: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

If a Quotient is raised to an exponent, distribute the exponent to every factor in the numerator and denominator.

Ex. 1 $\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$ Ex. 2 $\left(\frac{2}{3}\right)^2$ Ex. 3 $\left(\frac{2x^3}{3y^2}\right)^3$

6. Zero Exponents: $x^0 =$

Any power with an exponent of zero is equal to one.

Ex. 1 $(-2)^0 =$ Ex. 2 $-2^0 = -(2^0)$ Ex. 3 $(-237x^3y^7)^0 =$

Proof:

$3^2 \div 3^2$ $3^2 \div 3^2$

So, $3^0 = 1$

7. Negative Exponents: $x^{-m} = \frac{1}{x^m}$

A negative in the exponent of a power means to 'flip the base' or 'take the reciprocal'. A negative exponent has nothing to do with the sign of the number.

Ex. 1 $x^{-2} =$ Ex. 2 $4^{-2} =$ Ex. 3 $\left(\frac{4}{5}\right)^{-3} =$ Ex. 4 $\left(\frac{1}{3}\right)^{-2} =$

Ex. 4 Simplify first, then evaluate using x = 2.

$(x^{-3})(x^2)(x^5)$ When x = 2,

Pg. 349 - 351 # 1, 3-6,7ac, 9ac, 10 ac, 12 ace, 13 ace

Lesson 2: More Exponent Laws

8. Powers of the form: $x^{\frac{1}{n}}$

The exponent $\frac{1}{n}$ means to take the n[th] root. i.e. $x^{\frac{1}{n}} = \sqrt[n]{x}$

Ex 1. $x^{\frac{1}{2}}$ Ex. 2 $x^{\frac{1}{3}}$ Ex. 3 $x^{\frac{1}{12}}$ Ex. 4 $81^{\frac{1}{2}}$

Ex. 5 $(-27)^{\frac{1}{3}}$ Ex. 6 $(-64)^{\frac{1}{4}}$ Ex. 7 $(64)^{\frac{1}{3}}$ Ex. 8 $(64)^{\frac{1}{6}}$

9. Powers of the form: $x^{\frac{m}{n}}$

The exponent $\frac{m}{n}$ means to take the n[th] root and raise the answer to an exponent m.

i.e. $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{\left(x^m\right)}$

Ex 1. $x^{\frac{3}{4}}$ Ex. 2 $x^{\frac{2}{3}}$ Ex. 3 $81^{\frac{3}{4}}$ Ex. 4 $(-125)^{\frac{2}{3}}$

<u>Quiz next class</u> – no notes!

Pg. 359 # 1 – 8

REVIEW FOR QUIZ

Total: /27

Mrs. Behnke MAP 4CI **Algebraic Models** Name:____________________

1. Simplify using the exponent laws, then evaluate. Give your answer as an integer or a fraction.

/6

a) $5^{-3} \times 5^2 \times 5^4$ b) $4^{17} \div 4^{15}$ c) $\left(10^4\right)^{-1}$

2. Simplify the following exponential expressions using the Exponent Laws (remember: no negative exponents in your answers).

/6

a) $x^5\left(x^3\right)^2$ b) $\dfrac{d^{-5}}{d^2}$ c) $\left(x^{-2} \times x^5\right)^3$

d) $\left(x^8 y^7\right) \div \left(x^4 y^{-1}\right)^2$ e) $\left(3xy^9\right)^3$ f) $\left(-123x^{-137}y^9\right)^0$ g) -5^0

/9

3. Express in radical form, then evaluate exactly.

a) $(-27)^{\frac{1}{3}}$ b) $625^{\frac{3}{4}}$ c) $(9)^{\frac{3}{2}}$

/6

MAP 4CI : Unit 6 Algebraic Models
Unit 6 Lesson 3 – Exponential Equations Part 1

Definition of an Exponential Equation: ____

____

Example : ____________________

____

Methods to Solve

1. Common Base: ____

____

2. Systematic Trial: ____

____

3. Graphing: ____

____

4. Logarithms: ____

____

Unit 6 Lesson 3 (Continued)

Method #1 : Common Base

Looking for a common base:

Express each number as a power

a. 8 as a power of 2.

b. 81 as a power of 9

c. 81 as a power of 3

d. 0.25 as a power of 2

Using a common base to solve exponential equations

- **Step 1 – find common base on both sides of equation.**
- **Step 2 – set exponents equal to each other and solve.**

