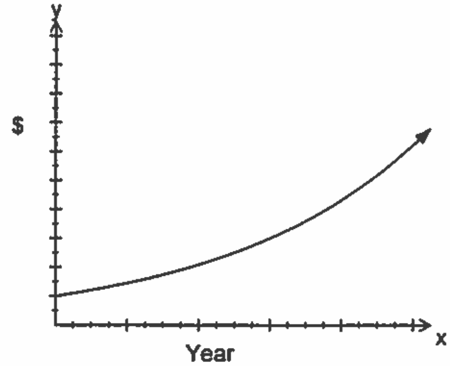
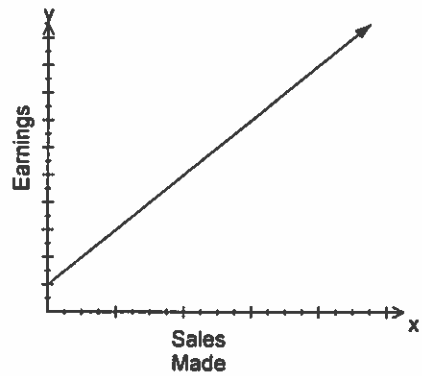
MAP4CI UNIT 5 GRAPHICAL MODELS Lesson 1.

C: DESCRIBING RELATIONSHIPS IN GRAPHS

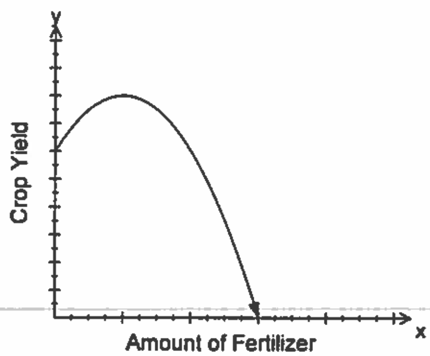
UNIT 5 TEST DATE:



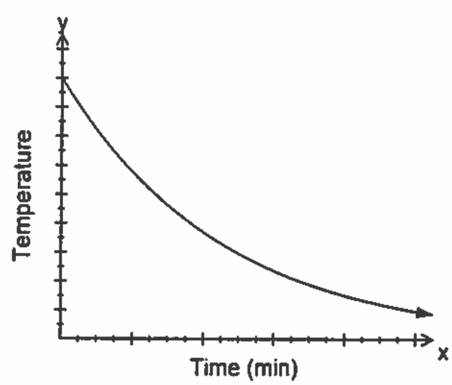
1. Value of a Compound Interest Investment



2. Jack's earnings at the sporting goods store



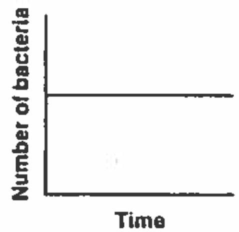
3. Crop yield versus amount of fertilizer used.

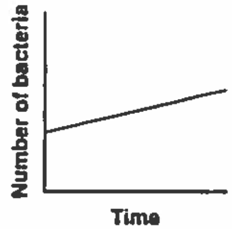


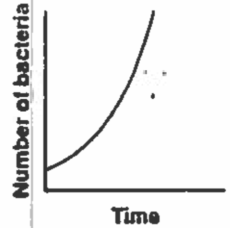
4. Temperature of a cooling coffee cup.

In questions 1 to 4, choose the graph that best represents the given description. Justify your choice.

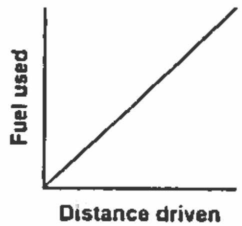
1. The number of bacteria in a laboratory colony increases over time, slowly at first and then more rapidly.

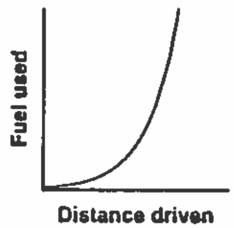
a)


b)


c)


2. The fuel used increases steadily as the distance driven increases.

a)


b)


c)


3. As the price increases, the revenue earned increases, reaches a maximum, then decreases.

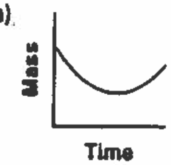
a)

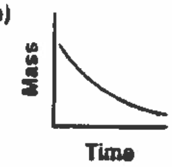

b)

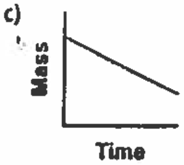

c)

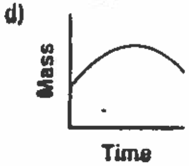

d)


4. The radioactive substance decayed over time, rapidly at first and more slowly.

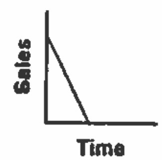
a)


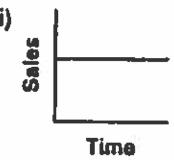
b)


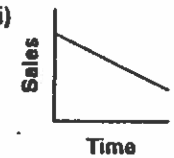
c)


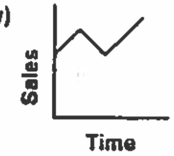
d)


5. Match each graph with the statement that best describes it. Which words gave clues about the shape of the graph?

i)


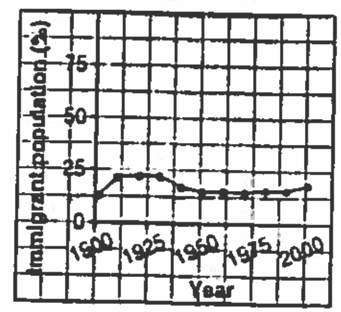
ii)


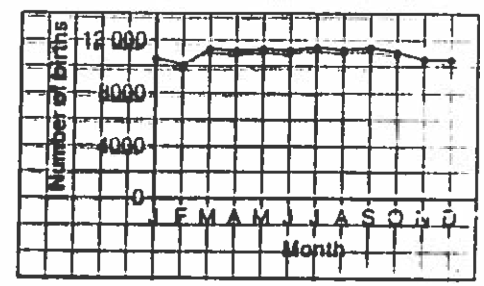
iii)


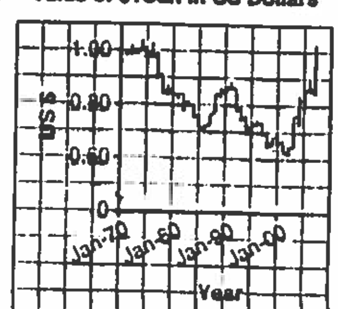
iv)


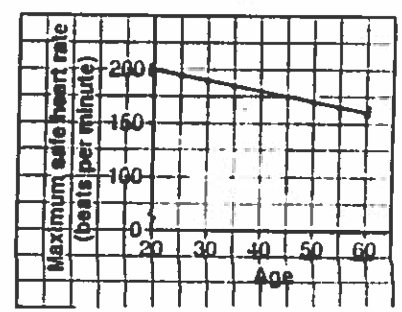
a) Sales have fallen dramatically over the last year.
b) Sales have fallen steadily over the last year.
c) Sales have remained constant over the last year.
d) Sales have fluctuated over the last year.

