U9D8_T_Optimization of a Cylinder

Wednesday, May 30, 2018   1:00 PM

U9D8_T_O
ptimizatio...

MPM1DI U9D8 (9.5/9.6)

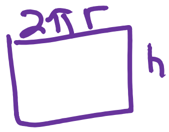
Optimization of a Cylinder

Investigation A: How can you compare the volumes of cylinders with the same surface area?

Many products come in cylinders. Your task is to design a cylindrical juice can that uses no more than 375 cm² of aluminum. The can should have the greatest capacity possible.

1. To investigate the volume of the cylinder as its radius changes, you will need an expression for the height in terms of the radius, given that the surface area is 375 cm².

$A = 2\pi r^2 + \underline{2\pi r h}$



$2\pi r^2 + 2\pi r h = 375$

$2\pi r h = 375 - 2\pi r^2$

$h = (375 - 2\pi r^2) \div (2\pi r)$

2. Complete the table below by calculating the height and volume of each cylinder.

$\pi r^2 h$

d	Radius (cm)	Height (cm)	Volume (cm³)	Surface Area (cm²)
2	1	58.68	184	375
4	2	27.84	350	375
6	3	16.89	478	375
8	4	10.92	549	375
10	5	6.94	545	375
12	6	3.95	447	375
14	7	1.53	236	375

3. **REFLECT: Summarize your Findings**

a) What is the maximum volume for the cans in your table? And what are the radius and height of the can with the volume?

549 cm^3, $r = 4 \text{cm}$, $h \doteq 11 \text{cm}$

b) What relationship do you notice between the radius and height?

The height is closest to twice the radius (closest to $h = d$, $h = 2r$ or $r = \frac{1}{2}h$)

c) Do these dimensions give the optimal volume for the surface area of 375 cm^2? How could you extend your investigation to determine the dimensions of a can with a volume greater than the value in the table? How can you solve for the dimensions algebraically?

We think a cylinder with $h = 2r$ will have a greater volume.

$$2\pi r^2 + 2\pi r h = 375, \quad h = 2r$$

$$2\pi r^2 + 2\pi r(2r) = 375$$

$$2\pi r^2 + 4\pi r^2 = 375$$

$$6\pi r^2 = 375$$

$\div 6\pi$

$$r^2 = 19.89436\ldots$$

$\sqrt{\ }$

$$r \doteq 4.4603 \text{ OR } \cancel{-4.46}$$

$$V = \pi r^2 h, \quad h = 2r$$

$$V = \pi (4.4603)^2 (2 \times 4.4603)$$

$\boxed{V = 557.5 \text{ cm}^3}$ OPTIMAL when $h = 2r$

Pg 508 #1-4 Pg 513 #1, 2, 5, 6

Example 1 Maximize the Volume of a Cylinder

a) Determine the dimensions of the cylinder with maximum volume that can be made with 600 cm^2 of aluminum. Round the dimensions to the nearest hundredth of a centimetre.

$2\pi r^2 + 2\pi r h = 600$, $h = 2r$ (for optimal)

$6\pi r^2 = 600$

$\div 6\pi$

$r^2 = 31.83098\ldots$

$\sqrt{\ }$

$r \doteq 5.6419$

$r \doteq 5.64\,cm$, $h = 11.28\,cm$

b) What is the volume of this cylinder, to the nearest cubic centimetre?

$V = \pi r^2 h$

$V = \pi (5.64)^2 (11.28)$

$\boxed{V \doteq 1127\,cm^3}$ ($1128\,cm^3$ is more accurate)

Pg 508 #1-4 Pg 513 #1, 2, 5, 6

Investigation B: How can you compare the surface areas of cylinders with the same volume?

Your task is to design a cylindrical juice can that must hold 1000mL of juice.

1. To investigate the surface area of the cylinder as its radius changes, you will need an expression for the height in terms of the radius, given that the volume is 1000mL. $1 cm^3 = 1 mL$

$$\pi r^2 h = 1000$$

$$h = \frac{1000}{\pi r^2}$$

$$h = 1000 \div (\pi \times r^2)$$

2. Complete the table.

	Radius (cm)	Height (cm)	Volume ($1cm^3$=1mL)	Surface Area (cm^2) $2\pi r^2 + 2\pi rh$
	1	318	1000	2004
	2	79.6	1000	1025
	3	35.4	1000	724
	4	19.9	1000	601
d=10	5	12.7	1000	556
d=12	6	8.8	1000	558
	7	6.5	1000	594

Pg 508 #1-4 Pg 513 #1, 2, 5, 6

REFLECT: Summarize your Findings

a)Describe the dimensions of the cylinder with the least surface area. Are these dimensions the optimal ones?

The least surface area occurs when the height is closest to twice the radius. This is not the most optimal since $h \neq 2r$.

b) Given a specific volume, how could you determine the optimal dimensions of a cylinder algebraically?

$V = \pi r^2 h$, $h = 2r$ for optimal.

$\pi r^2 h = 1000$ (given $V = 1000$ mL)

$\pi r^2 (2r) = 1000$

$2\pi r^3 = 1000$

$r^3 = \frac{1000}{2\pi}$

$r = \sqrt[3]{\frac{1000}{2\pi}}$

$r \doteq 5.41926$ cm, $h = 10.83852$ cm

Pg 508 #1-4 Pg 513 #1, 2, 5, 6

Example 2 Minimize the Surface Area of a Cylinder

a) Determine the least amount of aluminum required to construct a cylindrical container with a 4L capacity, to the $\boxed{\text{nearest tenth}}$ of a square centimetre.

$\hookrightarrow 4000\ cm^3$

$V = \pi r^2 h$, $h = 2r$, $V = 4000$, solve for r.

$2\pi r^3 = 4000$

$r^3 = \frac{4000}{2\pi}$

$r = \sqrt[3]{\frac{2000}{\pi}}$

$r \doteq 8.6$

for optimal

$SA = 6\pi r^2$

$= 6\pi(8.6)^2$

$= \boxed{1394\ cm^2}$

$\therefore$ $1394\ cm^2$ is the least amount of aluminum required. (more accurate: $1395\ cm^2$)

b) Describe any assumptions you made.

no extra for seams or waste.

Pg 508 #1-4 Pg 513 #1, 2, 5, 6