U9D1_T_ Volume of 3_D Shapes

Tuesday, May 22, 2018 7:13 PM

U9D1_T_
Volume o...

MPM 1DI U9D1

Warm Up: What is Volume? The amount of space a 3-D object takes up.

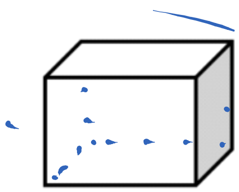
Volume of 3-D Shapes

Polyhedron: A three-dimensional object with faces that are polygons.

Prism:

A prism is a three-dimensional solid (a polyhedron). The top and bottom (the bases) are parallel, identical polygons. The lateral faces are rectangles; they meet the bases at right angles. A prism ~~are~~ is named by the shape of its bases, for example, rectangular prism, triangular prism, square-based prism.

Volume of any Prism: $V = A_{base} \times \text{height}$



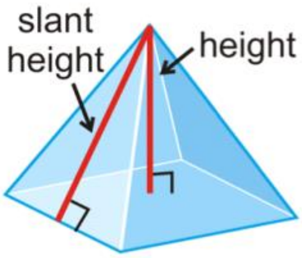
Pyramid:

A pyramid is a three-dimensional solid (a polyhedron) with a polygon-shaped base. The remaining sides are triangles that come to a point at the top.

https://www.youtube.com/watch?v=qXC8uzy_HFw

Volume of any Pyramid:

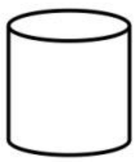
$V = \frac{1}{3}(A_{base} \times \text{height})$ **or** $V = A_{base} \times \text{height} \div 3$

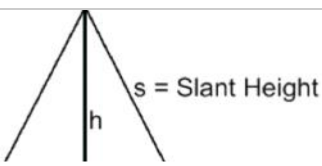


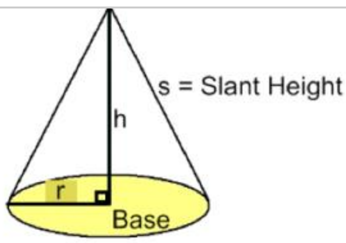
A ***cylinder*** is a three-dimensional solid with identical parallel circular bases. The lateral surface is curved and extends from one base to the other base.

Volume of a Cylinder is the same as a prism:

$V = A_{base} \times \text{height}$ **or** $V = \pi r^2 h$

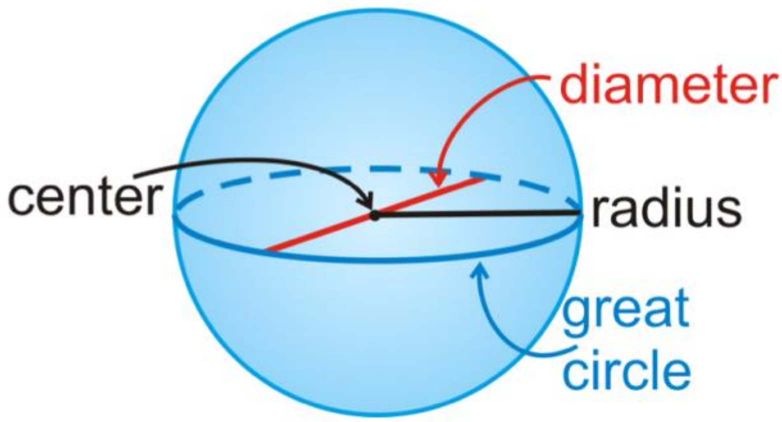






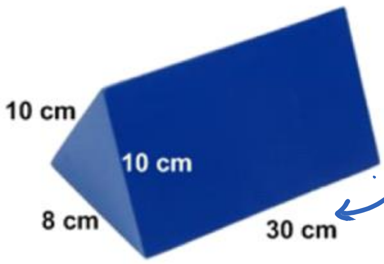
Volume of a Cone = $\frac{1}{3}$ A$_{base}$ × height or $V = \frac{1}{3}\pi r^2 h$

A *sphere* is a round ball-shaped three dimensional solid. Every point on the surface of the sphere is the same distance from the centre of the sphere.



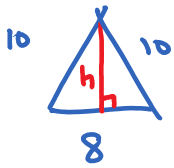
Volume of a Sphere: $V = \frac{4}{3}\pi r^3$ or $V = 4\pi r^3 \div 3$

Example 1: Calculate the volume of the following triangular-based prism.



length 30 cm is actually the height of the prism.

$$V = A_{base} \times H$$
$$= \frac{bh}{2} \times H$$
$$\doteq \frac{8(9.165)}{2} \times 30$$
$$= \boxed{1099.8\ cm^3}$$



h 10 4

$h^2 = 10^2 - 4^2$

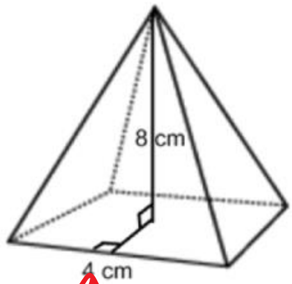
$h^2 = 84$

$h = \pm\sqrt{84}$

$h \doteq 9.165$

OR ~~$h \doteq -9.165$~~

Example 2: Calculate the volume of the following square-based pyramid.