6. Describe the trends in each graph.

a) Percent of Immigrants in the Population of Canada


b) Number of Births in Each Month in 2004


c) Value of $1Can in US Dollars


d) Maximum Safe Heart Rate During Exercise


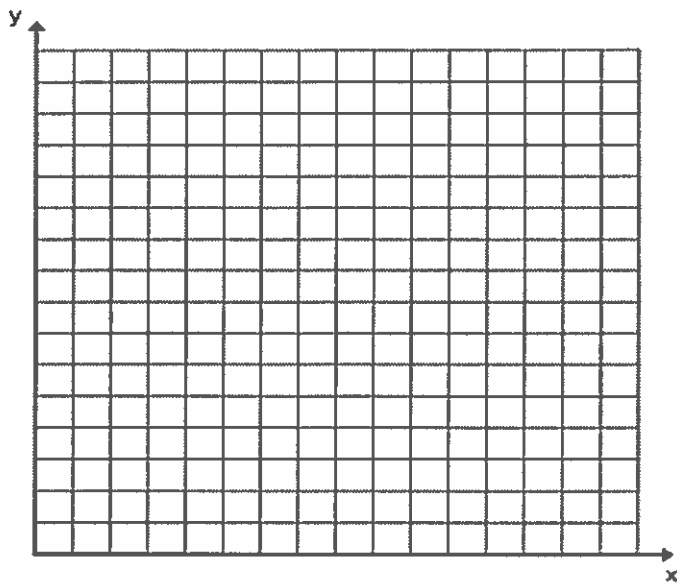
MAP 4CI Unit 5 lesson 2   <u>5.1 LINEAR MODELS</u>   Date:____________________

Rate of change____

____

Ex.1 Danika drove at a constant speed from Peterborough to Ottawa.
The table shows the distance she travelled over time.

Time (h)	Distance (km)	Rate of Change
0.0	0	
0.5	42	
1.0	84	
1.5	126	
2.0	168	
2.5	210	
3.0	252	
3.5	294	



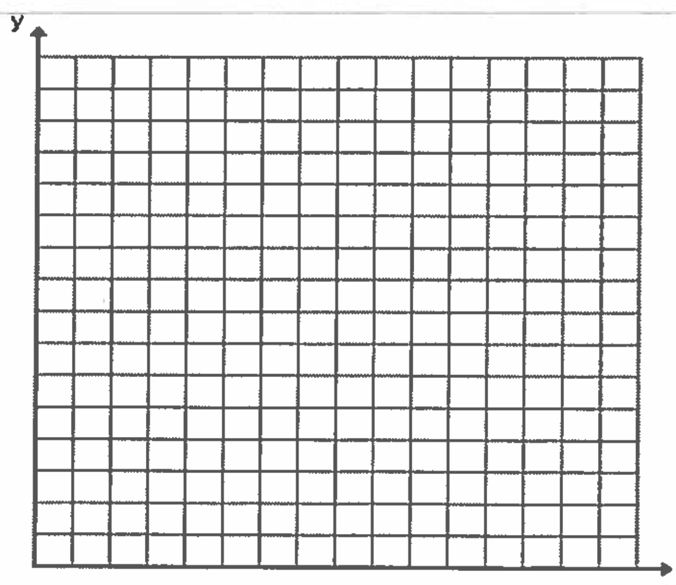
a) Draw a graph of the data. Describe the shape of the graph.

b) Does the rate of change appear to be increasing, constant, or decreasing?

c) Determine the rate of change. Include appropriate units.

Ex. 2 **Linear Regression**
The table shows how the mass of a liquid is related to its volume.
a) Use a graphing calculator to graph the data or DESMOS.
b) Use linear regression to determine the equation of the line of best fit.

Volume (mL)	Mass (g)
0	90
25	110
50	129
75	148
100	168
125	188
150	207



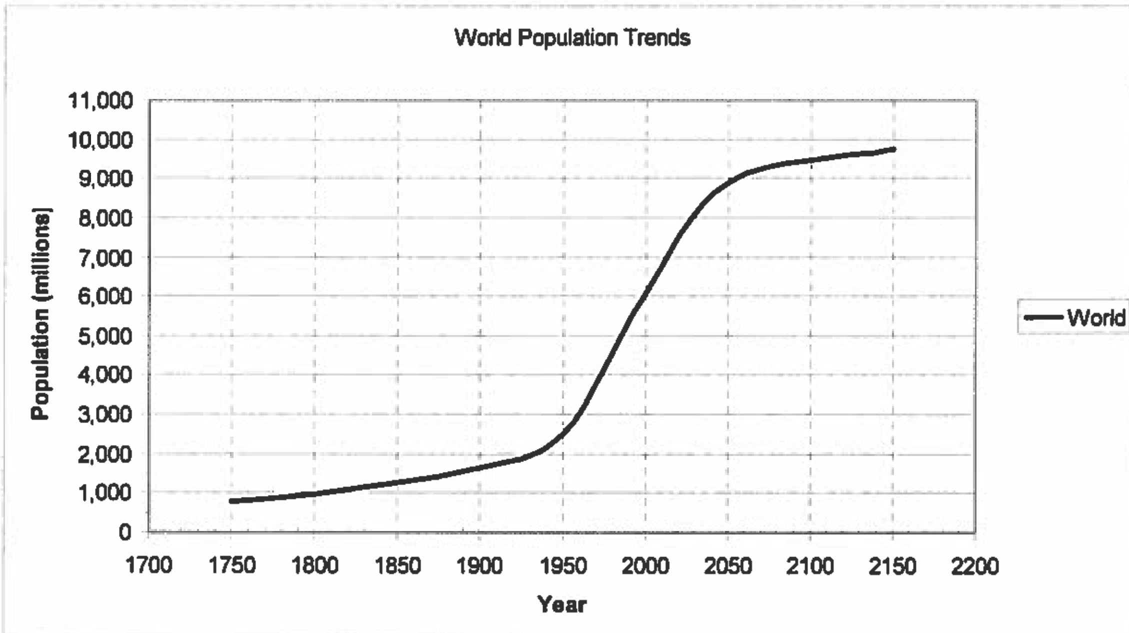
Ex.3 Complete Pg 277 # 6 using a graphing calculator

Practice: Pg 278 # 8, 9, 11 Pg. 275 # 1-5, 7

Unit 5 Lesson 3 :

Rates of Change

The average rate of change = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$



