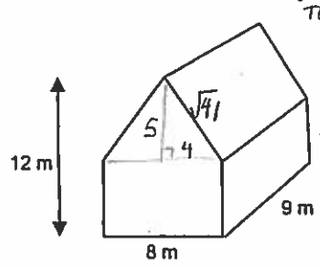
Review: Choosing the correct formula Grade 9

For each question, determine the shape(s) and formula(s) required to answer the question. Solving is not necessary.

1. What is the volume of this barn?



$V_{TOTAL} = V_{\square} + V_{\triangle}$

$= lwh + \frac{bh}{2} \times H$

$= 8(9)(7) + \frac{8(5)}{2}(9)$

$= 684\ m^3$

2. The exterior of the barn in question #1 is to be painted (including the tin roof). If one can of paint covers 37m^2, how many cans need to be purchased to paint the barn?

$A_{TOTAL} = 2A_{base} + A_{4\square's}$

$= 2\left(lw + \frac{bh}{2}\right) + (7 + \sqrt{41} + \sqrt{41} + 7) \times 9$

$= 2\left[8(7) + \frac{8(5)}{2}\right] + 241.256$

$= 152 + 241.256 = 393.256$

Number of cans $= \frac{393.256}{37} = 10.6$

∴ you would need 11 cans of paint.

3. Some hay bales are rectangular prisms. Others are cylindrical. A rectangular bale is 80 cm by 50 cm by 30 cm. A cylindrical bale has base diameter 150 cm and length 120 cm. Calculate the volume of each type of hay bale.

$V_{\square} = lwh$

$= 80(50)(30)$

$= 120\,000\ cm^3$

$V_{\bigcirc} = \pi r^2 h$

$= \pi (75)^2 (120)$

$= 2120\,575\ cm^3$

4. A can of soup has a diameter of **64 mm** and a height of **4 inches**. Determine the amount of aluminum needed to manufacture this can to the nearest **square cm**. Include a diagram.

1 inch = 2.54 cm

4 inches = 10.16 cm

64 mm = 6.4 cm

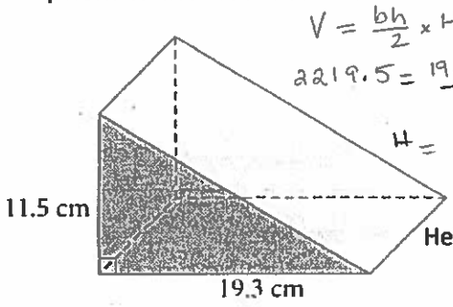
$r = 3.2$ cm

$A_{TOTAL} = 2\pi r^2 + 2\pi rh$

$= 2\pi (3.2)^2 + 2\pi (3.2)(10.16)$

$\doteq 269\ cm^2$

5. The **volume** of this right triangular prism is 2219.5 cm^3. **Determine the height** of the prism to the nearest cm.



$V = \frac{bh}{2} \times H$

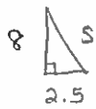
$2219.5 = \frac{19.3(11.5)}{2} \times H$

$H = \frac{2219.5 \times 2}{19.3(11.5)}$

$H \doteq 20\ cm$

6. A conical paper cup at a water dispenser has a diameter of 5 cm and a height of 8 cm. How much paper, to the nearest square centimetre, is used to make the cup?

$r = 2.5$ cm



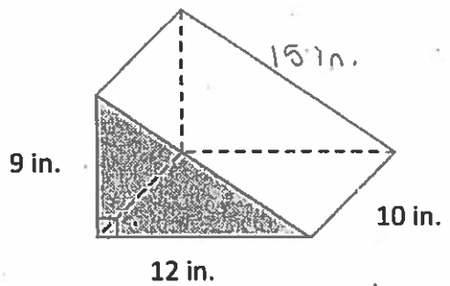
$s = \sqrt{70.25}$

$A_{TOTAL} = \pi rs$

$= \pi (2.5)\sqrt{70.25}$

$= 65.82\ldots \doteq 66\ cm^2$

7. Calculate the **surface area** of this right triangular prism to the **nearest square inch**.

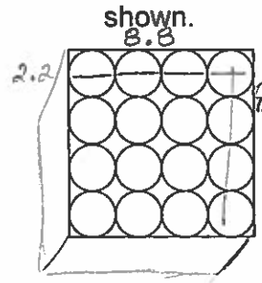


$A_{TOTAL} = A_{2\triangle's} + A_{3\square's}$

$= 12(9) + (12+9+15)(10)$

$= 468\ in^2$

8. A standard bouncy ball has a diameter of 2.2 cm. Determine the surface area of a square-based prism box that could just hold 16 bouncy balls, arranged in four rows of four balls, one row deep as shown.



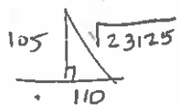
Note: this diagram is looking down on the box... it is a square-based prism

$A_{TOTAL} = A_{4\square's} + A_{2\square's}$

$= 4(2.2)(8.8) + 2(8.8)^2$

$= 232.32\ cm^2$

9. Sneferu's North Pyramid at Dahshur, Egypt is a square based pyramid. The side length of the square base is 220 m and its height is 105 m. Calculate the surface area of **the lateral faces** of this pyramid.