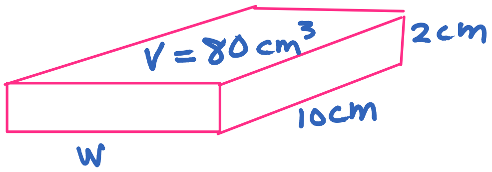


base is a square

$A_{base} = 4 \times 4$

$$V = \frac{(A_{base})(Height)}{3}$$
$$V = \frac{b^2h}{3}$$
$$V = \frac{4^2(8)}{3}$$
$$\boxed{V \doteq 42.7\ cm^3}$$

Example 3: A box of chocolates has a volume of 80 cm^3. If its length is 10 cm and its height is 2 cm, what is its width?



$V = lwh$

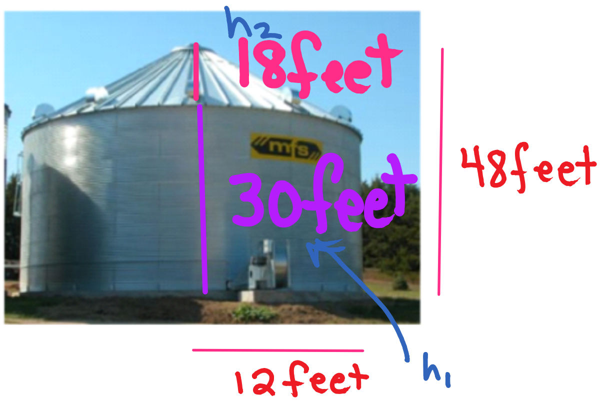
$80 = 10(w)(2)$

$80 = 20w$

$w = 4$

$\therefore$ the box is 4cm wide.

Example 4: A grain bin has a radius of 12 ft and a height of 48 ft. How much grain will the farmer need to order to fill the bin? (Note: 1 kg of grain fills 1 ft^3 of space. Also, assume grain (oats) is ordered in tonnes (1 metric ton = 1000kg).) (Note: the cone portion has a height of 18 feet)



$V = V_{\text{cylinder}} + V_{\text{cone}}$

$V = \pi r^2 h_1 + \pi r^2 h_2 \div 3$

$V = \pi(12)^2(30) + \dfrac{\pi(12)^2(18)}{3}$

$V = 4320\pi + 864\pi$

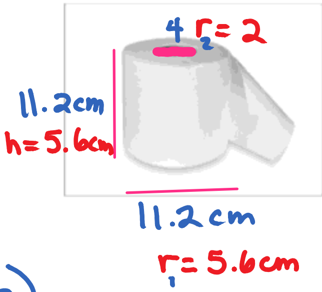
$V \doteq 13571.68 + 2714.33$

$V \doteq 16\,286.0\ ft^3$

need 16286 kg.

$\therefore$ we would order 16 tons (or 16.2 tons).

Example 5: A roll of toilet paper has a height and diameter of 11.2cm. If the inner cardboard roll is 4cm in diameter, what is the volume of toilet paper on the roll?



$V = V_{\text{cyl}} - V_{\text{cyl}}$

$V = \pi r_1^2 h - \pi r_2^2 h$

$= \pi(5.6)^2(11.2) - \pi(2)^2(11.2)$

$\doteq 962.684\ldots$

$\doteq 962.7$

$\therefore$ there is $962.7 cm^3$ of paper.

Example 6: The radius of a sphere is tripled. How does this affect the volume of the sphere? Explain. $V_{sphere} = \frac{4}{3}\pi r_1^3$

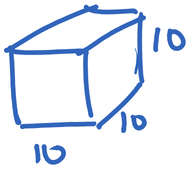
$\frac{4}{3}\pi(3r_1)^3$

$= \frac{4}{3}\pi \underbrace{(3)^3}_{3^3 = 27}(r_1)^3$

So, tripling the radius makes the volume 27 times bigger.

Example 7: A spherical gemstone just fits inside a plastic cube with edges 10 cm.

a) Calculate the volume of the gemstone, to the nearest cubic centimetre.



diameter of sphere 10cm

$r = 5$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(5)^3$ 523.598...

$V \doteq 524$

$\therefore$ the volume of the gem is $524 cm^3$.

b) How much empty space is there?

$V_{space} = V_{cube} - V_{sphere}$

$= 10 \times 10 \times 10 - 524$

$= 476$ $\therefore$ the empty space is $476 cm^3$.