1. What is the average rate of change over the following periods:
   a. 1750 – 1950, b. 1950 – 2050, c. 2050 – 2150?

WHO GROWTH CHARTS FOR CANADA

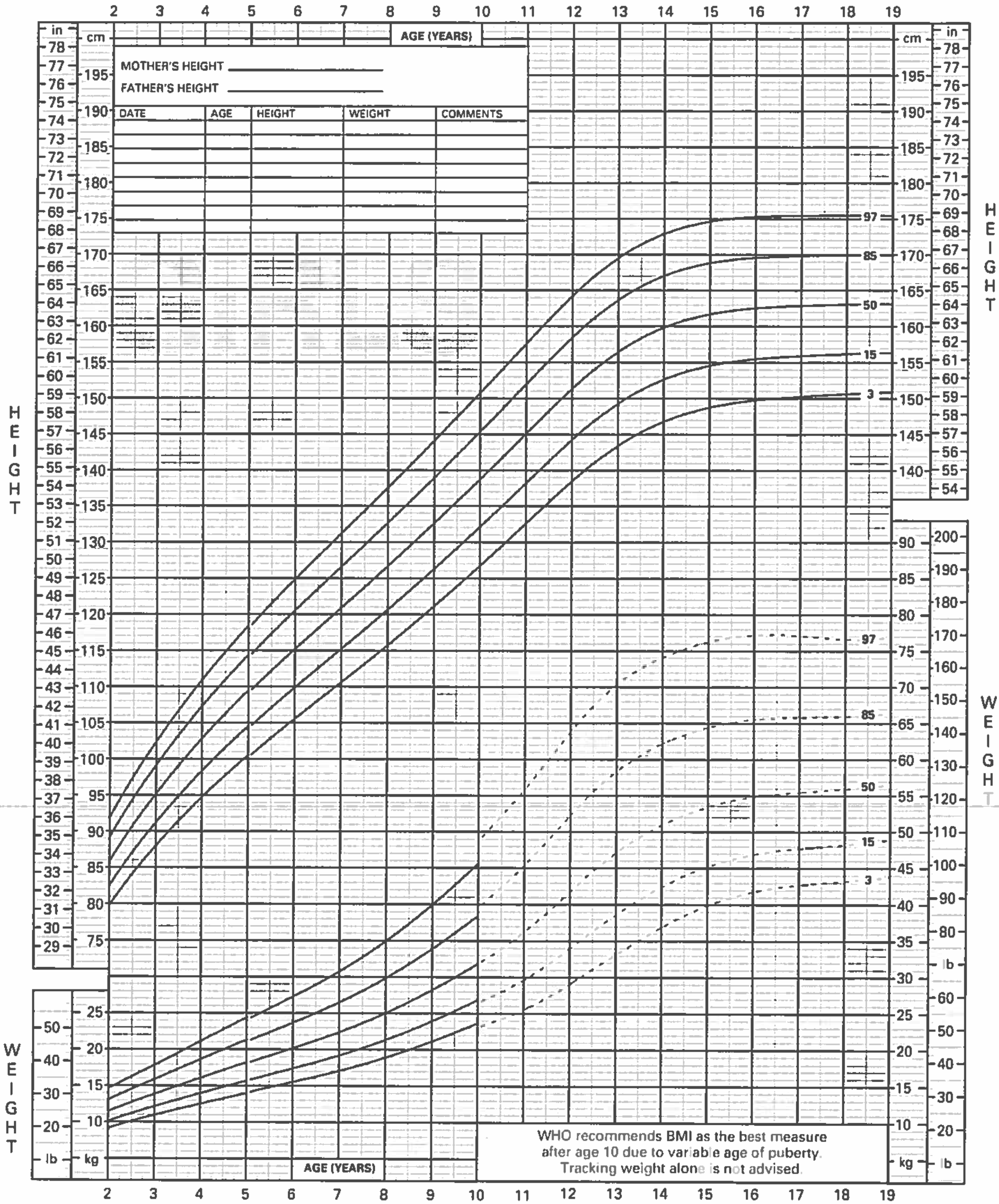
GIRLS

2 TO 19 YEARS: GIRLS

Height-for-age and Weight-for-age percentiles

NAME: ____________________

DOB: ____________ **RECORD #** ________



SOURCE: The main chart is based on World Health Organization (WHO) Child Growth Standards (2006) and WHO Reference (2007) adapted for Canada by Canadian Paediatric Society, Canadian Pediatric Endocrine Group (CPEG), College of Family Physicians of Canada, Community Health Nurses of Canada and Dietitians of Canada. The weight-for-age 10 to 19 years section was developed by CPEG based on data from the US National Center for Health Statistics using the same procedures as the WHO growth charts.