Solve the following exponential equations

a. $3^x = 3^7$

b. $2^x = 32$

c. $7^{3x-4} = 49$

d. $9^{2x-1} = 27^{3x}$

HW Pages 365-367 #1-7, 10

Unit 6 Lesson 4 **Exponential Equations Part 2**

Method #2 : Systematic Trial

Used when you cannot find a common base:

Example1:

a. solve $3^x = 7$ to 1 decimal place

b. solve $2^{x+1} = 5$ to 1 decimal place

Example 2:

Justin has $1000 in savings to invest, and he wants $1200 to use to buy a new laptop in a few years. If he earns 4.3% per year, compounded annually, the equation that describes Justin's investment is $1200 = 1000(1.043)^n$, where n is the number of years for which the money is invested. Solve the equation for n to determine how long it will take before he can buy the laptop. Round your answer to one decimal place.

Unit 6 Lesson 4 **Exponential Equations Part 2**

Method #3 : Graphing

This is the least accurate of the three methods. It can be used when you cannot calculate rational or decimal exponents.

Method – create a table of values, graph and estimate the solution.

Example:

a. $3^x = 35$

x	y

HW: Pg 373 – 374 # 3, 4, 8, 9, 10

MAP 4CI Unit 6 Lesson 5 <u>Construct and Apply Exponential Models</u>

Method – create a table of values, graph and estimate the solution.

Example 1: Simple and Compound Interest

Jason has \$500 to invest and is considering two investment options.

- Option A: A treasury bond that pays 8% simple interest. The amount, *A*, after *n* years is given by the equation $A = 500 + 40n$
- Option B: A savings account that pays 6.5% per year, compounded annually. The amount, *A*, after *n* years is given by the equation $A = 500(1.065)^n$

a) Graph each relation on the same set of axes. **Use** Desmos **to help you.** Describe each relation.

b) Compare the options. Which is the better investment? Why?

n	A_1	A_2

Unit 6 Lesson 5 (continued)

Example 2: Half-life.

An important property of a radioactive substance is its **half-life**, the time it takes for a radioactive sample to decay to half its original mass. For example, iodine-131 is a radioactive substance with a half-life of eight days. This material is commonly used for thyroid analysis.

a) Complete the table of values for an initial dose of 100 units of iodine-131.

Time (Days)	Units Remaining in the Bloodstream	First Differences	Second Differences	Percent Differences (Ratios)

b) Is this relation linear or non-linear? Is this relation exponential? Explain.

c) Construct a scatter plot of the data. Does the trend confirm your answer to part b? Explain.

d) Determine an equation for the curve of best fit. ____________________

e) Determine how long it will take for the initial dose of iodine-131 to decay to one unit.__________

****Practice: Pg. 385–387 # 1 – 4, 7 ✓Answers Pg. 560**

Unit 6 Day 7

MAP4CI – Algebraic Models Review

A. Simplifying and Evaluating Exponents

1. Simplify, with no negative exponents:

a. $(m^5)(m^2)$ b. $t^4 \div t$ c. $(x^5)^3$ d. $\left(\dfrac{x}{y}\right)^{-3}$ e. $-(-x)^0$ f. m^{-2}

2. Evaluate the following when c=5 and d=-3.

a. c^2d^3 b. $\dfrac{c^2d^3}{c^4d}$ c. $\dfrac{4c^{1/2}d}{c^{3/2}}$ d. $c^{-1}d^2 \times c^3 \div c^2$

3. Evaluate, round to nearest 1000th if necessary.

a. $27^{\frac{2}{3}}$ b. $\left(\dfrac{36}{121}\right)^{\frac{3}{2}}$ c. $2.1^{-1.6}$

4a. Write in radical form: i. $a^{\frac{1}{3}}$ ii. $a^{\frac{2}{3}}$ iii. $a^{-\frac{1}{5}}$

4b. Write in exponential form: i. $\sqrt{x}$ ii. $\sqrt[3]{x^2}$ iii. $\dfrac{1}{\sqrt[4]{a}}$

5. The formula $B = 0.4089M^{\frac{3}{4}}$ gives the bird inhalation rate, B (cubic metres of air per day) for a bird with mass M (kilograms).
a. rewrite the formula using radicals
b. calculate the inhalation rate for a 4.5 kg bald eagle and a 8.0 kg Canada goose.
c. Determine the mass of a bird whose inhalation rate is twice that of a bald eagle.