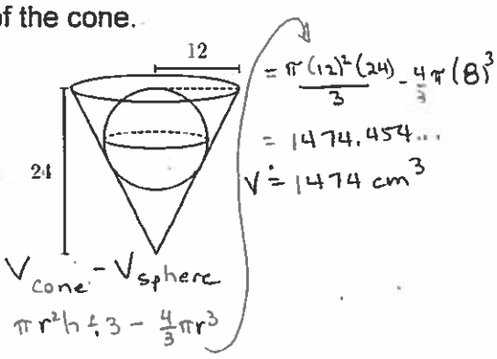
$A_{TOTAL} = A_{4\triangle's}$

$= 2bs$

$= 2(220)(\sqrt{23125})$

$\doteq 66910\ m^2$

10. A sphere is with radius of 8 cm is placed inside of a cone that has a radius of 12 cm and a height of 24 cm. Calculate the volume of empty space inside of the cone.



$V_{space} = V_{cone} - V_{sphere}$

$= \pi r^2 h \div 3 - \frac{4}{3}\pi r^3$

$= \frac{\pi (12)^2 (24)}{3} - \frac{4}{3}\pi (8)^3$

$= 1474.454\ldots$

$V \doteq 1474\ cm^3$

OPTIMIZATION

EX. 1. Cereal is packaged in a square-based prism box. The box contains 5564 cm^3 of cereal.

a) What dimensions for the box require the least amount of cardboard? Round the dimensions to the nearest tenth of a centimetre.

$V = x^3$

$x^3 = 5564$

$x = \sqrt[3]{5564}$

$x \doteq 17.7$ cm

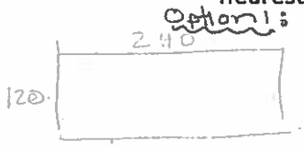
The least amount of cardboard would be needed if a cube was constructed with sides 17.7cm long.

b) Does cereal usually come in a box shaped like the one you found in part a)? Suggest reasons for this.

No.
- hard to hold
- greater surface area looks like more product

EX. 2. Dylan has a piece of plywood that measures 120 cm by 240 cm. He wants to construct a square-based prism box to hold his sports equipment in the garage. Dylan wants to maximize the volume of the box and to keep the waste of plywood to a minimum.

a) Determine the dimensions of the box with maximum volume, including a lid. Round to the nearest tenth of a centimetre if necessary.



Option 1: Assuming we can form a cube by glueing scrap pieces together.

$A_T = 120 \times 240$

$= 28800$

Optimal is cube (sides are 6 congruent squares)

$A_T = A_{6 \text{ squares}}$

$6x^2 = 28800$

$x^2 = 4800$

$x \doteq 69.282...$

69.2 × 69.2 × 69.2 cm Dimensions

SA = 28731.84 cm^2

waste: 68.16 cm^2

b) What is the volume of the box?

Option 1: $V = 69.2^3$

$V \doteq 331374 \text{ cm}^3$

The volume of the box is 331374 cm^3.

Option 2: no waste, not a cube, no glueing scraps together.

Square base 60cm × 60cm, height 90cm.

$SA = 2 \times 60^2 + 4 \times 60 \times 90$

$= 28800 \text{ cm}^2$ (no waste)

$V = 60 \times 60 \times 90 = 324000 \text{ cm}^3$ ← less V than cube with waste.

EX. 3. Canned pineapple is an example of an ideal cylinder. The volume of a can of pineapple is 398 mL.

$V = 398 \text{cm}^3$

a) What are its dimensions in cm?

$V = \pi r^2 (2r)$

$398 = 2\pi r^3$

$r = \sqrt[3]{\frac{398}{2\pi}}$

$r \doteq 3.198627...$

∴ the radius of the can is 4cm, height is 8cm.

b) How much sheet metal is needed to make the can?

$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi (4)^2 + 2\pi (4)(8)$

$= 301.6 \text{ cm}^2$

or $SA = 6\pi r^2$

$= 301.6 \text{ cm}^2$

∴ it will take 301.6 square centimetres of sheet metal.

EX. 4. The All Grow fertilizer company plans to sell a cylindrical bargain jug of concentrated liquid fertilizer.

a) Find the dimensions of the jug that would have a maximum volume if the company plans to construct it out of 1884 cm^2 of plastic.

$SA = 1884 \text{ cm}^2$ $h = 2r$

$h \doteq 20 \text{cm}$

$6\pi r^2 = 1884$

$r = \sqrt{\frac{1884}{6\pi}}$

$r \doteq 10 \text{ cm}$

The jug would be 20cm tall with a radius of 10cm.

b) What volume of fertilizer will the cylindrical jug hold?

$V = \pi r^2 h$

$= \pi (10)^2 (20)$

$= 6283 \text{cm}^3$

$1 \text{cm}^3 = 1 \text{mL}$

∴ the jug would hold 6283 mL.