WHO GROWTH CHARTS FOR CANADA

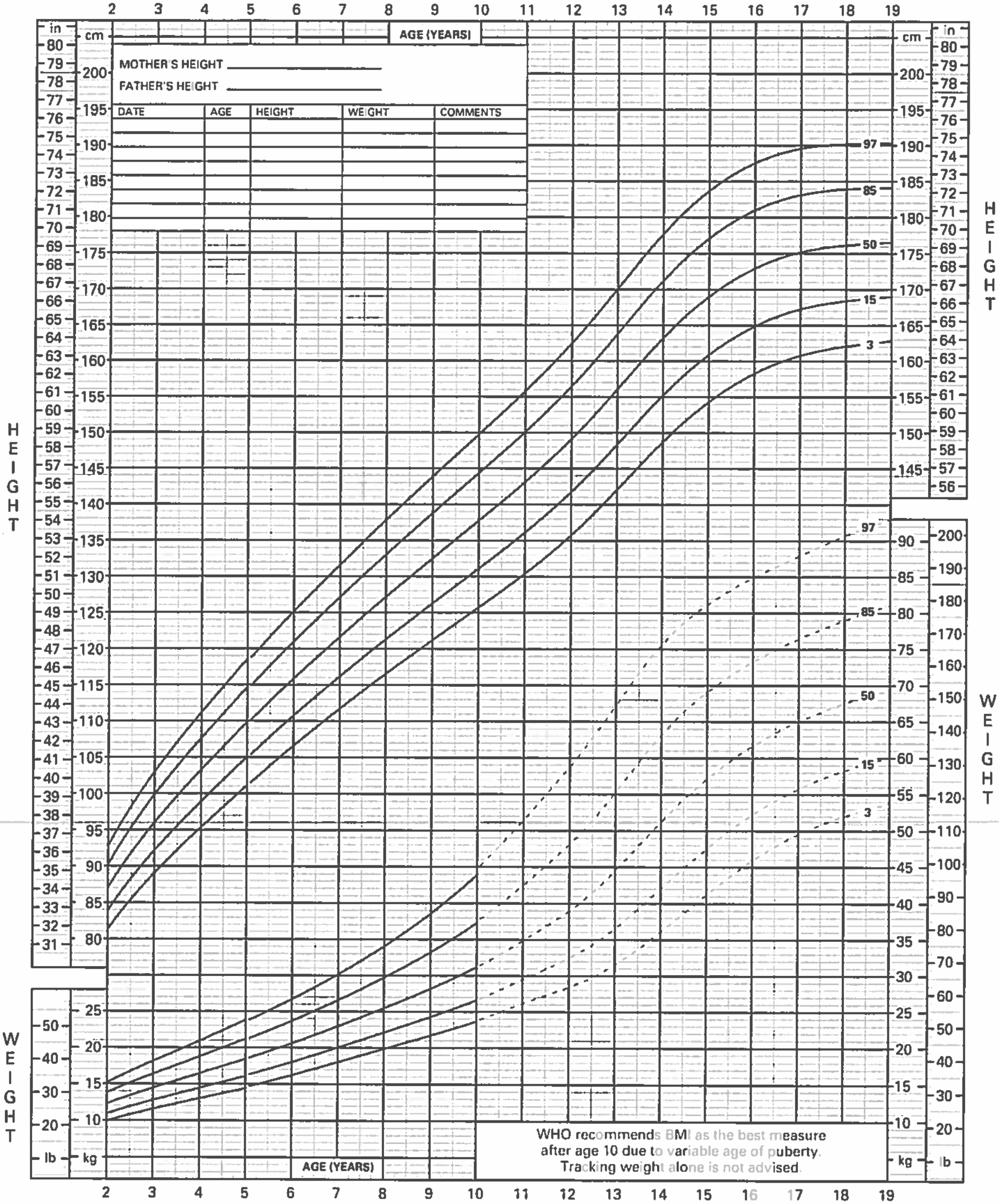
BOYS

2 TO 19 YEARS: BOYS

Height-for-age and Weight-for-age percentiles

NAME: ______________________

DOB: ______________ **RECORD #** ________



SOURCE: The main chart is based on World Health Organization (WHO) Child Growth Standards (2006) and WHO Reference (2007) adapted for Canada by Canadian Paediatric Society, Canadian Pediatric Endocrine Group (CPEG), College of Family Physicians of Canada, Community Health Nurses of Canada and Dietitians of Canada. The weight-for-age 10 to 19 years section was developed by CPEG based on data from the US National Center for Health Statistics using the same procedures as the WHO growth charts.

 www.whogrowthcharts.ca

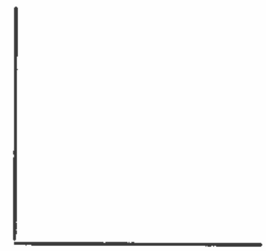
Unit 5 lesson 4: Review: Linear Relations

- A linear relationship means ______________________________________.
- To calculate First Differences, the independent variable (x-values) must be increasing or decreasing by ____________________________.
- The First Differences are the ____________________ ( ).
- Points on the graph lie along a____________________.

If rate of change is positive, the quantity is ______________.

If rate of change is negative, the quantity is ______________.

If rate of change is zero, the quantity is ______________.



The rate of change of a linear relation is ______________. For example, a car travelling at a constant speed will travel equal distances over equal time intervals.

... next Quadratic Model example (see next page)

Summary:

- Finite Differences = First and Second Differences
- To use Finite Differences the x-values must be increasing or decreasing by the same amount.
- If the First Differences are not constant, the relation is ________________.
- If the Second Differences are constant, it is a ________________________________.
- You can use quadratic regression on a graphing calculator to find the ______________ of the ______________.
- You can use an equation that models the data set to ________________________ about the data.

2. Calculate the first and second differences. Then, determine if each relation is linear, quadratic, or neither.

a)

x	y	First Differences	Second Differences
−1	16		
0	14		
1	8		
2	−2		
3	−16		

b)

x	y	First Differences	Second Differences
0	1		
1	2		
2	4		
3	8		
4	16		

c)

x	y	First Differences	Second Differences
−2	3		
−1	0		
0	−2		
1	−3		
2	−3		

d)

x	y	First Differences	Second Differences
−4	−1		
0	2		
4	5		
8	8		
12	11		

Practice: Pg. 289 # 1, 2, 3, 8 abcd CHECK Answers Pg. 553-554

Unit 5 Lesson 4

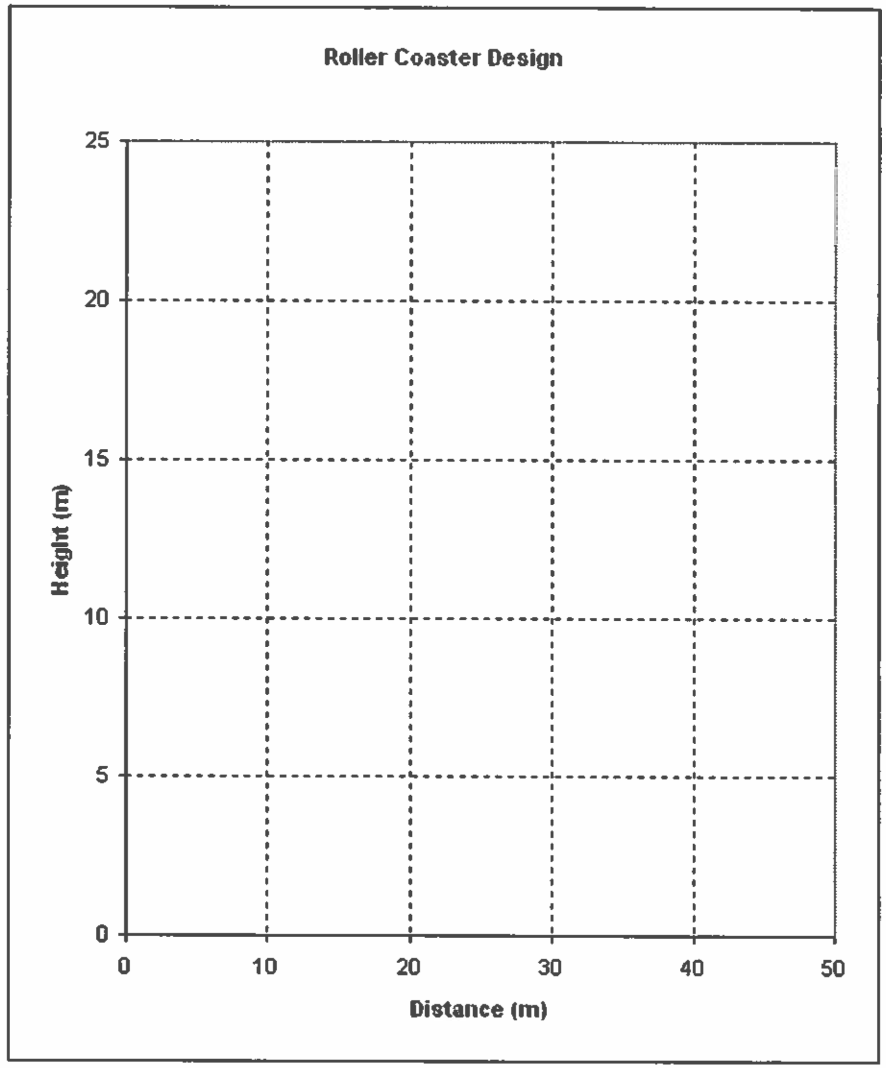
Quadratic Model : Roller Coaster

Distance (m)	Height (m)	1st Differences	2nd Differences
0	21		
5	14		
10	9		
15	6		
20	5		
25	6		
30	9		
35	14		
40	21		