B. Exponential Equations

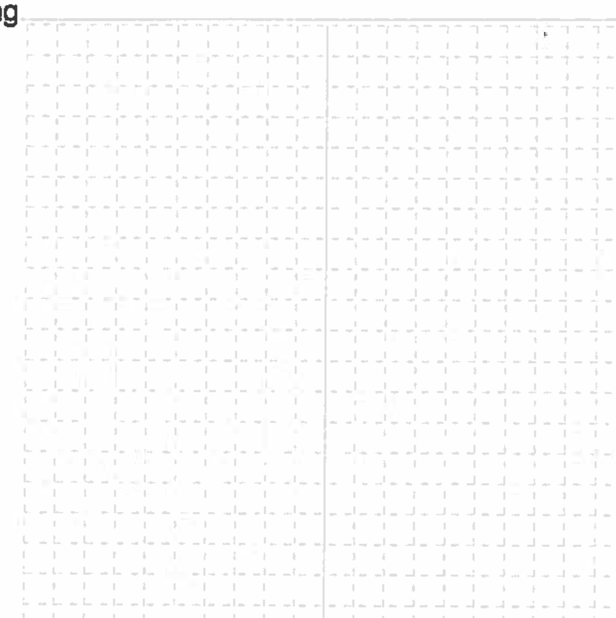
6. Solve the following equations algebraically (using common base). Check your answers.

a. $4^{2x} = 4^6$ b. $5^x = 625$ c. $3^{2x+1} = 9$
d. $10^{x+1} = 10^{2x-3}$ e. $4^{3x-2} = 32^{x+1}$ f. $25^{x+1} = 125^{x-2}$

7. Determine the value of y to the nearest tenth, using systematic trial.

a. $10^y = 125$ b. $3^y = 6$ c. $250(1.03)^y = 400$

8. In the equation $3^{z+1} = 99$ Solve for z by graphing



Unit 6 Day 7

C. Application Problems (Exponential Models)

9. The amount of medicine A remaining in a body after t hours can be calculated using the formula $A = 300(0.8)^t$.
a. Calculate the amount of medicine remaining in a body after 3 hours.
b. Determine the time it takes (to the nearest hour) so that there is only 1 mg of medicine remaining in a body.

10. \$1500 was invested for 2 years in an account that pays interest compounded annually. What was the interest rate if the investment was worth \$1800 after two years? Use the formula $A = P(1+i)^n$.

11. \$25000 was invested in an account that pays 5.0% interest compounded annually. How many years was the money in the account if the investment was worth \$28500 at the end of the term? (Hint – use systematic trial or graphing to solve this problem).

12. A ball is dropped and bounces several times, losing some of its rebound height after each bounce. The height reached, h, in metres, after n bounces is given by the equation
    $h=1.5(0.75)^n$.
    a) Graph the relation and describe the trend.
    b) What is the maximum height after i) the first bounce?
    ii) the second bounce?
    iii) the third bounce?

Answers:

1.a. m^7, **b.** t^3, **c.** x^{15}, **d.** $\frac{y^3}{x^3}$, **e.** -1, **f.** $\frac{1}{m^2}$,
2.a. -675, **b.** 0.36, **c.** -2.4, **d.**9,
3.a. 9, **b.** $\frac{216}{1331} = 0.1623$, **c.** 0.3051,
4a. i. $\sqrt[3]{a}$, ii.. $(\sqrt[3]{a})^2$ iii. $\frac{1}{\sqrt[5]{a}}$ **4b.** i. $x^{1/2}$, ii. $x^{2/3}$ iii. $\frac{1}{a^4}$
5a. $B = 0.4089\sqrt[4]{M^3}$, **b.** 1.26, 1.94, **c.** 11.34kg,
6a. 3, **b.** 4, **c.** 0.5, **d.** 4, **e.** 9, **f.** 8,
7a. 2.1, **b.** 1.6, **c.** 15.9
8. $z \approx 3.2$,
9a. 153.6, **b.** 25.56hrs
10. $i \approx 0.095$ or 9.5%,

11. $n \approx 2.7$ years

12. a) The height of successive bounces is decreasing exponentially. (As the number of bounces increases, the height decreases exponentially.)

b) i) 1.125 m ii) 0.84 m iii) 0.63 m

Also try, Pg. 390-391 # 1ac, 2ac, 3, 4a, 5, 6ab, 7abcd, 8, 9ace, 10, 12, 15, 16, 17