1. Calculate 1st and 2nd differences.
2. Is this relationship quadratic, how can you tell?

3. Plot the data on the given graph.
4. Where is the roller coaster closest to the ground?

MAP4CI : Graphical Models



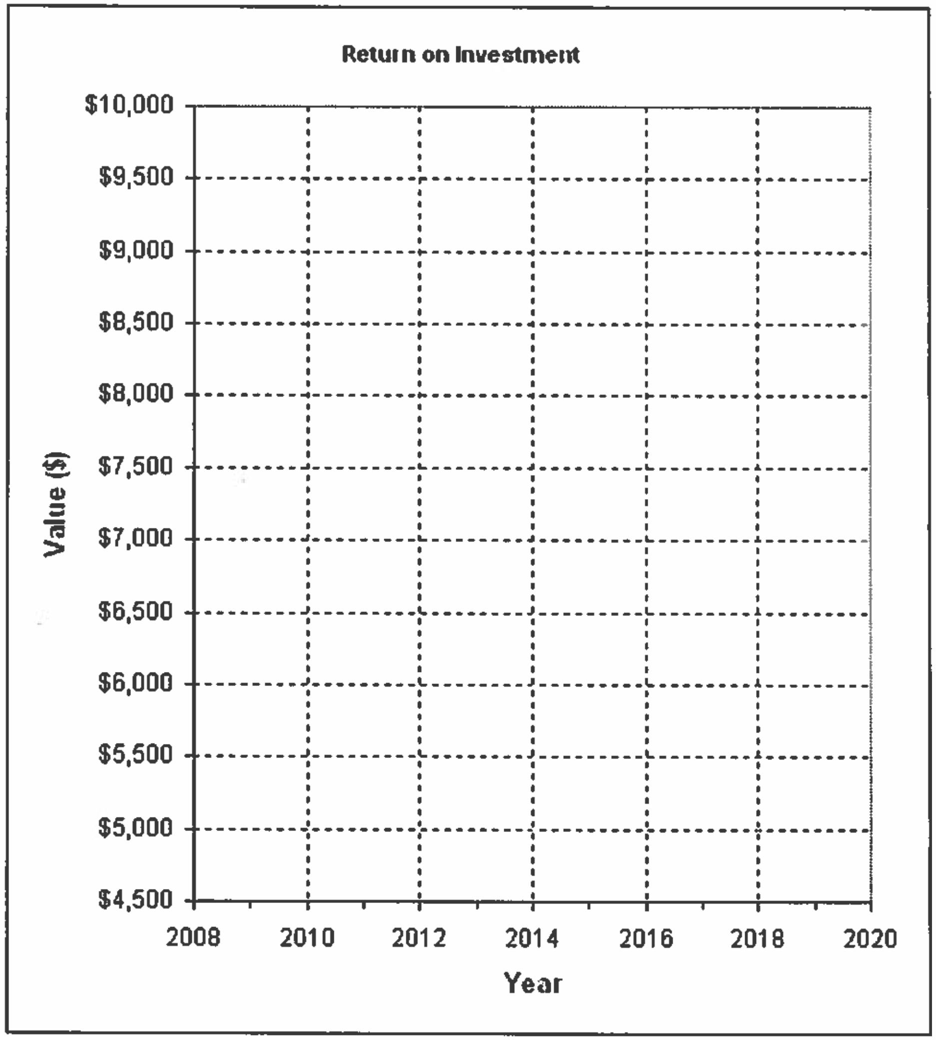
Unit 5 lesson 5

MAP4CI : Graphical Models

Exponential Model (Growth) - Return on Investment

You have invested $5000 in a GIC that earns 6.5% per year compounded annually. Complete the table below and graph the amount your investment is worth at the end of each year.

Date	Principle ($)	Ratio
2008	$5,000	
2009		
2010		
2011		
2012		
2013		
2014		
2015		
2016		



$y = ab^x$

1. Is this relationship exponential?

How can you tell?

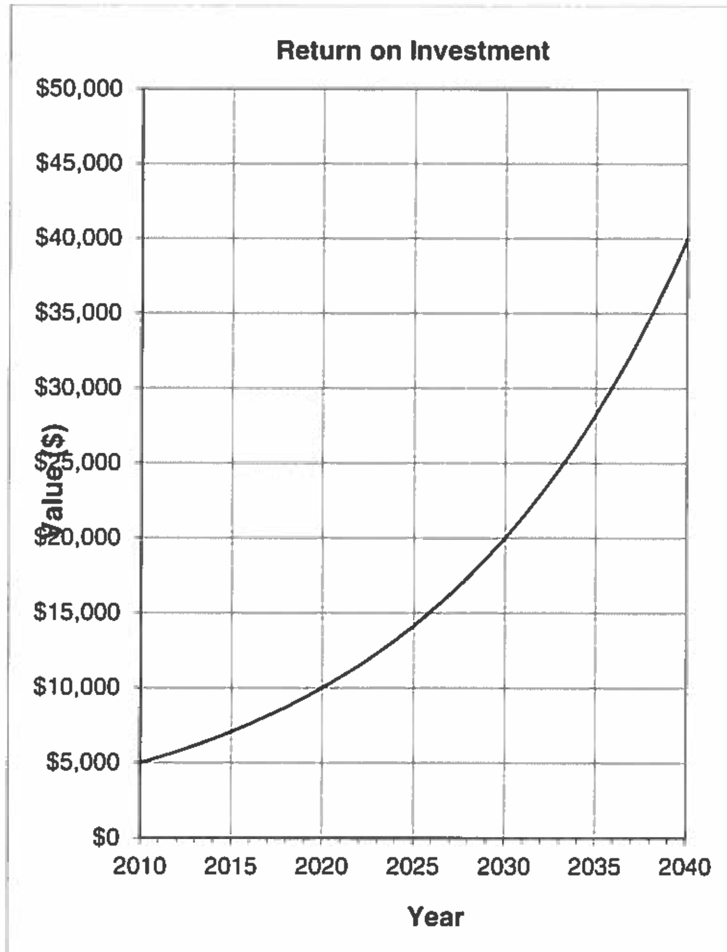
2. How long will it take for your investment to double in value?

U5D5 HW: pg. 301 #1-5

UNIT 5 Day 6:
Exponential Growth and Doubling Time

Doubling time refers to the amount of time for a quantity to double in value. For exponential relations, this doubling time is a constant value.

Date	Principle ($)
2010	$5,000
2011	$5,359
2012	$5,743
2013	$6,156
2014	$6,598
2015	$7,071
2016	$7,579
2017	$8,123
2018	$8,706
2019	$9,330
2020	$10,000
2021	$10,718
2022	$11,487
2023	$12,311
2024	$13,195
2025	$14,142
2026	$15,157
2027	$16,245
2028	$17,411
2029	$18,661
2030	$20,000
2031	$21,435
2032	$22,974
2033	$24,623
2034	$26,390
2035	$28,284
2036	$30,314
2037	$32,490
2038	$34,822
2039	$37,321
2040	$40,000



1. What is the doubling time for this investment?

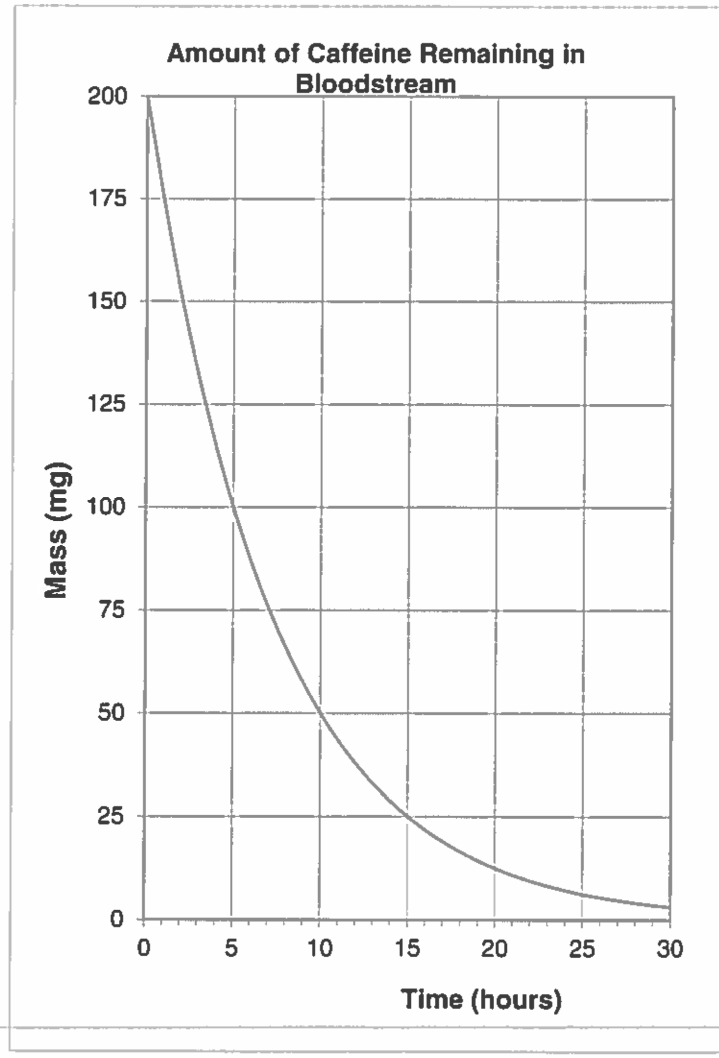
2. What is the multiplying factor for this investment?

3. What is the annual rate of return (percentage)?

Exponential Decay and Half Life Time

Half Life time refers to the amount of time for a quantity to divide in half (multiply by 0.5).
For exponential relations, the half life is a constant value.

Time (hrs)	Caffeine (mg)
0	200.00
1	174.11
2	151.57
3	131.95
4	114.87
5	100.00
6	87.06
7	75.79
8	65.98
9	57.43
10	50.00
11	43.53
12	37.89
13	32.99
14	28.72
15	25.00
16	21.76
17	18.95
18	16.49
19	14.36
20	12.50
21	10.88
22	9.47
23	8.25
24	7.18
25	6.25
26	5.44
27	4.74
28	4.12
29	3.59
30	3.12



Note : 1 small cup of coffee contains approximately 100 mg of caffeine

1. What is the half life for caffeine in the bloodstream?

2. What is the decay factor for caffeine in the bloodstream?

3. What is the percent decrease per hour for caffeine?

Worksheet: Exponential Models

1. According to a historical account, a servant asks to be rewarded by receiving one grain of rice for the first square of a checkerboard, two grains for the second square, four grains for the third square; the amount doubles with each square on the checkerboard.

a) How many grains of rice would the servant receive for the tenth square of the checkerboard?

b) Explain why an exponential model can be used for the relationship between the square on the checkerboard and the number of grains of rice.

2. Does each number sequence represent exponential growth? Explain.

a) 10, 100, 1000, 10000

b) 3, 6, 9, 12

c) ¼, ½ , 1, 2, 4

3. Calculate the finite differences and the ratios. Is each relation linear, quadratic or exponential?

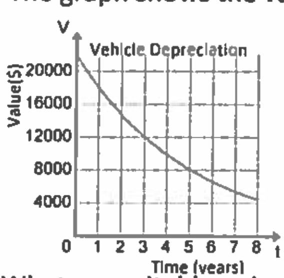
a)

x	y	First Differences	Second Differences	Ratios
-3	-64			
-2	-16			
-1	-4			
0	-1			
1	-1/4			
2	-1/16			

b)

x	y	First Differences	Second Differences	Ratios
-2	6.0			
-1	4.9			
0	3.8			
1	2.7			
2	1.6			
3	0.5			

4. The graph shows the value of the vehicle over time.



a) Describe the relationship between time and the value of the vehicle.

b) Estimate the value of the vehicle after 4 years.

c) Describe the rate of change of the value of the vehicle with respect to time.

d) What are suitable units for the rate of change of the value of the vehicle with respect to time?

e) Is the rate of change of the value of the vehicle with respect to time increasing, constant, or decreasing? Explain.

Unit 5 Lesson 8 HW: pg. 316 #1, 2, 3, 5, 8

Date: ____________

Analyse Graphical Models

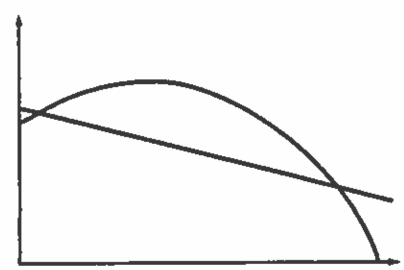
Section 5.4

1. Match each situation with its graph.
   a) The number of bacteria in Colony X remained the same over time. Colony Y started with 50 bacteria and doubled every half-hour.

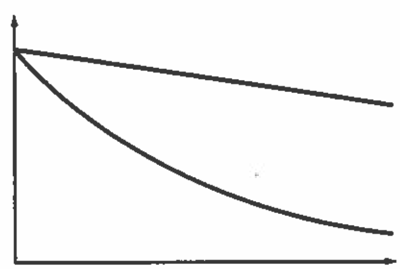
   b) Two cups of water were cooled in different controlled environments. Cup X cooled at a constant rate. The temperature of Cup Y decreased by one-half every 20 min.

   c) Ball X rolled down a ramp. Ball Y was thrown from a point above the ground.

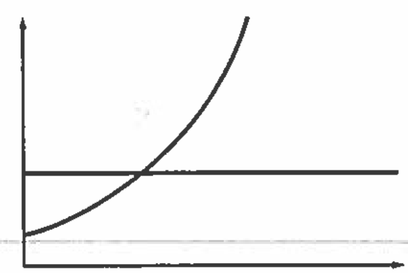
A



B



C



2. The population of Town X started at 90 000 and increased by 25 000 every year. The population of Town Y started at 4000 and doubled every year. Which statement is true?

   A The population of Town X is always greater than the population of Town Y.

   B The rate of change of the population of Town X is increasing.

   C The rate of change of the population of Town Y is increasing.

   D The population of Town Y is greater than the population of Town X after 4 years.

Date: ____________

Section 5.4

3. Refer to question 2.

a) Complete the table of values.

Year	Town X Population	Town Y Population
0		
1		
2		
3		
4		
5		

b) Determine an equation to model the population of each town.

c) In what year is the population of Town Y greater than the population of Town X?

4. Ing has the choice of two payment options for her new job.

Option A: Starting salary of $48 000, with a $1000 raise every following year.

Option B: Starting salary of $45 000, with a 2.5% raise every following year.

a) Complete the table of values.

Year	Option A Salary ($)	Option B Salary ($)
0		
1		
2		
3		
4		
5		
6		
7		

b) Which option should Ing choose? Why?

Unit 5 Day 8: Analyzing Real Life Data – Population of British Columbia. The data given below is the population of British Columbia since 1921.

Year	Pop'n (millions) actual	1st Diff	2nd Dif		Ratio
1921	0.52				
1931	0.69				
1941	0.82				
1951	1.17				
1961	1.63				
1971	2.18				
1981	2.82				
1991	3.37				
2001	4.08				

1. Complete the table above filling in the first and second differences and the ratios.
2. Which model fits the data the best?

Regression Analysis: The regression equations for this data are given by:

linear $y=0.0455x+.1007$

quadratic $y=0.0004x^2+0.0121x+0.4901$

exponential $y=0.5252(1.02718)^x$

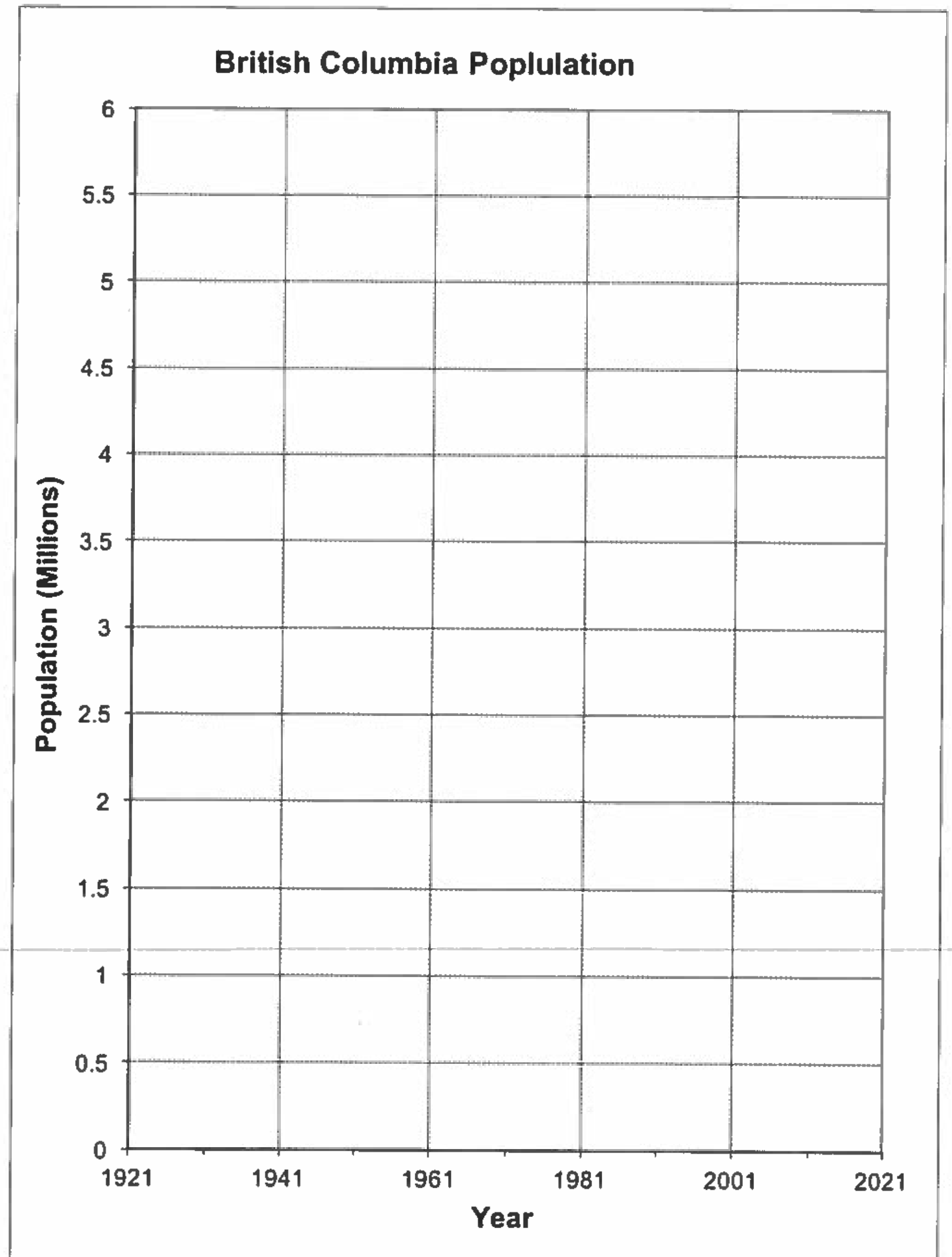
where x is the number of years since 1921.

3. Calculate the population for each year, using the given regression equations.

Year	Actual Pop	Calculated Population		
		Linear	Quadratic	Exponential
1921	0.52			
1931	0.69			
1941	0.82			
1951	1.17			
1961	1.63			
1971	2.18			
1981	2.82			
1981	3.37			
2001	4.08			
2011	est			

4. Estimate the population of BC in 2011 based on the data, then look up the actual 2011 population on-line.

5. Plot the actual population and the calculated populations on the same graph.

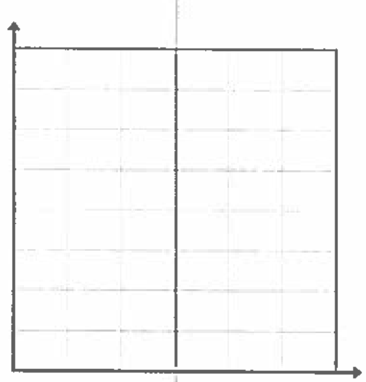


6. Which equation best describes the relationship how the population of BC changes over time?

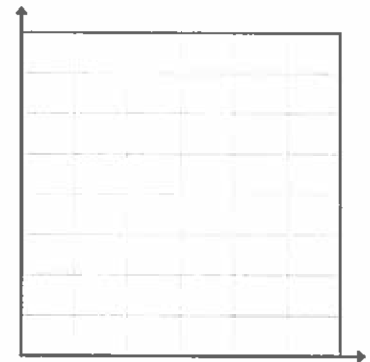
Summary: Rate of Change =

Compare distance-time graphs (for equal time intervals) for a car that is:

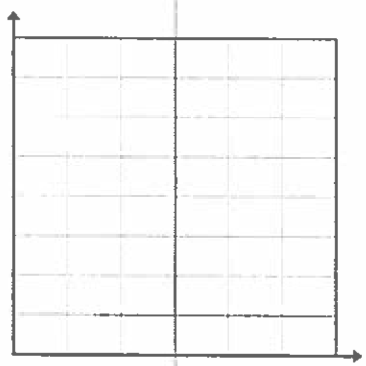
a) moving at a constant speed   rate of change is ____________



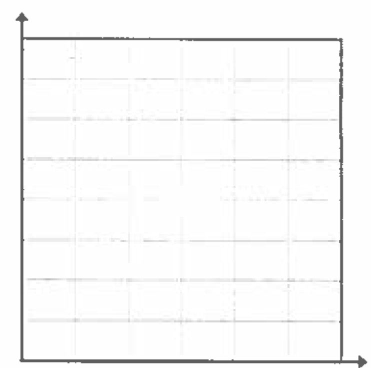
b) accelerating   rate of change is ____________



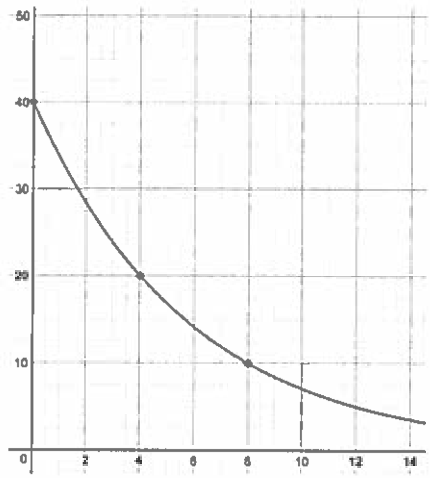
c) stopped (eg. 20 km from home) rate of change is ____________



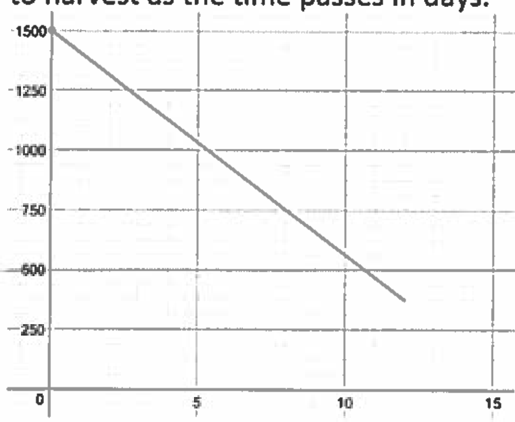
d) slowing down   rate of change is ____________



1. Identify the type of relationship (linear, quadratic, or exponential) of the following graph. Explain how you got your answer.

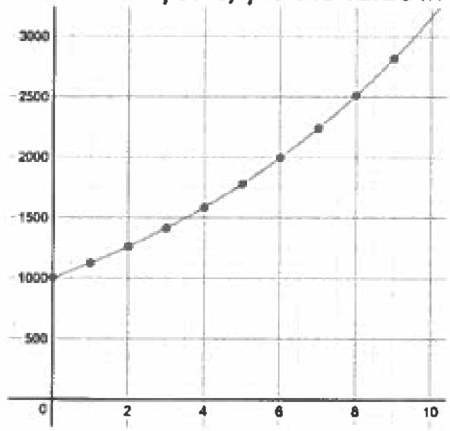


2. Walter grows hay on 1500 acres of land. Hay is harvested in the summer. The graph shows the number of acres Walter has left to harvest as the time passes in days.



a) Describe the relationship between the area Walter has left to harvest and the number of days.
b) Use the graph to estimate the area remaining after four days.
c) By how much does the area decrease each day?
d) What are suitable units for the rate of change of area remaining with respect to the number of days?
e) Predict the number of days required to complete the harvest.

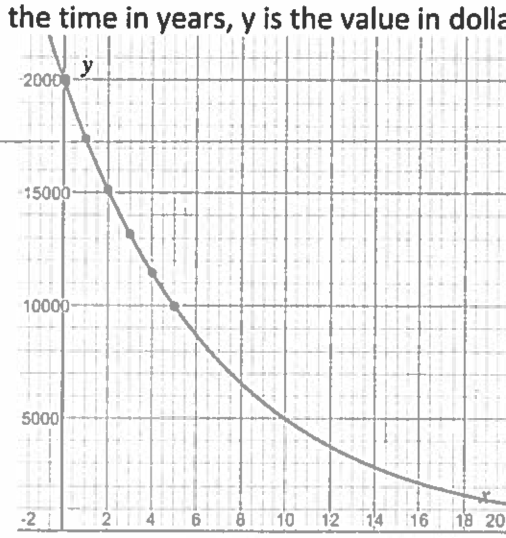
3. The following graph shows the growth of an investment over time. It can be modeled by the equation $y = 1000(1.122)^x$, where x is the time in years, y is the value in dollars.



What is the annual growth for this investment expressed as a percent?

What is the doubling time for this investment?

4. The following graph shows the depreciation of a car over time (value of the car decays over time). It can be modeled by the equation $y = 20000(0.87)^x$, where x is the time in years, y is the value in dollars.



What is the annual decay rate expressed as a percent?

What is the half-life for the value of the car?